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DEVELOPMENT OF AMPLIFIED SEISMIC RESPONSE SPECTRA FOR STRUCTURAL ANALYSIS OF DECOUPLED SMALL BORE PIPES

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Abstract. *In various applications, not at least within the nuclear industry, piping systems are required to be structurally verified for seismic loads. Piping systems often have connected small bore pipes which also have requirements for seismic qualification. If the ratio of run-to-branch pipe moment of inertia is large, the small bore pipes can often be decoupled from the main pipes when performing the structural analysis. There are many reasons why decoupling is preferred in comparison to performing a coupled analysis. Firstly, from a project perspective, it is desirable to finish the design and verification of the main pipes before the work on the small bore pipes is started. Secondly, with decoupled pipes the analysis models become smaller, meaning reduced analysis time and reduced risk of encountering convergence difficulties for dynamic loads. Finally, decoupling the small bore pipes makes it possible to distinguish between primary and secondary loads, which will take away unnecessary conservatism in the evaluation of the small bore pipes. Though there are advantages of decoupling, the analyst will be faced with the question of how to consider the impact of the main pipe oscillations in the analysis of the small bore pipe. Seismic loads are generally applied to pipes as response spectrum loads derived in a seismic analysis of the supporting building structure. A clear disadvantage of a response spectrum analysis is that it only calculates bounds for forces, moments, displacements and accelerations and that it does not result in any frequency dependent quantities. There is no established industrial practice how the impact on the small bore pipe from the main pipe should be considered at the decoupling point and in many cases simplifications that give non-conservative results are made. This study presents a method to create an amplified response spectrum at the small bore piping connection point making use of modal analysis results and response spectrum input data. The method is validated by comparison with results from coupled analyses. The benefit of the proposed method is also demonstrated by applying it in evaluation of a small bore pipe.*

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1 INTRODUCTION

In various applications, not at least within the nuclear industry, piping systems are required to be structurally verified for seismic loads. Generally, the seismic loads are applied to pipes as response spectrum loads derived in a seismic analysis of the supporting building structure. The structural behavior of the piping is then obtained by performing a response spectrum analysis in which the modal responses of the piping system are combined.

Piping systems often have connected small bore pipes which also have requirements for seismic qualification. If the ratio of run-to-branch pipe moment of inertia is large, the small bore pipes can often be decoupled from the main pipes when performing the structural analysis.

There are many reasons why decoupling is preferred in comparison to performing a coupled analysis. Firstly, from a project perspective, it is desirable to finish the design and verification of the main pipes before the work on the small bore pipes is started. Secondly, with decoupled pipes the analysis models become smaller, meaning reduced analysis time and reduced risk of encountering convergence difficulties for dynamic loads. If the small bore pipe is very small in comparison to the main pipe, the modal analysis could also result in twin modes and an unrealistically high response can be obtained if an unsigned modal combination method is used, a phenomenon addressed in [1]. Finally, decoupling the small bore pipes makes it possible to distinguish between primary and secondary loads, which will take away unnecessary conservatism in the evaluation of the small bore pipes.

Though there are advantages of decoupling, the analyst will be faced with the question of how to consider the impact of the main pipe oscillations in the analysis of the small bore pipe. A clear disadvantage of a response spectrum analysis is that it only calculates bounds for forces, moments, displacements and accelerations and that it does not result in any frequency dependent quantities.

While different decoupling criteria have been developed for systems and subsystems in general and main and small bore pipes in particular and considerable attention is given to their accuracy, the main focus is always on the main system or pipe. No guidance how the impact from the main system on the subsystem or small bore pipe should be considered is given. The lack of guidance for the analysis of subsystems is also pointed out in [2], in which an overlap approach is proposed as an alternative to decoupling. Antaki [3], also points out the lack of uniform industrial practice and that the practice is to model the connection point as an anchor and apply anchor motions and an envelope of the spectra for the main pipe supports at this point.

It is obvious that the latter method is inexact and in most cases non-conservative. The accuracy of the method becomes lower, the more flexible the main pipe is. In the overlapping method, the main pipe is included up to a point at which each of the three translational directions has been blocked at least twice. Though the validity of this method is justified in [2], it has other drawbacks. It is not a pure analysis decoupling since there are still needs for modelling of the main pipe. In some cases a rather large part of the main pipe needs to be modelled in order to fulfil the blocking criteria. Furthermore, with the overlapping method, it is not possible to distinguish between primary and secondary loads in the small bore pipe.

The purpose of this study is to present a method to create amplified response spectra to be used as input for analyzing decoupled small bore pipes. The method has been developed as a research project involving an M.Sc. thesis project resulting in [4].

2 DESCRIPTION OF THE METHOD

2.1 Construction of a response spectrum

A response spectrum is derived by looking at the response of a single degree-of-freedom (SDOF) oscillator (illustrated in Figure 1) subjected to an input motion. There are many books and articles describing the derivation of response spectra. Chopra [5], for example, presents the following stepwise procedure for deriving a response spectrum for a given input motion $\ddot{u}_g(t)$:

1. Numerically define $\ddot{u}_g(t)$
2. Select natural frequency ω_n and damping ζ of a SDOF oscillator
3. Compute the deformation response $u(t)$ of this SDOF system due to the input motion $\ddot{u}_g(t)$
4. Determine u_0, the peak value of $u(t)$
5. Calculate the spectral ordinates $D = u_0$, $V = \omega_n D$ and $A = \omega_n{}^2 D$
6. Repeat steps 2 to 5 for a range of ω_n and ζ values covering all possible systems of engineering interest
7. Plot *D*, *V* or *A* vs ω_n

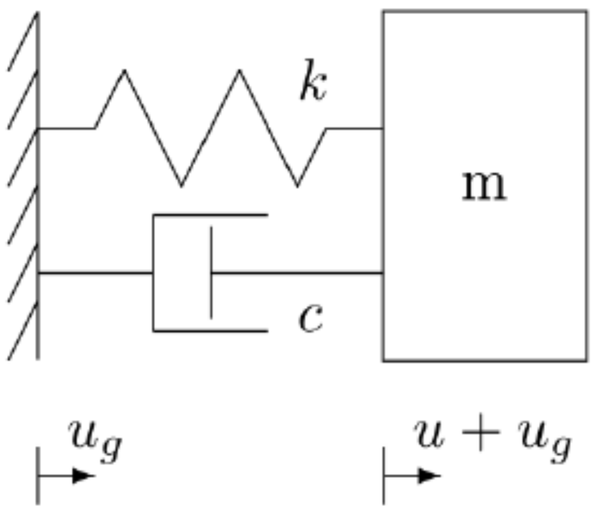

Figure 1: Single degree-of-freedom oscillator.

2.2 Computation of the deformation response

For the connection point of a small bore pipe, the pipe motions would be the input for generating response spectra. Since the main pipe seismic loads are given as response spectra, no time history is available. However, the mode shapes of the main pipe eigenmodes are known and each eigenmode can be considered as an oscillation of the form

$$u_g(t) = u_{g0}\sin(\omega t) \tag{1}$$

For an oscillator with natural frequency ω_n, the dynamic amplification based on the relative motion of the mass can be expressed as

$$\frac{u_0}{u_{g0}} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \tag{2}$$

The dynamic amplification calculated by this expression is shown in Figure 2.

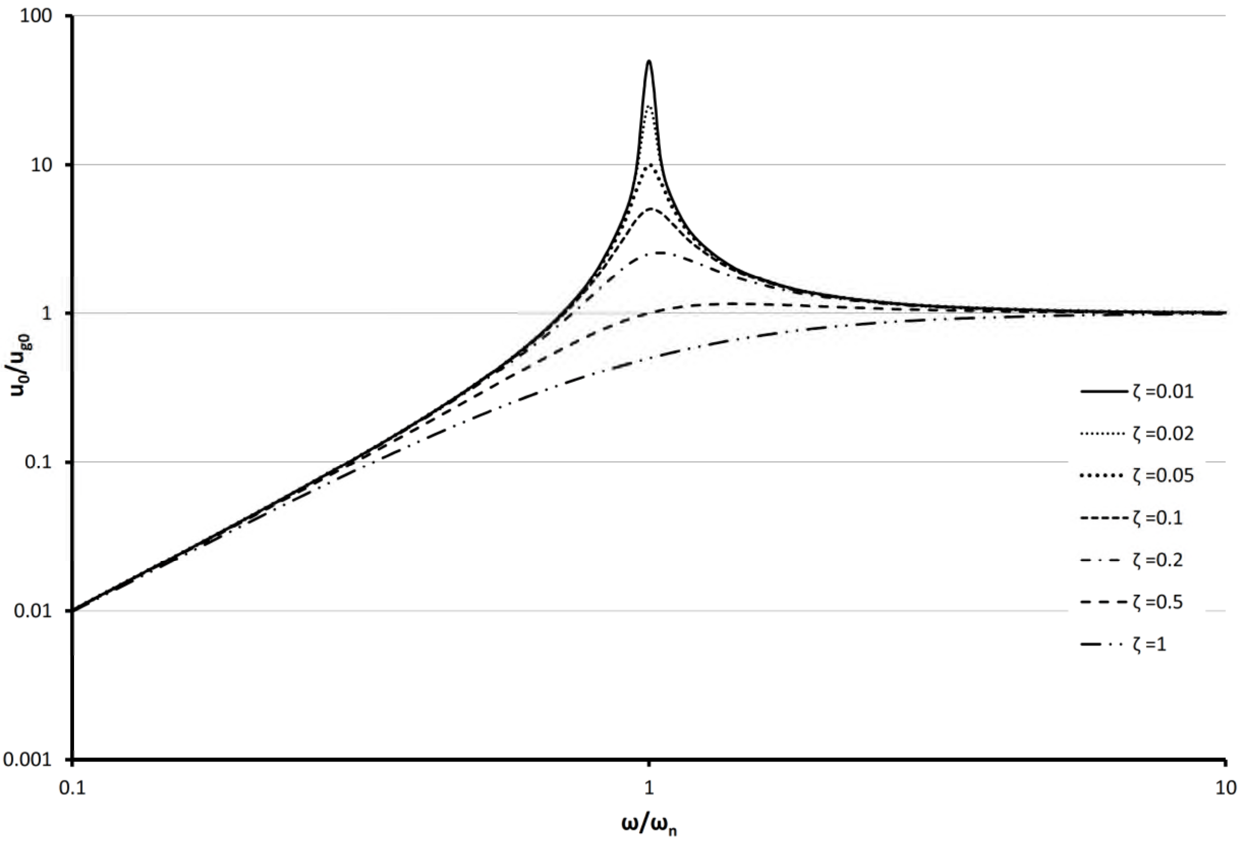

Figure 2: Dynamic amplification based on relative motion.

Hence, for each pair of main pipe natural frequency ω_i and subsystem frequency ω_n and damping ratio ζ a dynamic amplification factor $DF(\omega_n, \omega_i, \zeta)$ can be computed as

$$DF(\omega_n, \omega_i, \zeta) = \frac{\left(\frac{\omega_i}{\omega_n}\right)^2}{\sqrt{\left(1-\left(\frac{\omega_i}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega_i}{\omega_n}\right)\right)^2}} \tag{3}$$

The peak value u_0 of the deformation of an oscillator with frequency ω_n due to a main pipe harmonic oscillation with amplitude u_{g0} may be expressed as

$$u_0 = DF \cdot u_{g0} \tag{4}$$

For a system with all pipe mode shapes in the same coordinate direction subjected to a uniform response spectrum, the maximum displacement of the main pipe in the *i*th mode is obtained by

$$\{\mathrm{u}\}_i = \{\Phi\}_i \,\Gamma_i \frac{1}{\omega_i^2} A_i \tag{5}$$

where $\{\Phi\}_i$ is the mode shape of mode i, Γ_i is the participation factor of mode *i* and A_i is the spectral acceleration corresponding to mode *i* obtained from the response spectrum exciting the main pipe. At the decoupling point, where the input motion for generation of a response spectrum is denoted u_g, the maximum displacement the *i*th mode thus becomes

$$u_{g0,i} = u_{\Phi,i} \Gamma_i \frac{1}{\omega_i^2} A_i \tag{6}$$

where $u_{\Phi,i}$ is the mode shape coordinate of mode *i* at the decoupling point. The maximum deformation response for any oscillator eigenmode *n* with natural frequency ω_n can thus be calculated as

$$u_{0,i} = DF(\omega_n, \omega_i, \zeta) u_{\Phi,i} \Gamma_i \frac{1}{\omega_i^2} A_i \tag{7}$$

In this way the maximum response for any pair of main pipe frequency ω_i and oscillator frequency ω_n can be calculated. Normally, several modes contribute to the main pipe response. To obtain the maximum response for a subsystem frequency ω_n considering a main pipe response to which several modes contribute, a combination of the oscillator responses $u_{0,i}$ due to all contributing main pipe modes *i* has to be made. Several methods for combining modes exist, some of which are described in paragraph 2.5. Normally the combination methods are expressed in terms of a general modal quantity R_i which can be displacement, force, acceleration etc. for mode *i*. Let the maximum combined modal response regardless of combination method be

$$R_{\mathrm{mod}} = \sum_{i=1}^{1} R_{i,\max} \tag{8}$$

where the summation in this case does not mean algebraic summation but summation according any method. If Eq. (8), which is written in terms of the general modal quantity is rewritten as the deformation response for the oscillator as given by Eq. (7), the combined response for a subsystem of frequency ω_n can be expressed as

$$u_0 = \sum_{i=1}^{I} u_{0,i} = \sum_{i=1}^{I} DF(\omega_n, \omega_i, \zeta) u_{\Phi,i} \Gamma_i \frac{1}{\omega_i^2} A_i \tag{9}$$

The deformation response thus calculated corresponds to step 4 in the procedure for constructing a response spectrum described in paragraph 2.1. Table 1 shows in an illustrative way how the oscillator deformation response is constructed for a number of oscillator frequencies for a chosen damping ratio ζ. For each oscillator frequency the spectral ordinates D, V and A can be calculated according to step 5 in paragraph 2.1 and plotted vs frequency to obtain a plot of the response spectrum. The procedure can be repeated for any damping ratio ζ.

Oscillator frequency	**Deformation response of oscillator**				
	$u_0(\omega_{i=1})$	$u_0(\omega_{i=2})$	$u_0(\omega_{i=\dots})$	$u_0(\omega_{i=I})$	u_0
$\omega_{n=1}$	$DF(\omega_{n=1},\omega_{i=1}) \cdot u_{\Phi,1}\Gamma_1 \frac{A_1}{\omega_{i=1}^2}$	$DF(\omega_{n=1},\omega_{i=2}) \cdot u_{\Phi,2}\Gamma_2 \frac{A_2}{\omega_{i=2}^2}$	$\dots$	$DF(\omega_{n=1},\omega_{i=I}) \cdot u_{\Phi,I}\Gamma_I \frac{A_I}{\omega_{i=I}^2}$	$\sum_{i=1}^{I} DF(\omega_{n=1},\omega_i) \cdot u_{\Phi,i}\Gamma_i \frac{A_i}{\omega_i^2}$
$\omega_{n=2}$	$DF(\omega_{n=2},\omega_{i=1}) \cdot u_{\Phi,1}\Gamma_1 \frac{A_1}{\omega_{i=1}^2}$	$DF(\omega_{n=2},\omega_{i=2}) \cdot u_{\Phi,2}\Gamma_2 \frac{A_2}{\omega_{i=2}^2}$	$\dots$	$DF(\omega_{n=2},\omega_{i=I}) \cdot u_{\Phi,I}\Gamma_I \frac{A_I}{\omega_{i=I}^2}$	$\sum_{i=1}^{I} DF(\omega_{n=2},\omega_i) \cdot u_{\Phi,i}\Gamma_i \frac{A_i}{\omega_i^2}$
$\omega_{n=\dots}$			$\dots$		
$\omega_{n=N}$	$DF(\omega_{n=N},\omega_{i=1}) \cdot u_{\Phi,1}\Gamma_1 \frac{A_1}{\omega_{i=1}^2}$	$DF(\omega_{n=N},\omega_{i=2}) \cdot u_{\Phi,2}\Gamma_2 \frac{A_2}{\omega_{i=2}^2}$	$\dots$	$DF(\omega_{n=N},\omega_{i=I}) \cdot u_{\Phi,I}\Gamma_I \frac{A_I}{\omega_{i=I}^2}$	$\sum_{i=1}^{I} DF(\omega_{n=N},\omega_i) \cdot u_{\Phi,i}\Gamma_i \frac{A_i}{\omega_i^2}$

Table 1: Tabular construction of oscillator response

2.3 Excitation and response in three directions

For a general case with mode shapes in the three dimensional space and spectrum excitation in all three directions an oscillator in each coordinate direction can be simulated by extending Eq. (4) to

$$\{\mathrm{u}_0\} = \begin{Bmatrix} x_0 \\ y_0 \\ z_0 \end{Bmatrix} = DF \cdot \{\mathrm{u}_{g0}\} = DF \cdot \begin{Bmatrix} x_{g0} \\ y_{g0} \\ z_{g0} \end{Bmatrix} \tag{10}$$

The corresponding maximum modal deformation response for each mode then becomes

$$\{\mathrm{u}_0\}_{i,d} = DF(\omega_n,\omega_i,\zeta)\{\mathrm{u}_\Phi\}_i \,\Gamma_{i,d} \frac{1}{\omega_i^2} A_{i,d} \tag{11}$$

where d denotes the direction of spectrum excitation (x, y or z), $\{\mathrm{u}_0\}_{i,d} = \{x_{0,d} \quad y_{0,d} \quad z_{0,d}\}_i^T$ is the deformation response for mode i and load application direction d, $\{\mathrm{u}_\Phi\}_i = \{x_\Phi \quad y_\Phi \quad z_\Phi\}_i^T$ is the mode shape coordinates at the decoupling point, $\Gamma_{i,d}$ is the participation factor of mode i under loadings in direction d and $A_{i,d}$ is the spectral acceleration for mode i in direction d. Combining the modal the response according to Eq. (8) gives

$$\left\{ \mathrm{u}_0 \right\}_d = \sum_{i=1}^{I} \left\{ \mathrm{u}_0 \right\}_{i,d} = \sum_{i=1}^{I} DF(\omega_n, \omega_i, \zeta) \left\{ \mathrm{u}_\Phi \right\}_i \Gamma_{i,d} \frac{1}{\omega_i^2} A_{i,d} \tag{12}$$

The contribution from the three directions of load application can then be combined by square root of sum of square (SRSS) combination:

$$\left\{ \mathrm{u}_0 \right\} = \begin{Bmatrix} x_0 \\ y_0 \\ z_0 \end{Bmatrix} = \begin{Bmatrix} \sqrt{x_{0,x}^2 + x_{0,y}^2 + x_{0,z}^2} \\ \sqrt{y_{0,x}^2 + y_{0,y}^2 + y_{0,z}^2} \\ \sqrt{z_{0,x}^2 + z_{0,y}^2 + z_{0,z}^2} \end{Bmatrix} \tag{13}$$

The above expression gives the maximum deformation response for each direction. By varying the oscillator frequency the spectral ordinates D, V and A can be computed for each direction and plotted vs frequency.

2.4 Multi-level excitation

So far the method presented is based on a uniform spectrum application or single point spectrum analysis. The same method can be applied to a multi-point spectrum analysis. The maximum modal deformation response for each mode i, direction d and level l can be obtained as

$$\left\{ \mathrm{u}_0 \right\}_{i,d,l} = DF(\omega_n, \omega_i, \zeta) \left\{ \mathrm{u}_\Phi \right\}_i \Gamma_{i,d,l} \frac{1}{\omega_i^2} A_{i,d,l} \tag{14}$$

where $\Gamma_{i,d,l}$ is the participation factor for spectrum excitation of i under loadings in direction d at level l. $A_{i,d,l}$ is the spectral acceleration for mode i in direction d at level l. Depending whether the support points (levels) are considered to move in phase, stochastically independent, or counter phase, the combination of different levels is made by algebraic, square root of summed squares (SRSS) or absolute summation. For L levels of independent support excitations, the combination of level responses becomes

$$\left\{ \mathrm{u}_0 \right\}_{i,d} = \sum_{l=1}^{L} \left\{ \mathrm{u}_0 \right\}_{i,d,l} \tag{15}$$

where the summation in this case does not mean algebraic summation but summation according any method. The results from Eq. (15) can then be inserted in Eq. (12). Using these results in Eq. (13) finally gives the total response and the procedure to create response spectra can be continued.

2.5 Modal combination method

To obtain the maximum response for a subsystem frequency ω_n considering a main pipe response to which several modes contribute, a combination of the subsystem responses $u_{g0,i}$ due to all contributing main pipe modes i has to be made. Since all information on time and sign relations between modes are lost in a response spectrum analysis, combinations are usually made by statistical methods. Several methods exist. A few common methods are described below expressed in terms of a general modal quantity R_i which can be displacement, force, acceleration etc. for mode i.

1. **Absolute summation:** All modes are assumed to have their maxima at the same time which will be an upper bound for the response. The combined modal response can be expressed as

$$R_{\mathrm{mod}} = \sum_{i=1}^{I} \left| R_{i,\max} \right| \tag{16}$$

where I is the highest mode considered to contribute to the response.

2. **Square root of sum of squares (SRSS):** The modes are considered to be statistically independent, i.e. no correlation between the individual modal responses:

$$R_{\mathrm{mod}} = \sqrt{\sum_{i=1}^{I} R_{i,\max}^2} \tag{17}$$

3. **NRC ten percent method:** In this method, defined in [6], modes k with frequencies separated by more than 10% of each other are combined with SRSS as above. However, modes whose frequencies are spaced within 10% of each other are combined by absolute summation. This can be expressed as

$$R_{\mathrm{mod}} = \sqrt{\sum_{k=1}^{I} R_{k,\max}^2 + 2\sum \left| R_{i,\max} R_{j,\max} \right|} \quad i \neq j \tag{18}$$

where $1 \le i \le j \le I$. The first summation is done for modes with separated frequencies and the second summation is done for modes with closely spaced frequencies. Closely spaced modes i and j are defined as modes with frequencies within 10% of each other according to

$$\frac{\omega_j - \omega_i}{\omega_i} \le 0.1 \tag{19}$$

4. **Complete quadratic combination (CQC):** A modal correlation coefficient between each pair of modes is considered in the summation of responses according to:

$$R_{\mathrm{mod}} = \sqrt{\left(\sum_{i=1}^{I} \sum_{j=1}^{I} R_{i,\max} \rho_{ij} R_{i,\max} \right)} \tag{20}$$

where i and j are any two modes up to the highest contributing mode I and $i \neq j$. ρ_{ij} is the correlation coefficient. A few different methods for calculating the correlation coefficient exist of which the methods according to Rosenblueth and Der Kiureghian are the most frequently used. In [7], Rosenblueth defines the following equations for the correlation coefficient:

$$\rho_{ij} = \frac{1}{1 + \epsilon_{ij}^2} \tag{21}$$

where

$$\epsilon_{ij} = \frac{\omega_i\sqrt{1-\zeta_i^2} - \omega_j\sqrt{1-\zeta_j^2}}{\zeta_i'\omega_i + \zeta_j'\omega_j} \qquad \zeta_i' = \zeta_i + \frac{2}{\omega_i s} \tag{22}$$

and s is the duration of the strong phase of the earthquake excitation. ζ_i and ζ_j are the modal damping ratios for mode i and j.
The correlation coefficient according to Der Kiureghian [8] is

$$\rho_{ij} = \frac{8\left(\zeta_i\zeta_j\omega_i\omega_j\right)^{1/2}\left(\zeta_i\omega_i + \zeta_j\omega_j\right)\omega_i\omega_j}{\left(\omega_i^2 - \omega_j^2\right)^2 + 4\zeta_i\zeta_j\omega_i\omega_j\left(\omega_i^2 + \omega_j^2\right) + 4\left(\zeta_i^2 + \zeta_j^2\right)\omega_i^2\omega_j^2} \tag{23}$$

The choice of method may be governed by design codes or it can be based on engineering judgement for the application in consideration. However, for obtaining the total subsystem response it would be reasonable to use the same method as used to obtain the main pipe response.

2.6 Order of combination

In paragraph 2.4, it was shown how the total response could be obtained by combining the level response followed by modal combination and finally combining the directional responses. If the modal combination is performed with the CQC method for which the sign of the modal responses is required, this order of combination is only possible if the level responses are combined algebraically. For other combination methods, the modal combination needs to be performed first according to

$$\{\mathrm{u}_0\}_{d,l} = \sum_{i=1}^{I}\{\mathrm{u}_0\}_{i,d,l} \tag{24}$$

Then the levels can be combined:

$$\{\mathrm{u}_0\}_{d} = \sum_{l=1}^{L}\{\mathrm{u}_0\}_{d,l} \tag{25}$$

Finally the results of this summation can be inserted in in Eq. (13) for combination of the loading directions.

2.7 Residual rigid response

It is not practical to calculate all mode shapes and frequencies for piping systems. For high frequencies, the modes behave rigid and are not excited by support motions. The frequency above which modes can be considered rigid is normally referred to as Zero Period Acceleration (ZPA) frequency and the corresponding acceleration is called ZPA. Hence, this frequency is normally used as cut-off frequency when performing modal extraction and modal combination. However, the mass associated with the modes above the cut-off frequency is then missing in the calculation of the response. To account for this, the residual rigid response of the missing mass can be obtained in a static analysis in which the missing mass is multiplied by the spectrum ZPA. This residual rigid response or missing mass correction can then be added to the solution from the combination of modal responses performed according to the methods described in paragraph 2.5 to obtain the complete system response.

Since amplified response spectra are created for frequencies up to the ZPA, the missing mass term will not affect the results. However, it will be included in the analysis examples in Section 3 and 4.

3 VERIFICATION

To verify the method, results from a decoupled analysis and a coupled analysis are compared. The model of the piping system is shown in Figure 3. The model consists of a main pipe with an outer diameter of 275 mm and a thickness of 20 mm. It has a small bore pipe with an outer diameter of 48.26 mm and a thickness of 5.08 mm attached to it.

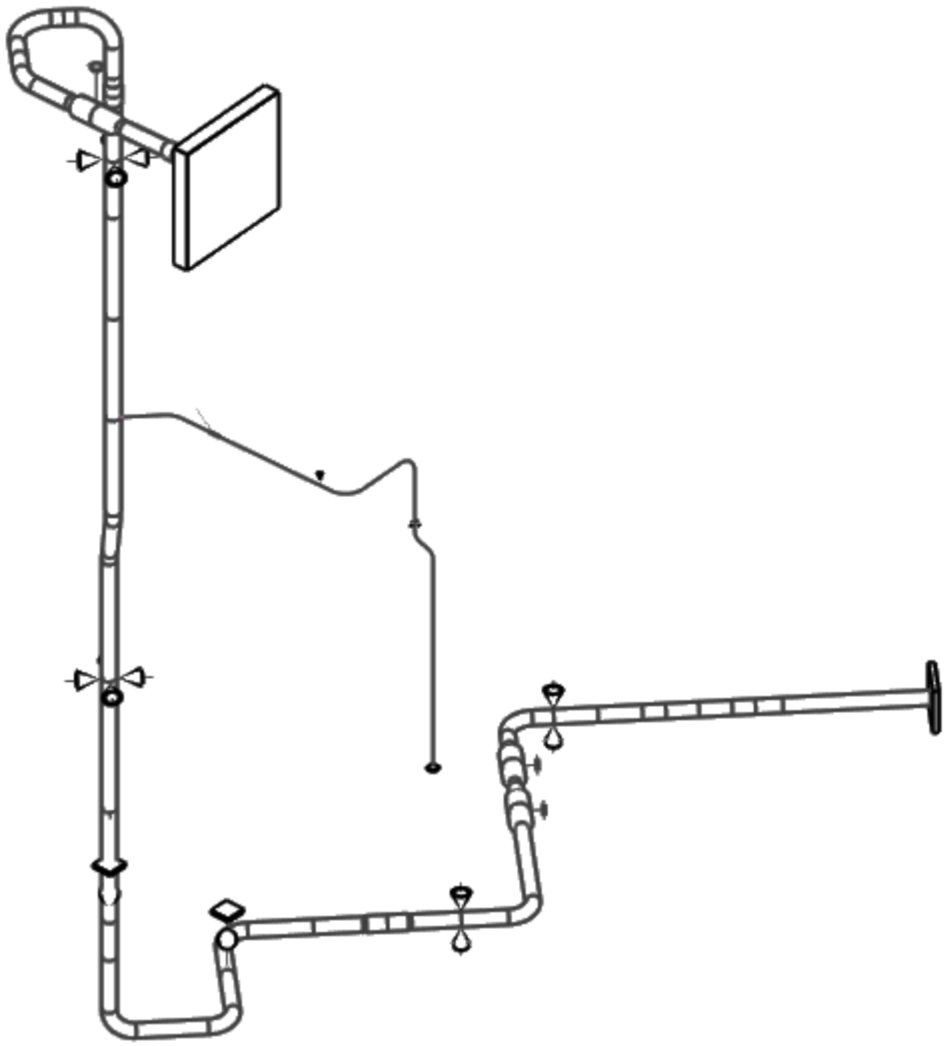

Figure 3: Model of coupled piping system.

3.1 Loads and analysis procedure

The system is supported at several points by restraints, spring hangers and fix supports at its terminal ends and subjected to four different levels of earthquake excitation making it a multi-level spectrum analysis. The applied response spectra are shown in Figure 4 and Figure 5. These are spectra for 5% damping, which is considered in the analysis. Levels LV=2 and LV=5 act on the main pipe supports whereas levels LV=3 and LV=4 act on the small bore pipe supports. The analysis is performed with a cut-off frequency of 60 Hz. The modes are combined by the CQC method with correlation coefficients according to Der Kiureghian. The contributions of the different levels are combined by absolute summation and the spectral directions by the SRSS method. Finally, the residual response is added although in this case, it is very small.

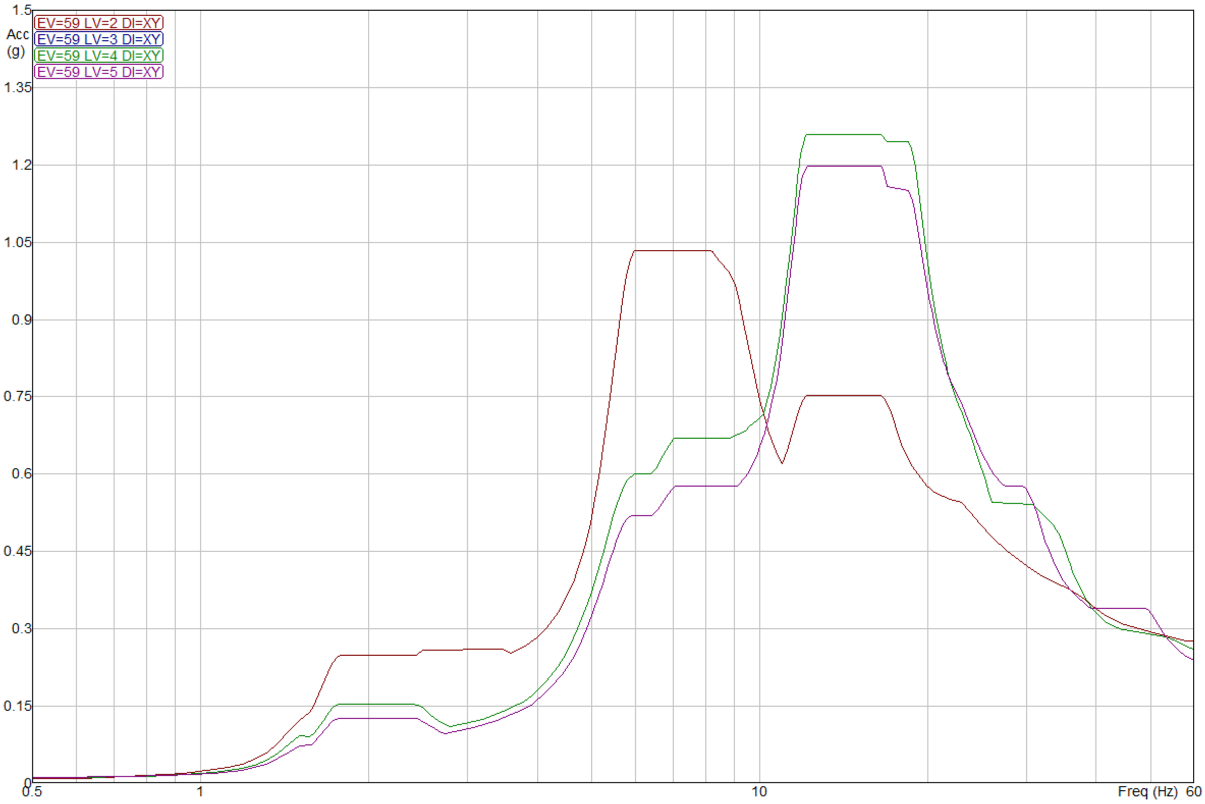

Figure 4: Applied response spectra in x and y directions.

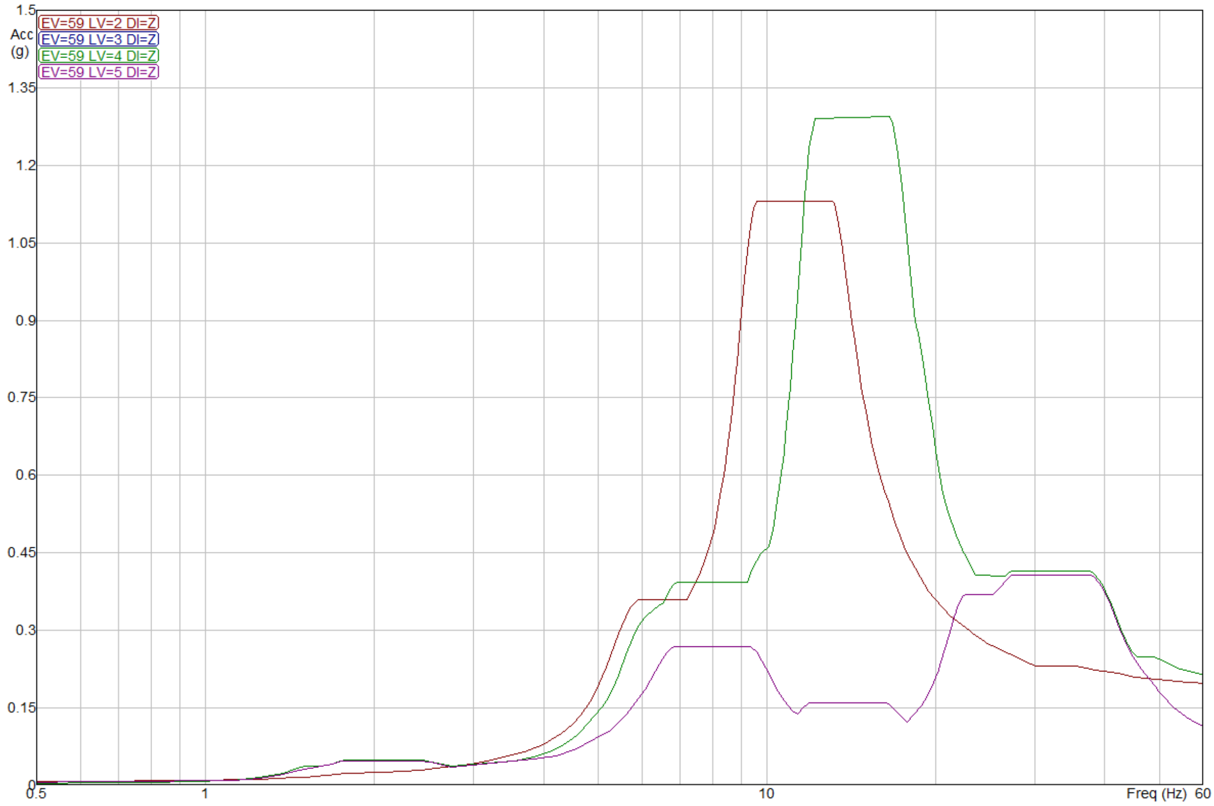

Figure 5: Applied response spectra in *z* direction.

In the decoupled analysis, the main pipe is analyzed first. The model is the same as for the coupled analysis though with the small bore pipe removed. Using the modal results a response spectrum for each direction is constructed for 5% damping according to the procedure in Section 2. The same combination methods are used as in the coupled analysis. The spectra thus generated are shown in Figure 6. Normally spectrum peaks are broadened in the frequency direction to account for inaccuracies in the calculation of natural frequencies. A plot of the generated spectra with peaks broadened ±15% is shown in Figure 7. Since there is no broadening effect in the main pipe excitation of the small bore pipe in the coupled analysis, a comparison using unbroadened spectra at the decoupling point would be of interest. Hence, the comparison is based both on results obtained by broadened and unbroadened amplified response spectra.

The model of the decoupled small bore pipe is the same as for the coupled analysis though with the main pipe removed. The decoupling point is modelled as a fix point. To simulate the stiffness of the omitted main pipe, the diagonal elements of the stiffness matrix of the main pipe have been calculated for force and moments applied at the decoupling point. These stiffness values have been applied to the fix point of the decoupled small bore pipe model. The model of the decoupled small bore pipe is shown in Figure 8, where also point labelling has been included.

In the coupled analysis, the small bore pipe behavior is a combination of inertial response and quasi-static response due to the motion of the main pipe. When the small bore pipe is analysed separately by applying response spectrum loads at the decoupling point, only the inertial response is obtained. In order to compare the results, the maximum displacements obtained at the decoupling point of the main pipe model have been applied in the analysis of the decoupled small bore pipe. The quasi-static results are then added to the inertial response by SRSS summation.

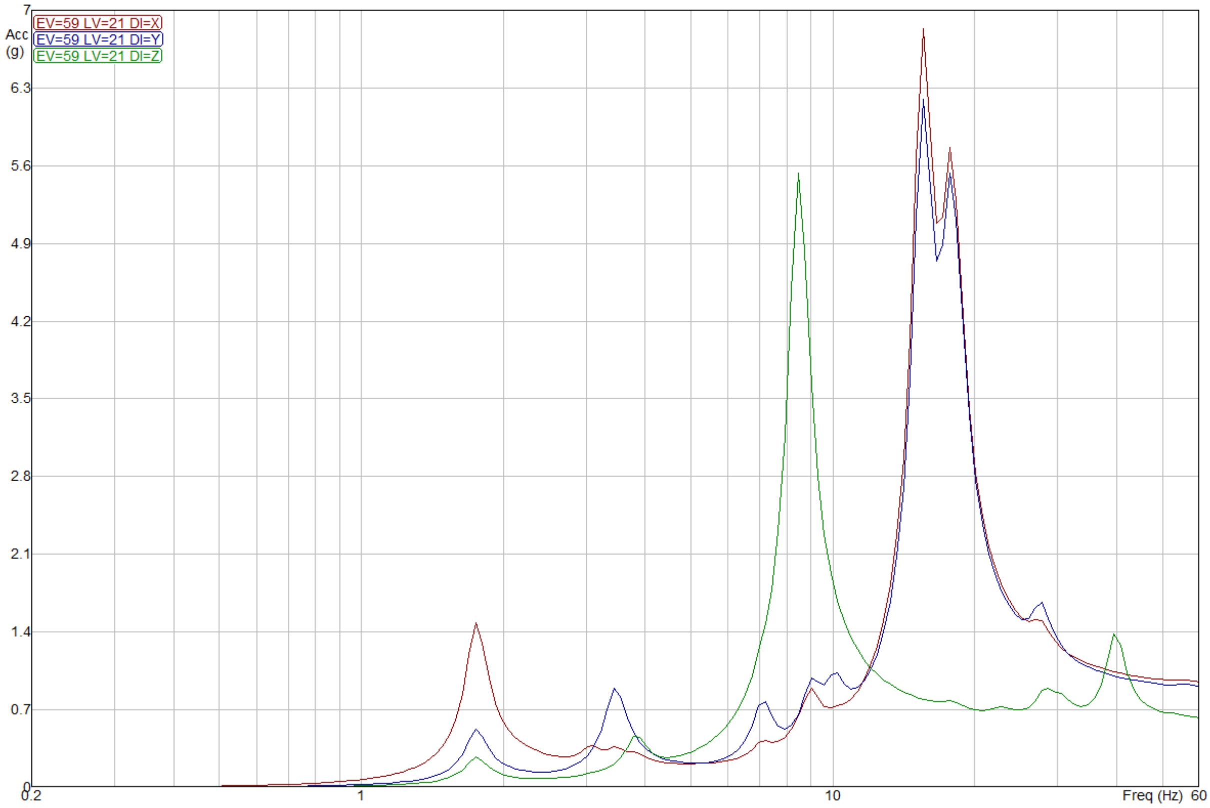

Figure 6: Generated amplified spectra at decoupling point.

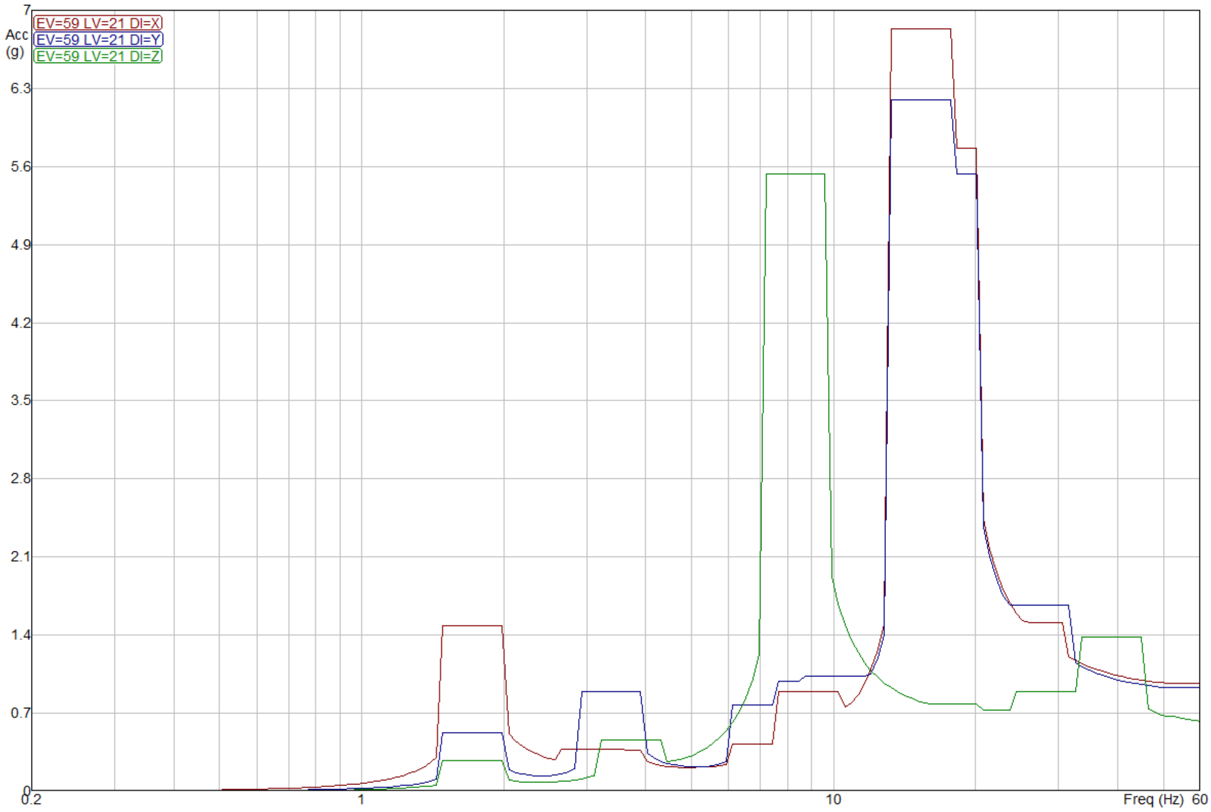

Figure 7: Generated amplified spectra at decoupling point with 15% peak broadening.

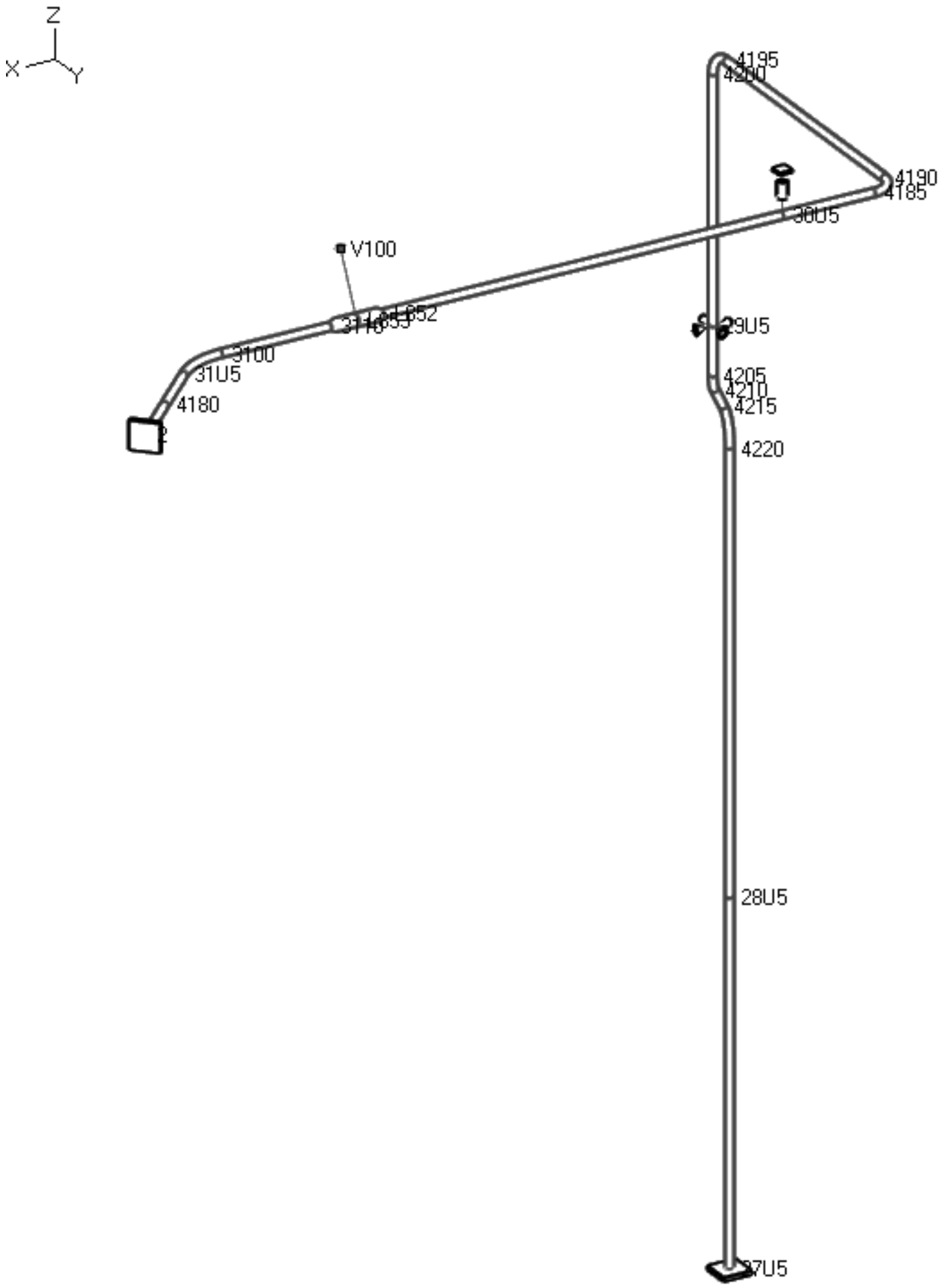

Figure 8: Model of decoupled small bore pipe.

3.2 Results

The results are compared by looking at the displacements at the points indicated in Figure 8. Table 2 shows the directional displacements u_x, u_y and u_z for each point in the coupled analysis and the decoupled analysis with unbroadened spectra applied at the decoupling point. Also the resultant displacements r_c for the coupled model and r_d for the decoupled model are presented. The difference between the resultant displacements is presented both as displacement values and as a percentage. Table 3 shows the same comparison but based on the analysis with broadened spectra applied at the decoupling point.

Point	Coupled model				Decoupled model				Δr [mm]	$\Delta r/ r_c$ [%]
	u_x [mm]	u_y [mm]	u_z [mm]	r_c [mm]	u_x [mm]	u_y [mm]	u_z [mm]	r_d [mm]		
2	8.26	6.07	4.82	11.32	8.26	6.07	4.82	11.32	0.00	0
3100	10.90	6.43	7.45	14.69	9.63	7.25	6.73	13.81	-0.88	-6.00
3110	10.90	6.53	9.64	15.95	9.63	8.20	8.27	15.11	-0.83	-5.23
4180	9.08	5.91	5.31	12.07	8.55	6.27	5.15	11.78	-0.29	-2.36
4185	10.91	5.41	7.53	14.32	9.63	5.63	6.32	12.82	-1.50	-10.44
4190	10.89	6.11	5.89	13.80	9.56	5.97	5.21	12.42	-1.39	-10.05
4195	12.30	6.11	1.99	13.87	10.40	5.96	2.04	12.16	-1.72	-12.37
4200	9.31	5.02	1.39	10.67	7.71	4.74	1.22	9.13	-1.54	-14.41
4205	1.04	0.69	1.39	1.87	0.87	0.64	1.22	1.63	-0.24	-12.86
4210	2.54	1.62	0.88	3.14	2.17	1.52	0.76	2.75	-0.39	-12.47
4215	3.01	2.12	0.48	3.71	2.58	1.95	0.41	3.26	-0.45	-12.19
4220	4.25	3.15	0.01	5.29	3.63	2.90	0.01	4.64	-0.65	-12.23
27U5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-
28U5	6.07	4.81	0.00	7.74	4.72	3.94	0.00	6.15	-1.59	-20.56
29U5	0.00	0.00	1.39	1.39	0.00	0.00	1.22	1.22	-0.17	-12.48
30U5	10.91	5.51	7.94	14.57	9.63	6.24	6.74	13.31	-1.26	-8.67
31U5	10.30	6.14	6.32	13.56	9.24	6.71	5.90	12.85	-0.70	-5.19
L852	10.90	6.33	10.46	16.38	9.63	8.49	8.86	15.60	-0.78	-4.79
L853	10.90	6.43	10.06	16.17	9.63	8.35	8.57	15.36	-0.81	-4.99
V100	11.06	8.73	9.64	17.07	10.69	9.47	8.18	16.46	-0.62	-3.62

Table 2: Displacement comparison between coupled and decoupled analysis results for unbroadened spectra.

Point	Coupled model				Decoupled model				Δr [mm]	Δr/ rc [%]
	u_x [mm]	u_y [mm]	u_z [mm]	r_c [mm]	u_x [mm]	u_y [mm]	u_z [mm]	r_d [mm]		
2	8.26	6.07	4.82	11.32	8.26	6.07	4.82	11.32	0.00	0.00
3100	10.90	6.43	7.45	14.69	10.02	7.76	7.59	14.77	0.08	0.55
3110	10.90	6.53	9.64	15.95	10.02	8.86	9.95	16.67	0.72	4.54
4180	9.08	5.91	5.31	12.07	8.65	6.37	5.28	11.97	-0.10	-0.83
4185	10.91	5.41	7.53	14.32	10.02	6.48	8.12	14.43	0.11	0.80
4190	10.89	6.11	5.89	13.80	10.41	6.58	6.24	13.80	0.00	0.01
4195	12.30	6.11	1.99	13.87	12.96	6.58	2.13	14.68	0.81	5.84
4200	9.31	5.02	1.39	10.67	10.01	5.37	1.45	11.45	0.78	7.31
4205	1.04	0.69	1.39	1.87	1.10	0.73	1.46	1.97	0.10	5.17
4210	2.54	1.62	0.88	3.14	2.64	1.79	0.91	3.32	0.17	5.57
4215	3.01	2.12	0.48	3.71	3.08	2.29	0.50	3.88	0.16	4.44
4220	4.25	3.15	0.01	5.29	4.33	3.37	0.01	5.49	0.19	3.66
27U5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-
28U5	6.07	4.81	0.00	7.74	6.20	4.94	0.00	7.92	0.18	2.37
29U5	0.00	0.00	1.39	1.39	0.00	0.00	1.46	1.46	0.06	4.38
30U5	10.91	5.51	7.94	14.57	10.02	7.20	8.73	15.11	0.54	3.72
31U5	10.30	6.14	6.32	13.56	9.54	7.01	6.38	13.45	-0.11	-0.81
L852	10.90	6.33	10.46	16.38	10.02	9.11	10.91	17.39	1.01	6.15
L853	10.90	6.43	10.06	16.17	10.02	8.99	10.44	17.03	0.87	5.37
V100	11.06	8.73	9.64	17.07	11.57	11.31	9.78	18.90	1.82	10.69

Table 3: Displacement comparison between coupled and decoupled analysis results for broadened spectra.

3.3 Discussion of the results

First, it should be pointed out that an exact match of results would never be expected since information is always lost when decoupling is made. For example, adding the quasi-static solution to the dynamic response is an approximation in comparison to the coupled analysis which gives an exact solution. In spite of this, the comparisons show fairly good agreement. The decoupled analysis based on unbroadened spectra underestimates the response in the entire model with a maximum deviation of 21%. The decoupled analysis based on broadened spectra produce results similar to the coupled analysis or somewhat overestimates the results. The maximum deviation is 11%. Upon decoupling, there will never be an exact match of natural frequencies in comparison with the coupled analysis. With narrow peaks in the response spectrum, a small deviation in frequency will have a significant impact on the response. With broadened decoupled spectra, the effect of deviating natural frequencies due to approximations in the modelling can be accounted for.

This verifying example shows that the proposed method for generating and using amplified response spectra can be used when decoupling is desired. However, it is important to broaden the amplified spectra to account for deviations in natural frequencies. Even though broadened spectra are used, sources for errors should be reduced as much as possible. Hence, it is recommended to implement stiffness values at the decoupling point.

4 ANALYSIS EXAMPLE

One of the benefits of decoupling is the possibility to separate primary stresses from secondary in the stress evaluation. This will be shown in the following sample stress evaluation, which is performed on a coupled and decoupled system.

4.1 Code criteria

Several piping design codes exist and in many cases the equations for stress evaluation are similar. For this example, the class 2 criteria of the nuclear section of the ASME Boiler and Pressure Vessel Code [9] will be used. For dynamic loads, evaluation criteria exist both for non-reversing and reversing dynamic loads. Earthquake loads can be considered as reversing dynamic loads so the criteria for reversing dynamic loads are used. The earthquake considered is a safe shutdown earthquake which is a faulted condition event (Level D). Sub-paragraph NC-3654.2(b) of the code gives the following equations for stress evaluation:

$$B_2 \frac{D_0}{2I} M_W \leq 0.5 S_h \tag{26}$$

$$B_1 \frac{P_E D_o}{2t} + B_2' \frac{D_o}{2I} M_E \leq 3 S_h \tag{27}$$

$$C_2 \frac{M_{AM} D_o}{2I} < 6 S_h \tag{28}$$

$$\frac{F_{AM}}{A_M} < S_h \tag{29}$$

where Eq. (26) limits the sustained stress due to weight loading, Eq. (27) limits the stress due to weight and inertial loading (primary stress) whereas Eqs. (28) and (29) limits the stresses due to dynamic anchor motions (secondary stresses). For a coupled analysis, the secondary stresses cannot be separated from the primary stresses so they will be included in the evaluation w.r.t. Eq. (27) and no evaluation will be made w.r.t. Eqs. (28) and (29).

4.2 Analysis models and loads

The same analysis models as were used in the validation in Section 3 are used for the stress evaluation. However, in this analysis more severe response spectra are applied to the model. The spectra derived for application at the decoupling point are broadened. As an interesting comparison, also envelopes of the spectra acting on the main pipe are derived and applied at the decoupling point. Both the amplified and enveloped spectra are shown in Figure 9.

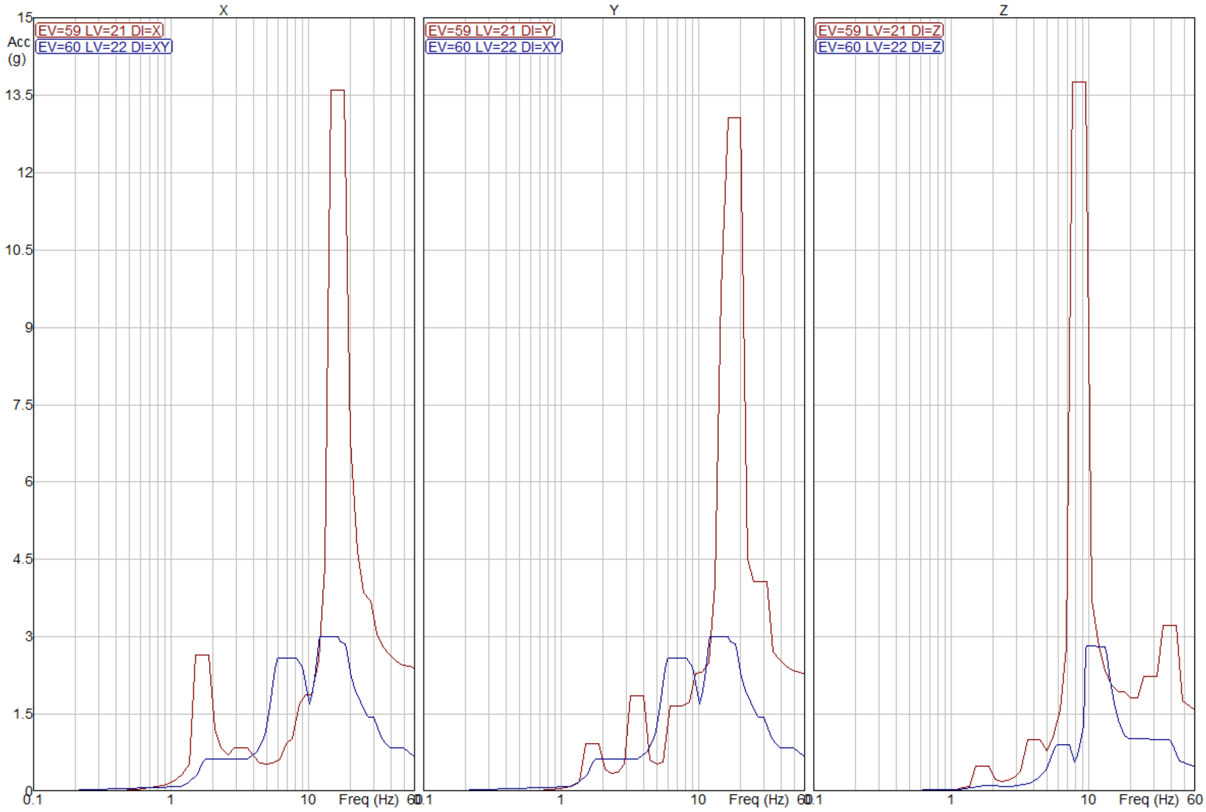

Figure 9: Amplified response spectra (EV=59) and enveloped response spectra (EV=60).

4.3 Analysis results

The highest stress results from the analyses are shown in Table 4. For each equation, the results are shown as stress usage ratios, i.e. the stress calculated according to any of the equations divided by the allowable stress. Hence, a stress usage ratio above 1.0 means failure to qualify the system. In the coupled analysis, Eq. (27) is not fulfilled. The reason for this is the fact that the stress in the coupled analysis is not only due to inertial loading since it also includes the stress in the small bore pipe caused by the movement of the main pipe . By decoupling, the primary and secondary stresses can be separated and assessed w.r.t. the appropriate equations. This results in a qualified system. Not unsurprisingly, the amplified spectra give higher results than the envelope spectra.

Analysis	Stress usage ratio			
	Eq. (26)	Eq. (27)	Eq. (28)	Eq. (29)
Coupled	0.89	1.11	N/A	N/A
Decoupled with amplified spectra	0.94	0.92	0.28	0.05
Decoupled with envelope spectra	0.94	0.72	0.28	0.05

Table 4: Results from the example analysis.

4.4 Discussion

The results of the analysis works a good example showing that decoupling can be useful in qualifying a piping system since the separation in primary and secondary stresses can be made.

The practice to model the connection point as an anchor and apply anchor motions and an envelope of the spectra for the main pipe supports at this point as was stated in [3] clearly underestimates the stress and gives non-conservative results. Looking at the stress only due to the inertial response and not together with pressure and weight stress as in Eq. (27), the difference between the amplified and envelope methods is even greater. This is not surprising if the spectra in Figure 9 are compared. The spectral peaks are significantly higher in the amplified spectra and the frequency contents are different. The amplified spectra reflect the dynamic characteristics of the main pipe whereas envelope spectra lack influence of the main pipe on the small bore pipe response. No validating proof supporting the envelope method exists; it is only used for convenience reasons. However, this example clearly shows that it is not a suitable method since the risk is high that it will give non-conservative results.

5 CONCLUSION

The method to create amplified response spectra at the small bore piping connection point shows good agreement when applied in a decoupled analysis and comparing the results with a coupled analysis. However, it is recommended to broaden the spectra to account for modelling errors when decoupling.

Any errors in decoupled analyses are most likely due to the decoupling and not due to the generation of amplified response spectra. Existing decoupling criteria mainly focus on the influence of the small bore pipe on the main pipe and not the opposite. More investigations will be needed to provide decoupling criteria that are appropriate for response spectrum analysis of small bore pipes.

When it can be justified to decouple small bore pipes, it is a useful technique since secondary stresses can be separated from primary stresses and unnecessary conservatism can be removed from the analysis. However, it is important to create amplified spectra. The practice to apply envelope of the spectra acting on the main pipe at the decoupling point is not a suitable method. It is clearly shown in the example analysis that this is not a conservative method.

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