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A DISCRETE MACRO-MODEL USING HOMOGENIZATION WITH STRAIN-RATE DEPENDENCY FOR THE OUT-OF-PLANE STUDY OF MASONRY PANELS SUBJECTED TO IMPACT LOADING

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Abstract. *In recent decades, a great deal of effort has been made to develop solutions to reduce destructive damage and casualties due to blast loads and impacts, also in light of a major protection of the built heritage against terrorist attacks. In the present study, a simple and reliable Homogenization approach coupled with a Rigid Body and Spring Model (HRBSM) accounting for high strain rate effects is utilized to analyse masonry panels subjected to impact load.*

The homogenization approach adopted relies into a coarse FE discretization where bricks are meshed with a few elastic constant stress triangular elements and joints are reduced to interfaces with elastic-plastic softening behaviour including friction, a tension cut-off and a cap in compression. Strain rate effects are accounted for assuming the most meaningful mechanical properties in the unit cell variable through the so-called Dynamic Increase Factors (DIFs), with values from literature data. The HRBS model, which has been implemented at structural level in the commercial code ABAQUS resorts on a discretization into rigid quadrilateral elements with homogenized bending/torque non-linear springs on adjoining edges.

The model is tested on a masonry parapet subjected to a standardized impact. A number of previous results obtained by literature models are available for comparison, as well as experimental data. Satisfactory agreement is found between the present results and existing literature in the field, both experimental and numerical.

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1 INTRODUCTION

Cultural heritage buildings are structures that outline a set of particular values which symbolically allow them to become part of a given culture identity and continuity [1]. Masonry is an ancient and still widespread construction material [2]. It is known that existing Unreinforced Masonry buildings (URM) present a high seismic vulnerability [3] even for low forces applied [4]. This aspect can be explained by: (i) the poor out-of-plane strength that is directly associated with the low tensile strength of URM; (ii) lack of capacity to dissipate energy; (iii) absence of seismic requirements in the time of its construction [3][5]; (iv) the lack of proper connections between structural elements [6]; (v) the flexibility of floors [7]; (vi) the material deterioration over time [8].

This highlights the importance of their maintenance and conservation, but also the need of mitigating excessive risk making them more prone to resist dynamic actions such as impacts or other possible extreme loading cases. A scientific-based intervention is less exposed to inadequate actions and thus in the evaluation of its safety, the numerical modelling is a valuable tool. Insomuch, it is mandatory to understand the behaviour of unreinforced masonry (URM) structures due to fast non-linear dynamic analyses [9–12].

The present paper addresses a simple two-step procedure within the scope of homogenization approach. The purpose is the macroscopic study of the out-of-plane dynamic behaviour of masonry walls under fast dynamics, suitable for impacts. It is known that URM walls subjected to dynamic loads can resist accelerations higher than their static strength and thus, according to the applied load strain rate, material properties may exhibit a dynamic enhancement [13]. In this regard, a bespoke simple Homogenized rate-dependent model coupled within a novel Rigid Body and Spring Model (hereafter, HRBSM), extended to a commercial software package (ABAQUS [14]), is described. The strategy is intended to be a fast and an accurate predictor tool. Its validation will be achieved through comparison with experimental results on masonry structures subjected to blast and impact loads ([15] and [16], respectively).

2 MESO-SCALE: HOMOGENIZED MODEL

2.1 Overview

A multi-scale approach is assumed for the study of masonry panels subjected to different load types. Such strategy lies on the periodicity feature of a given media and it is therefore a suitable strategy for masonry [17,18]. First, a micro-scale mechanical characterization on a representative volume element (hereafter, RVE) is achieved by solving a boundary value problem (BVP). Then, the macroscopic constitutive response is accomplished through the assemblage of these RVE units. The main features will be explained in what follows and, for further information of the quasi-static approach, the reader is referred to [19] and [20]. Please note that the description will be made for a running bond texture, see Fig. 1.

In brief, homogenization consists in deriving the upper-scale properties by introducing averaged quantities for macroscopic strain and stress tensors (E and Σ, respectively) obtained at a micro-scale on the RVE (Y, elementary cell). The main concept of the homogenization process implies that the macroscopic stress and strain tensors are calculated as Eq. (1).

$$\mathbf{E} = <\boldsymbol{\varepsilon}> = \frac{1}{V}\int_Y \boldsymbol{\varepsilon}(\boldsymbol{u})\,dY \quad ; \quad \boldsymbol{\Sigma} = <\boldsymbol{\sigma}> = \frac{1}{V}\int_Y \boldsymbol{\sigma}\,dY \tag{1}$$

where <*> is the average operator, $\boldsymbol{\varepsilon}$ is the local strain value, which is directly dependent of the displacements field $\boldsymbol{u}$, $\boldsymbol{\sigma}$ is the local stress value and V is the volume of each elementary cell. The latter is governed by the Hill-Mandel principle [21,22] that establishes the energy equivalence between the macroscopic stress power with the micro-scale stress power over the volume of the RVE. All the mechanical quantities are considered as additive functions and periodicity conditions (local periodicity) are imposed on the stress field σ and the displacement field u [23] so that:

$$\boldsymbol{\sigma} \text{ periodic on } \partial Y \text{ and } \boldsymbol{\sigma n} \text{ antiperiodic on } \partial Y_1 \quad (2)$$

$$\boldsymbol{u}^{per} \text{ periodic on } \partial Y_1 \quad (3)$$

In the present model, the RVE is modelled as a continuum FE model, whereas joints are reduced to interfaces with zero thickness and bricks are discretized by means of a mesh constituted by plane-stress elastic triangles, Fig. 1. The formulation assumes that cracking and all non-linearity of each RVE are concentrated exclusively on interfaces between adjoining elements, both on brick and joints. The elastic domain of joints is bound by a composite yield surface that includes tension, shear and compression failure with softening (see Fig. 1). A multi-surface plasticity model is adopted, with softening, both in tension and compression. The parameters ft and fc are, respectively, the tensile and compressive Mode-I strength of the masonry or mortar–brick interfaces, c is the cohesion, F is the friction angle, and Y is the angle which defines the linear compression cap.

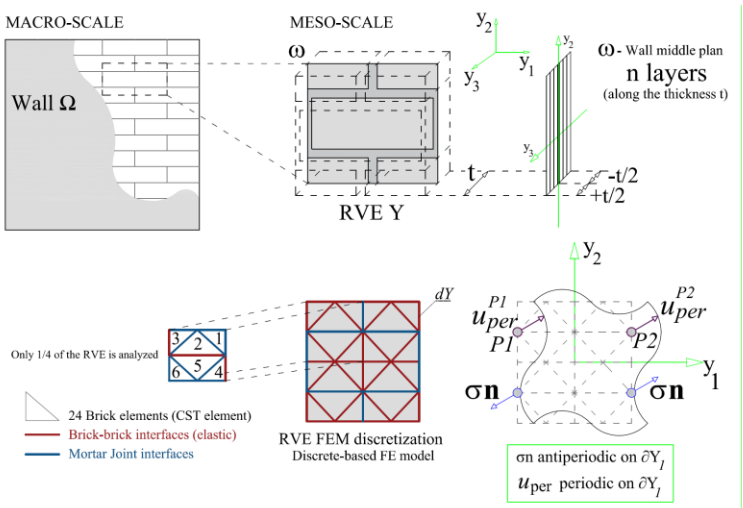


Fig. 1. Micro-mechanical model adopted for the present homogenized model and strength domain for mortar joints reduced to interfaces.

The response of the RVE under out-of-plane actions is obtained subdividing along the thickness the unit cell into several layers (40 subdivisions). A displacement driven approach is adopted, meaning that macroscopic curvature increments $\Delta\chi_{11}$, $\Delta\chi_{22}$, $\Delta\chi_{33}$ are applied through suitable periodic boundary displacement increments.
In this way, homogenized curves are approximated to define the nonlinear behaviour of the interfaces. For each interface at a structural scale, the cross-section equilibrium is iteratively calculated aiming to obtain the M-θ homogenized curves. The strategy to derive these curves is based on the macroscopic mode-I and mode-II stresses. Such assumption is plausible because masonry failure mechanisms tend to be mainly governed by joints failure due to its low tensile strength. The interface orientations accounted at a meso-scale are guided by the discrete mesh representation at a structural scale.

Quasi-static range	
Young's modulus of the brickwork composite (MPa)	20000
Young's modulus of the mortar (MPa)	1500
Young's modulus of the brick (MPa)	20000
Poisson coefficient (-)	0.30
Density of the brickwork composite (kg/m3)	2295
Shear modulus (MPa)	600
Cohesion, c (MPa)	0.63
Tensile strength ft (MPa)	0.45
Compressive strength fc (MPa)	30.0
Friction angle (ϕ) (degrees)	30.0
Linearized compressive cap angle (ψ) (degrees)	50.0
Mode I fracture energy, G_f^I (N/mm)	0.012
Mode II fracture energy, G_f^{II} (N/mm)	0.019
K_n - axial truss (MPa)	174.43
K_n - torque truss (MPa)	130449

Table 1. Static Mechanical properties adopted for the homogenization step.

2.2 Strain-rate effect

Existing research proves that the use of static strength properties can lead to unreliable results for the masonry behaviour under fast dynamic actions, see [13,24]. Static strength material properties may exhibit an enhancement according to the strain rate level of the applied load. A useful and practical way to numerically represent the material properties change is to define dynamic increase factors (DIFs). The use of DIF laws is found suitable to study masonry structures subjected to fast loads application [24,25]. In this way, a homogenized model that may account the latter is relevant.

Focusing on the present homogenized approach, the material model reflects the dynamic characteristics of mortar and brick, and is derived from the static in-plane homogenized model (see also [20]). The values that define the elastic behaviour and the strength envelope of the unit cell, i.e. the parameters that directly rule the plasticity model, are strain-rate dependent. Specifically, the Young's modulus of the brick E_b, Young's modulus of the mortar E_m, tensile strength of the mortar f_{tm}, shear modulus of the mortar G_m and cohesion c. Compressive behaviour is in practice scarcely active in out-of-plane loaded periodic masonry and therefore, excluded from the present considerations on DIF for the sake of simplicity. According to the strain rate level, the values of the material properties are obtained through the product between the quasi-static property value and DIF, Eq. (4).

$$\begin{cases} E_b = DIF_{Eb} \times E_b \\ f_{tm} = DIF_{ftm} \times f_{tm} \\ E_m = DIF_{Em} \times E_m \\ c = DIF_c \times c \end{cases} \tag{4}$$

Therefore, it is required to define such DIFs by: (i) introducing strain-rate laws, typically logarithmic curves, for each selected parameter; or (ii) using a discrete DIF value, independent from the strain rate level, which is a priori assumed and adopted as a constant value. If the former yields more realistic values, it is also true that the latter is a straightforward, simple and more aligned with normative proposals.

For the present study, the information proposed by [24] is used to obtain rate dependent homogenized relations and so, the former strategy is adopted. The bending and torsional moment curves may be integrated along the thickness for each strain rate level. The nonlinear curvature-bending moment flexural and torsional behaviours of the interfaces are approximated using 5-node simplified curves, see for instance the curves defined in [20]. The implementation of this information in a finite element package at a macro-scale will allow to represent and study three-dimensional structures due to out-of-plane dynamic actions.

3 MACRO-SCALE: DISCRETE FE-MODEL

3.1 Overview

The adopted two-fold strategy relies into a homogenization approach at a meso-scale and on a discrete FE model at a macro-scale level. The dynamic out-of-plane analyses of masonry walls are performed using a novel discrete mechanical system at a macro-scale. Fig. 2 presents the model for a clear understanding and was already validated for quasi-static purposes, see [20]. The work by Kawai [26,27] serves as background for its formulation.

Briefly, the system is composed by quadrilateral rigid plates. On the interfaces and connecting these rigid elements, deformable truss and rigid beams are placed on each node. These truss-beam system mimics the presence of flexural and torsional springs, governing the deformation and damage of the equivalent continuum. Additionally, mid-span hinges placed on interfaces allow to fix the axis of rotation for torsion movements without compromising the deformed shape. It should be noted that a decoupled characterization of flexural and torsional actions is adopted and such behaviour is ruled by the mechanical and material information derived beforehand at a meso-scale level. Nodal mass elements are lumped on the centre of each rigid plate. These elements concentrate the mass of the equivalent basic cell of the system, see Fig. 2.

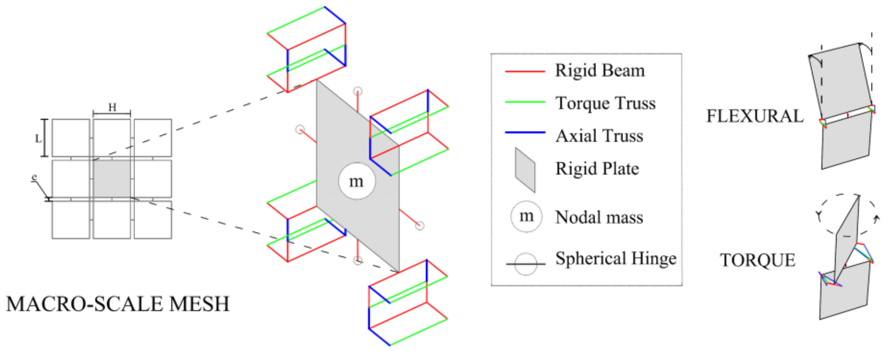


Fig. 2. Description of the novel discrete element system proposed.

3.2 Material properties

A proper constitutive model able to interpret the latter information obtained from a previous step and scale is essential. The model should be capable to allocate the mechanical information and, thus, mimicking effectively the elastic and inelastic response of the defined masonry RVE. The concrete damage plasticity (hereafter, CDP) model is selected, given the better representation of the inelastic laws. The reader is referred to e.g. [28,29] for a general overview of its main features and application.

The model combines a stress-based plasticity with strain-based scalar damage. It is able to assign different yield strengths, different stiffness degradation and recovery effect terms both in tension and compression regimes. Thus, effective stresses tend to govern the plastic part of

these models [30]. Moreover, it does consider the latter in the presence of material strain-rate dependency which may successfully idealize both the flexural and torsional interfaces dynamic and/or cyclic loading.
It may be noted that the strain-rate dependent homogenization model allows obtaining the macroscopic masonry material properties accounting for the strain softening regime. The novel HRBSM model implemented in the finite element package ABAQUS must be able to receive such data to fully represent the inelastic behaviour of masonry through stress-strain curves for axial and shear trusses of the system. But first, an identification of the desired mesh dimensions at a macro-scale and the geometrical characteristics of the masonry structure under study may be performed. This task is required to calibrate the elastic response of the input. The procedure described next converts the latter information into valid input data for the FE package used at a structural scale. Bearing in mind that rectangular-square elements are assumed for the sake of simplicity, two different angles are considered for the interfaces: 0 and 90 degrees. The behaviour of the interfaces is obviously orthotropic with softening, because it derives from the aforementioned homogenization strategy. The material orthotropy is reproduced at a structural level because the approach offers the possibility to reproduce different input stress-strain relationships according to the trusses plane. The conversion between moment and stress values is achieved by Eq. (5):

$$\sigma_{\text{axial truss}} = \frac{M \times L_{\text{influence}}}{(A_{\text{Axial}} \times t)}; \quad \sigma_{\text{torque truss}} = \frac{M \times L_{\text{influence}}}{(A_{\text{Torque}} \times H)} \tag{5}$$

Here, M is the bending moment, $L_{\text{influence}}$ is the influence length of each truss, t is the thickness of the wall, H the length of each quadrilateral panel, A_{Axial} is the axial truss area given by 0.25×t×H and A_{Torque} is the torque truss area given by 0.5×e×H, where e (value of 10 mm) is the gap between the rigid plates, which ideally should be zero but in practice is assumed small enough to be able to place trusses between elements.
At last, the stress homogenized input curves may be properly calibrated. An elastic calibration for the stress curves is conducted. The latter is guaranteed separately for both flexural and torsional movements and, therefore, a decoupled behaviour is derived. Briefly, by assuring the energy equivalence between the discrete mechanism and a homogeneous continuous shell element, it can be easily derived that, for both case studies, the Young's moduli of axial (E_{flexural}) and torque trusses (E_{torque}) are:

$$E_{\text{flexural}} = \frac{1}{6}\frac{e}{2L+e}\frac{E_{\text{masonry}}}{(1-\nu^2)} \text{ and } E_{\text{torque}} = \frac{1}{3}\frac{t^4}{(2L+e)H^2 e}\frac{E_{\text{masonry}}}{(1+\nu)} \tag{6}$$

For the simulations, the CDP model in ABAQUS requires that information regarding the post-failure stress-strain behaviour may be introduced for each element that features material non-linearity, i.e. the truss beams. Specifically, the cracking strain $\tilde{\varepsilon}_t^{ck}$ must be obtained for each point of the homogenized curve by Eq. (7):

$$\tilde{\varepsilon}_t^{ck} = \varepsilon_t - \varepsilon_o^{el} \tag{7}$$

where ε_o^{el} is the elastic strain corresponding to the undamaged material and ε_t is the total strain of the holonomic curve. Damage parameters d_t should also be introduced, which link the undamaged elastic modulus with that of the damaged material in the unloading phase, as $E_d = E(1-d_t)$, obtained through a plastic strain and total strain relation.

4 NUMERICAL MODEL VALIDATION

The interest of researchers regarding the dynamic analysis of masonry structures subjected to impacts has been growing, but experimental studies on the behaviour of these structures is

still scarce [11]. Recognizing the importance to develop further studies on the field, numerical models play an important role. In this framework, a discrete homogenized strain-rate sensitive model is proposed here and the main features of the discrete model were discussed in previous Sections.
To assess the ability of the approach proposed for the study of impact loading, a 'stretcher' bond masonry parapet tested by Gilbert et al. [31] is used. The wall is subjected to a low velocity impact load that tries to represent the impact of a vehicle. The comparisons will be performed by means of the experimental determined data [31], but also with numerical results collected from the studies of [25,32]. The selected parapets (from 21 tested) are designated as C6 and C7 and are replicates, see Fig. 3. Their assemblage was executed with strong concrete blocks and weak mortar (class iii mortar according to BS-5628). The walls and brick dimensions are 9150 x 1130 x 215 mm^3 (length x height x thickness) and 440 x 215 x 215 mm^3 (length x height x thickness), respectively.
At the wall base, the surface was coated with epoxy sand to reproduce the roughness of a given street floor. For the lateral supports, two abutment blocks connected to the walls through epoxy mortar were used. Numerically, the boundary conditions are considered to be fixed for each lateral edge and simple supported at the base of the walls. Aiming to model a car-like impact at both mid-height and length of the walls, a triangular out-of-plane load was applied through a steel plate. The load is idealized as a triangular time history distribution, in which the peak value is equal to 110 kN [31]. The deformation of the studied parapets was recorded in a node located 580 mm above the base and deviated 250 mm from the centre.

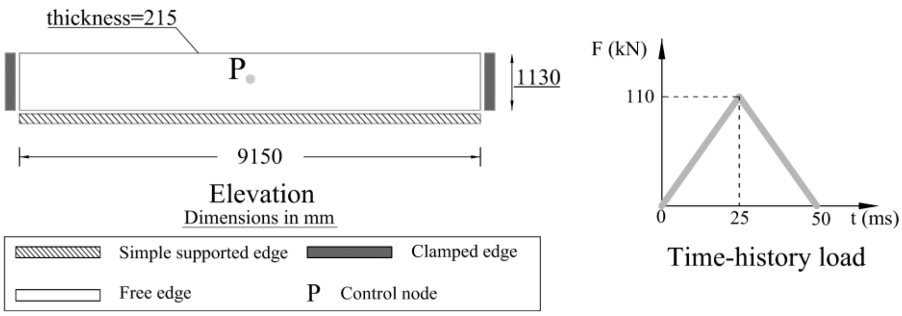
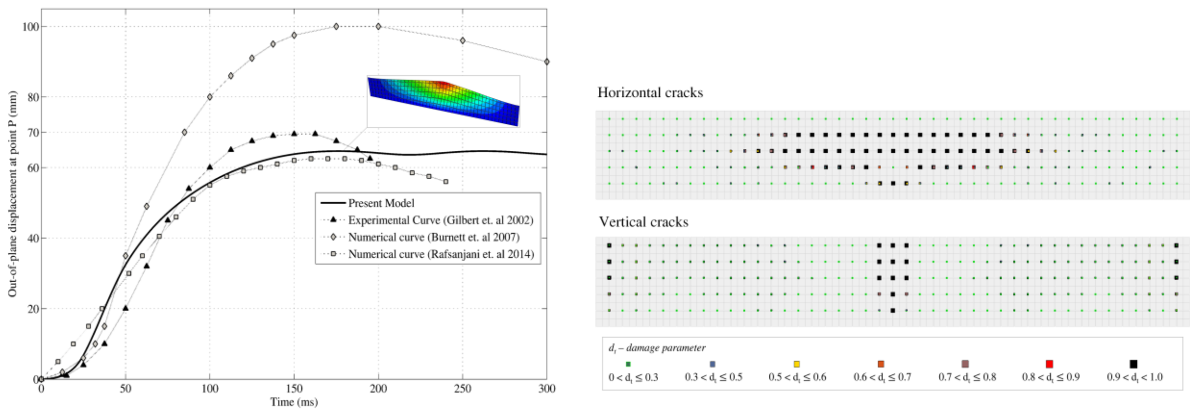


Fig. 3. Geometry of the running bond masonry parapets C6 and C7 studied by [31].

In the present model, the numerical simulation of the parapet walls will be accomplished through the aforementioned discrete element homogenized methodology. Firstly, the homogenization step at a meso-scale is performed to mechanically characterize the running bond masonry. The static material properties required are gathered in Table 1 and the rate-dependency issue is addressed. Hao and Tarasov [24] studied the experimental dynamic behaviour of a series of brick and mortar specimens under uniaxial compressive tests through a tri-axial static-dynamic apparatus. The analytical expressions required to describe the value of the DIFs derive from the latter research. It may be noted that the study by Rafsanjani et. al. in [32] that covers the current masonry parapets, use the same laws hereby presented to define the strain-rate dependency of masonry interfaces using a micro-modelling strategy.
The implementation of such laws in the homogenized model allows deriving stress-strain rate-dependent homogenized curves. Additionally, moment-curvature relationships are simply derived by the integration of the latter stress-strain curves through the thickness of the wall.
Fig. 4a reports the displacement magnitude with respect to time. A numerical model with a mesh size of 200 mm was adopted at a macro-scale. Curves resultant from (i) the experimental results [31], (ii) the numerical model by Burnett et. al [25], (iii) the numerical model

by Rafsanjami et. al [32] and, the simulation results of the discrete homogenized-based model are depicted. The curve by Burnett et. al [25] leads to excessive displacements (and understiff response) because it considers quasi-static values for the material parameters. Conversely, both the present and the micro-model by Rafsanjami et. al [32] are accurate in predicting the peak displacement, with a relative error of around 10%.
Fig. 4b indicates the observed damage pattern for the present model. Vertical cracks are clear around both the central area and the two supported edges. Also, horizontal cracks spread from the centre along the height of the masonry parapet. As expected, it is evident that damage tends to concentrate on the impact zone. This was implicitly concluded in the experiments tests [31].



(a) (b)

Fig. 4. (a) Time history of the out-of-plane displacement for the control node of parapets C6 and C7, with deformed shape for the instant 180 ms; (b) Damage pattern obtained for the applied load wall side: horizontal cracks and vertical cracks.

5 CONCLUSIONS

A two-step homogenization procedure has been presented for the out-of-plane dynamic study of masonry walls subjected to an impact load. The approach relies on an up-ward methodology, where a Homogenization scheme links the meso-scale orthotropic mechanical description of a masonry representative volume element with a novel Rigid Body Spring Model (HRBSM) developed at a structural scale.
At a meso-scale, a simple FE discretization of the unit cell through constant stress triangle elements, within a plane-stress analysis, and joints reduced to interfaces allows to obtain a good response at a cell level at a very low computational effort. The nonlinearity is concentrated on mortar joints within a plasticity model that accounts for a tension cut-off, a Coulomb friction and compressive cap model. Through a homogenization process, the macroscopic stress and strains are derived and, by considering a Kirchhoff-plate hypothesis, the macroscopic bending- and torsional- moment-curvature are found by integration over the masonry thickness. On the other hand, to account for the material properties enhancement that typically occurs in case of fast dynamic events such as those investigated, a strategy based on dynamic increase factors (DIFs) has been chosen. In this way, a procedure based on a simple strain-rate dependent homogenization approach has been developed and ready to be implemented at a macro-scale level.
At the upper-scale, a HRBSM model has been implemented into a standard advanced nonlinear finite element code, namely ABAQUS [14]. This strategy has inherent advantages due to the robust numerical procedures available in the software. The discrete model makes use of

rigid quadrilateral plates connected at the edges with a system of deformable truss and rigid beams that receives the information transferred between the different scales. These elements govern the deformation and damage propagation and have an independent constitutive model from the foregoing scale. Additionally, the imposition of both plasticity and damage on one-dimensional elements critically reduces the required computational costs to perform the analyses. It allows easy strain-rate calculations in uniaxial damage plasticity models to control the macro-model response.

For the model validation, dynamic analyses have been performed in masonry panels subjected to an impact load [31]. The adopted material properties stand in existent literature. Even if it is often recommended the use of an explicit procedure to solve dynamic problems, an implicit one that makes use of a Hilber-Hughes-Taylor solution method has been adopted. The authors do not report any issue regarding the numerical convergence.

The results show a satisfactory agreement with the ones obtained experimentally for both the cases studied, as far as the peak load and damage patterns are concerned. The considered simplified assumptions, in both scale-levels, make possible a good trade-off between accuracy and computational effort. In fact, the total required time to perform the analyses was 12:38 minutes. It may be finally noted that the time needed also depends on the fact that a full strain-rate description is undertaken instead of a constant DIF value approach.

6 ACKNOWLEDGEMENTS

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