

**INTEGRATING MODAL ANALYSIS AND SEISMIC
INTERFEROMETRY FOR STRUCTURAL DYNAMIC RESPONSE:
THE CASE STUDY OF GIOTTO'S BELL TOWER IN FLORENCE
(ITALY)**

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Abstract. *The preservation of historical building requires monitoring their long-term behaviour as well as to quickly detect possible damage. In this respect, dynamic characteristics (frequency, shapes mode and seismic wave velocity) can be identified and tracked in real-time combining modal analysis with seismic interferometry, as it is shown in this paper for the Giotto's Bell-Tower in Firenze (Italy) by using a temporary seismic network. Natural frequencies and modal shapes of the structure are calculated by classic Enhanced Frequency Domain Decomposition (EFDD). Changes of the modal frequencies may reflect changes in the dynamic properties of the building and/or in the soil-structure interaction effects. The effects of soil-structure interaction are analysed with the seismic velocity and transfer function of the structure estimated using seismic interferometry technique. Seismic wave velocity inside the building depends only on the structural properties of the building and it is uncoupled from the soil-structure interaction system. The results of seismic interferometry (i.e fixed-base frequency, pseudo-flexible body frequency) combined with the first modal frequency allow to define the degree of the soil-structure interactions on the dynamic behavior of the Giotto's bell tower. The full structural dynamic identification shows a weak contribution of the soil-structure interaction, that is supported by the high values of the horizontal and rocking stiffness, evaluated from the geotechnical properties of the soil. It is suggested that integrating the Operational Modal analysis and the seismic interferometry of the same ambient noise recording provides a unique and complete dynamic response of one of the worldwide important monument like the Giotto's Bell-Tower.*

Keywords: Dynamic Identification, Seismic Interferometry, Soil-Structure Interaction, Structural Health Monitoring.

1 INTRODUCTION

The interest towards the dynamic response of historical buildings is increasingly growing when its functionality and health status have to be estimated. This issue is of crucial interest when historical structures are in area of high-to-moderate seismic risk. The Giotto's Bell-Tower in Florence (Figure 1) is one of such an example, since it was subjected in the past to earthquakes up to $I_{\max} = \text{VIII}$ of the MCS scale ($ML=5$) [1].

Over seven hundred years after the its foundation, the Opera del Duomo is still active in preserving and enhancing the Cathedral, the Baptistery, the Bell-Tower and the Historical Museum, and in the last years, it has been planned a series of studies aimed to improve our knowledge on dynamic characteristic of the Giotto's Bell tower.

One of the main tool to check the health status of a heritage structure is the dynamic response analysis using ambient vibration (wind, traffic, sea waves etc.) [2, 3, 4]. These techniques, known as Operational Modal Analysis (OMA), are completely not invasive and provide useful information on the elastic behaviour of the building [5, 6, 7, 8].

Damage of ancient building, as produced by cracks, aging of materials and structural deformations [9], causes a decrease of stiffness, which in turn affects the natural frequencies and thus the mode shapes [3]. Therefore, relevant damage of the building may be detected through the evolution of its modal parameters [9, 3, 4].

However, environmental factors such as temperature, wind speed and rainfall are reported to also affect the modal parameters [10] indicating that changes in natural frequencies could be not, necessarily, related to a level of damage [11]. Furthermore, several studies using earthquake as well as ambient vibration records [11, 12] have demonstrated that, although without damage, the natural frequency can vary as much as 30 % due to nonlinearities in the response of the building, mainly related to soil-structure interactions.

In the last ten years, several authors, in addition to monitoring the modal frequencies, have presented new approaches to structural dynamic analysis based on seismic interferometry of ambient vibration [11, 16, 17, 18] to investigate the seismic wave propagation in the building. Seismic interferometry evaluates the impulse response function (IFR) of an elastic medium from one point to another. Wave phase velocity of impulse response functions (IRFs) is uncoupled with soil-structure interaction, being directly linked to the stiffness of the building [19, 20], and allows the evaluation of fixed-based frequency of the structure.

Therefore, by combining the results of seismic interferometry with the modal frequency obtained with OMA we are able to distinguish the effects of the soil-structure interactions (rocking and horizontal frequencies) and of structural frequency (fixed-based frequency) on the full dynamic response of the Giotto's bell Tower.

2 DYNAMIC IDENTIFICATION TEST

The Giotto's Bell-Tower has square base of 14.45 m, NS and WE oriented and it is 84.7 m height (Fig. 1). It presents four intermediate floors and four pillars marks the corners. The masonry of the bell-tower consists of Pietraforte stone (a quartz-calcareous turbidite sandstone, quarried in the near hills south of Firenze) and white Carrara Marble and other colored marble tiles form the revetment.

The seismic survey was carried out for a time interval of 36 hours, from 16 to 18 October 2013, by using ten seismic stations (Figure 1). Each seismic station was equipped with 3-component seismometers: six stations with Lennartz 3D/5s seismometers (sensitivity 400 V/m/s and with a 5 s eigenperiod); three stations with Guralp CMG-6T seismometer (sensitiv-

ity 2400 V/m/s with a 10 s eigenperiod) and one station with a Guralp CMG-40T seismometer (sensitivity 800 V/m/s with a 30 s eigenperiod). Seismic data were digitized using a 24 bits Guralp CMG24 Digitizer at 100 Hz, and the time synchronization between stations was achieved using GPS. The ten stations were installed at five different floors, from the ground level up to the roof. Five stations were installed on the North-Side (C02-C04-C06-C08-C10) and the other five on the South-Side (C01-C03-C05-C07-C09) of Giotto's Bell-Tower (Figure 1). Seismic monitoring provided an excellent set of data, which allowed us to combine the results from Operational Modal Analysis to seismic interferometry using the same ambient vibration noise.

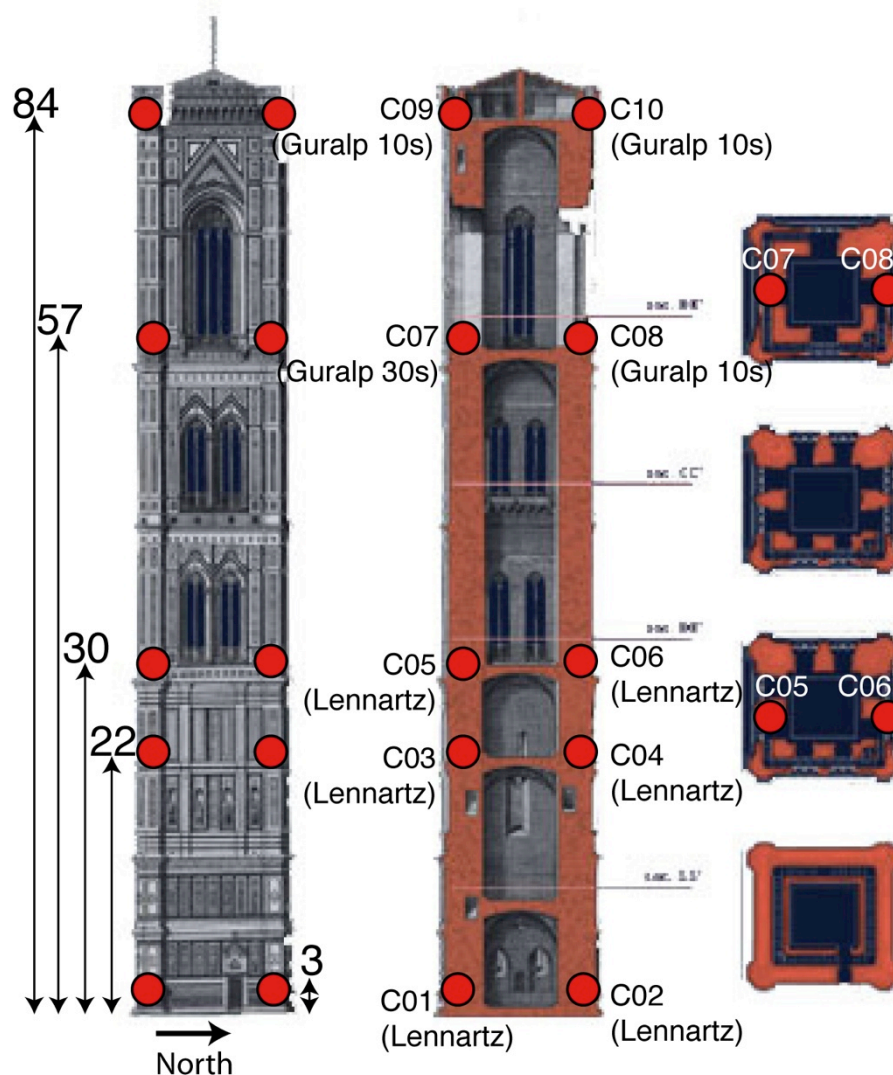


Figure 1. Vertical cross-section of the Giotto's Bell-Tower in Firenze (courtesy of "Opera di Santa Maria del Fiore") in the North-South direction, with the position and the height from the ground of ten seismic stations used for ambient vibration test.

3 OPERATIONAL MODAL ANALYSIS

The EFDD method is presented in literature by many authors [2, 3, 8, 13, 14, 15] as a frequency domain technique for operational modal analysis of structures. In the present study, we performed an automatic EFDD method, first presented by Brincker et al. (2007) [13] and

further developed by Rainieri and Fabbrocino et al. (2010) [2], to extract automatically the modal parameters of the structures.

The 36 hours long records has been divided into time windows 1800 s long and overlapping by 66%, for each of them; the spectral estimation was performed using the Welch's technique with Hanning weighting function and frequency resolution of 0.0031 Hz [15]. The automatic EFDD method allowed to detected the first five modes of the Giotto's Bell-Tower with natural frequency at 0.623 Hz, 0.647 Hz, 2.543 Hz, 3.081 Hz, 3.156 Hz and 5.731 (see Figure 2c). The estimated six modal shapes are shown in Figure 3 and it is relevant to note that the first modal shape is characterized by a motion trending $51^\circ - 231^\circ$ N direction (Figure 4a) with a frequency of 0.623 Hz, whereas the second mode shape is transversal and trends $141^\circ - 321^\circ$ N direction (Figure 4a), with a frequency of 0.647 Hz. Therefore, Giotto's Bell Tower exhibits a coupled behavior in the first two modes acting along the longitudinal and transverse directions at frequencies very close to each other resulting in a circular movement slightly eccentric in the N-S direction (Figure 4b). The third mode is torsional with a modal shape at 2.543 Hz. The fourth and fifth modes are instead bending modal shape at 3.081 Hz and 3.156 Hz, respectively (Figure 3). The sixth mode shows a bending modal shape at 5.731 Hz moving mainly in E-W direction (Figure 3).

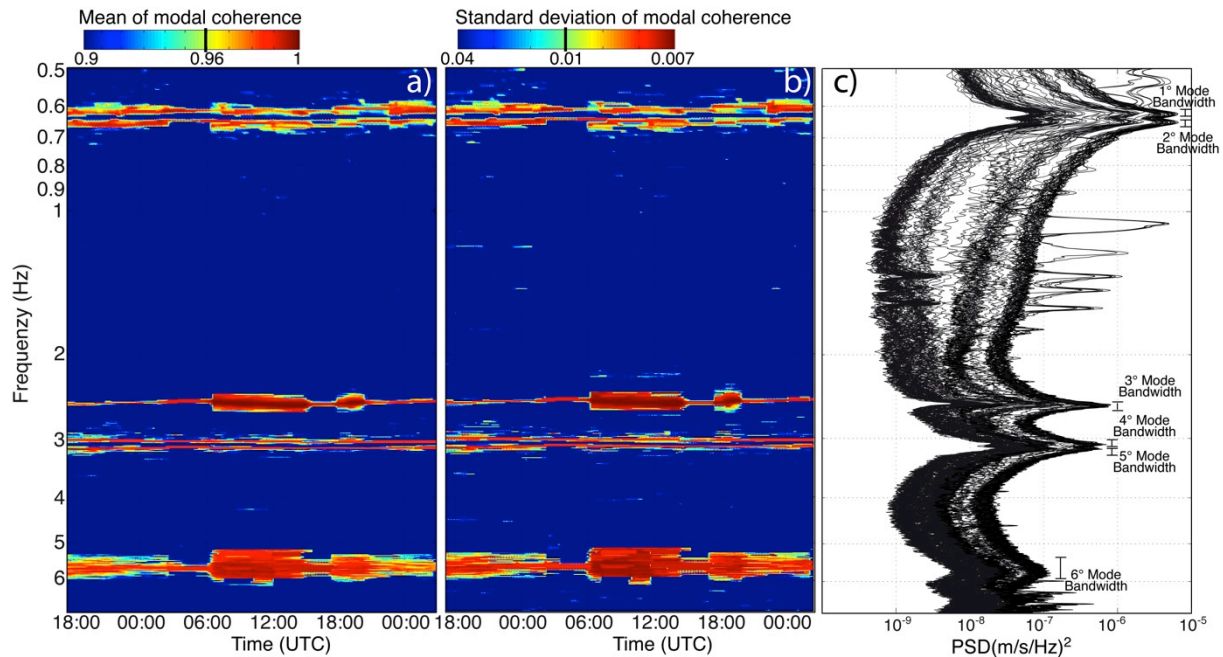


Figure 2. The automatic EFDD technique is based on detecting the modal bandwidth using the mean a) and the standard deviation b) of the modal coherence calculated during the seismic survey on 10 consecutive records, the white lines on the both colour bars represent the thresholds >0.96 and <0.01 used for the identification of mode bandwidth for mean a) and standard deviation of modal coherence b), respectively. c) first singular values estimated for all the observation time (36 hours).

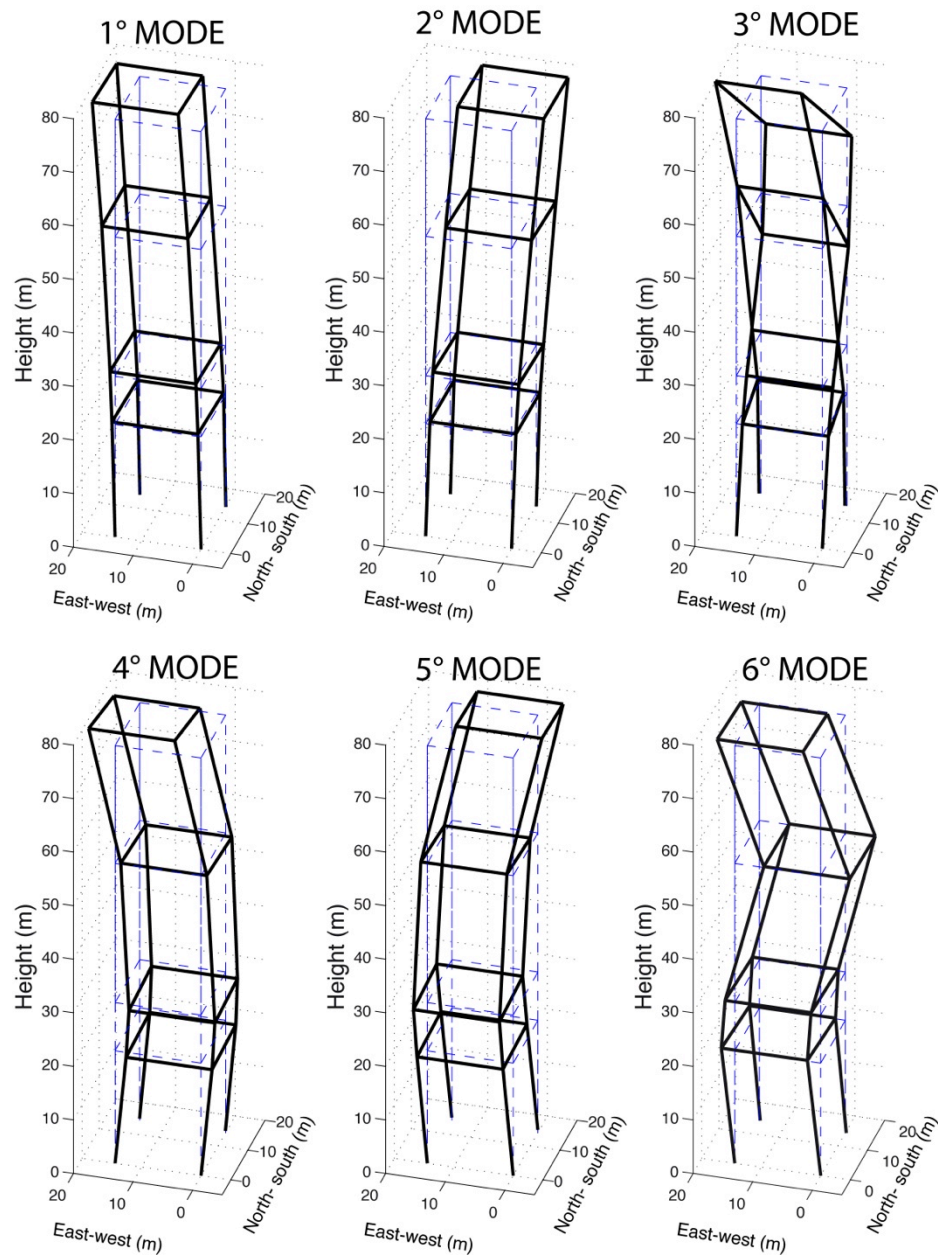


Figure 3. Three-dimensional representation of the six modes

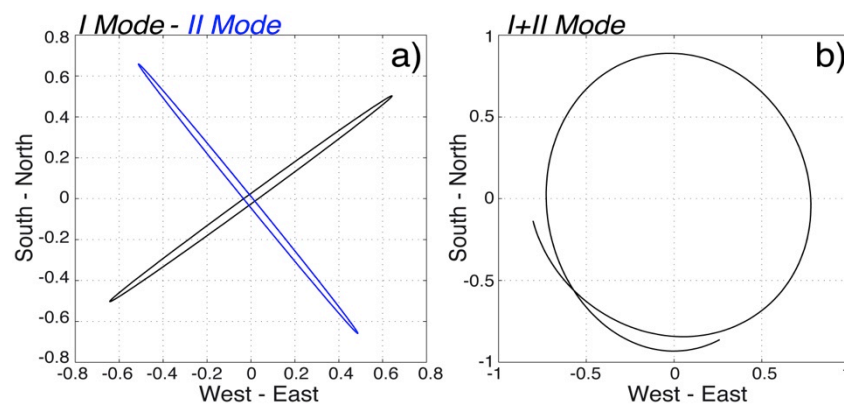


Figure 4. a) Particle motion evaluated at C10 station of the first mode (black) and second mode (blue) b) Vector sum of the two first modes produces a circular movement slightly eccentric in the N-S direction

4 SEISMIC INTERFEROMETRY

The building impulse response function (IRF) is estimated using seismic interferometry, which allows us to analyze the spatial distribution of the seismic waves velocity along the building. Seismic interferometry is based on the deconvolution $D(\omega)$ of the signal $u_{ref}(t)$ recorded at a reference station with the signal $u_i(t)$ recorded at a generic station (i), and it is represented in the frequency domain by [16, 17, 18]:

$$D(\omega) = \frac{u_i(\omega)u_{ref}^*(\omega)}{|u_{ref}(\omega)|^2 + \varepsilon} \quad (1)$$

where the asterisk denotes the complex conjugation and the parameter ε is set to 0.5 % of the mean amplitude $u_{ref}(\omega)$ spectral power. Deconvolution was applied to ambient noise using as the reference station the signal recorded at the top floor (C10 and C09 stations, in Figure 1) and then the resulting waveforms were filtered in broad band frequency of 0.2 – 20 Hz and stacked over the 1800 seconds intervals. The result is an up-going and down-going wave propagating through the building (Figure 5). The IRFs stacked every 1800 seconds, showing high signal-to-noise ratio (Figure 5) for the whole 36 hours long period of observation and indicates that this time period is long enough for interferometry analysis.

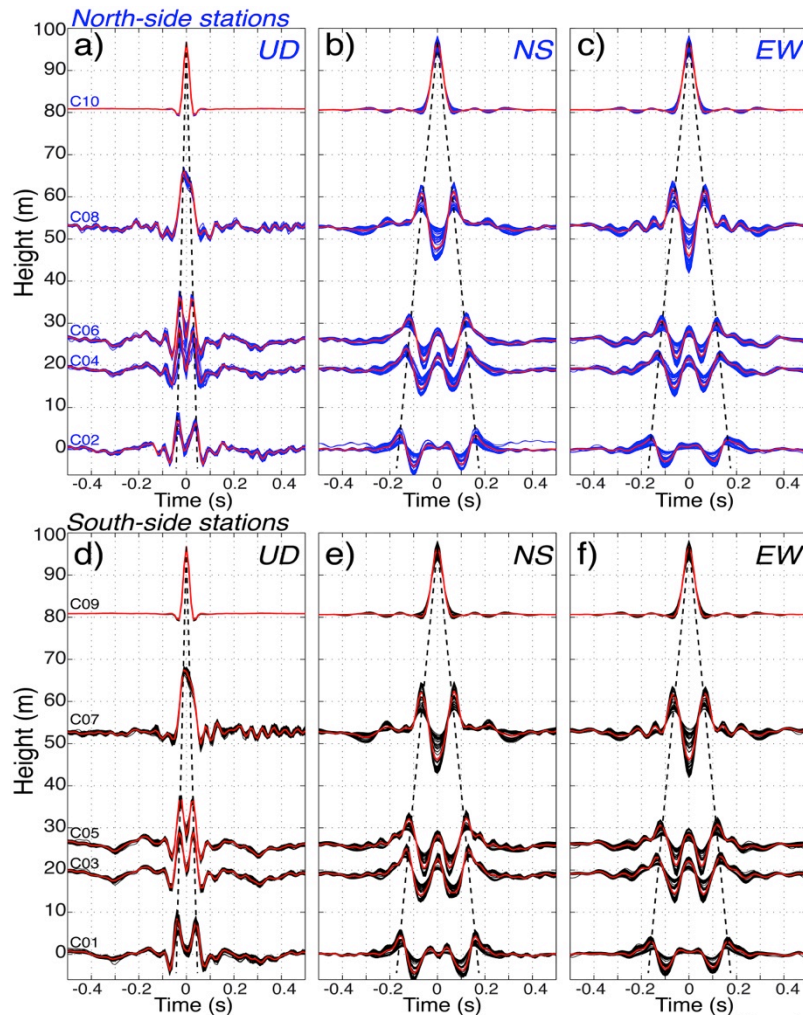


Figure 5. Stacking waveforms over 1800 seconds interval of the IRFs deconvolved with the motion on the top floor calculated for all the observation time (36 hours) and filtered in broad band frequency of 0.2 – 20 Hz, a,d)

UD component, b,e) NS component and c,f) EW component for North side (upper panel) and South Side (lower panel) stations

To estimate the three dimensional (Up-Down, NS, EW) propagation of the seismic waves, reference was made to the peak amplitude of the down-going wave.

For each waveform deconvolved, we detect automatically the arrival time of the peak amplitude at each floor and using a least squares fit we calculate the velocity of the seismic waves from the slope of the travel time (Figure 6).

This procedure is performed for the three components (Figure 6). The given errors are estimated considering the elevation of the stations to be error free and only errors in the arrival times are assumed [18].

The estimated compressional P-wave velocity is $V_p = 2040 \pm 40$ m/s with an error of $\pm 6.2\%$; the horizontal component was used to estimate the shear S-wave velocities V_s that are the same in both horizontal components of motion ($V_{NS} = 478 \pm 29$ m/s and $V_{EW} = 478 \pm 29$ m/s) with an error of $\pm 2\%$.

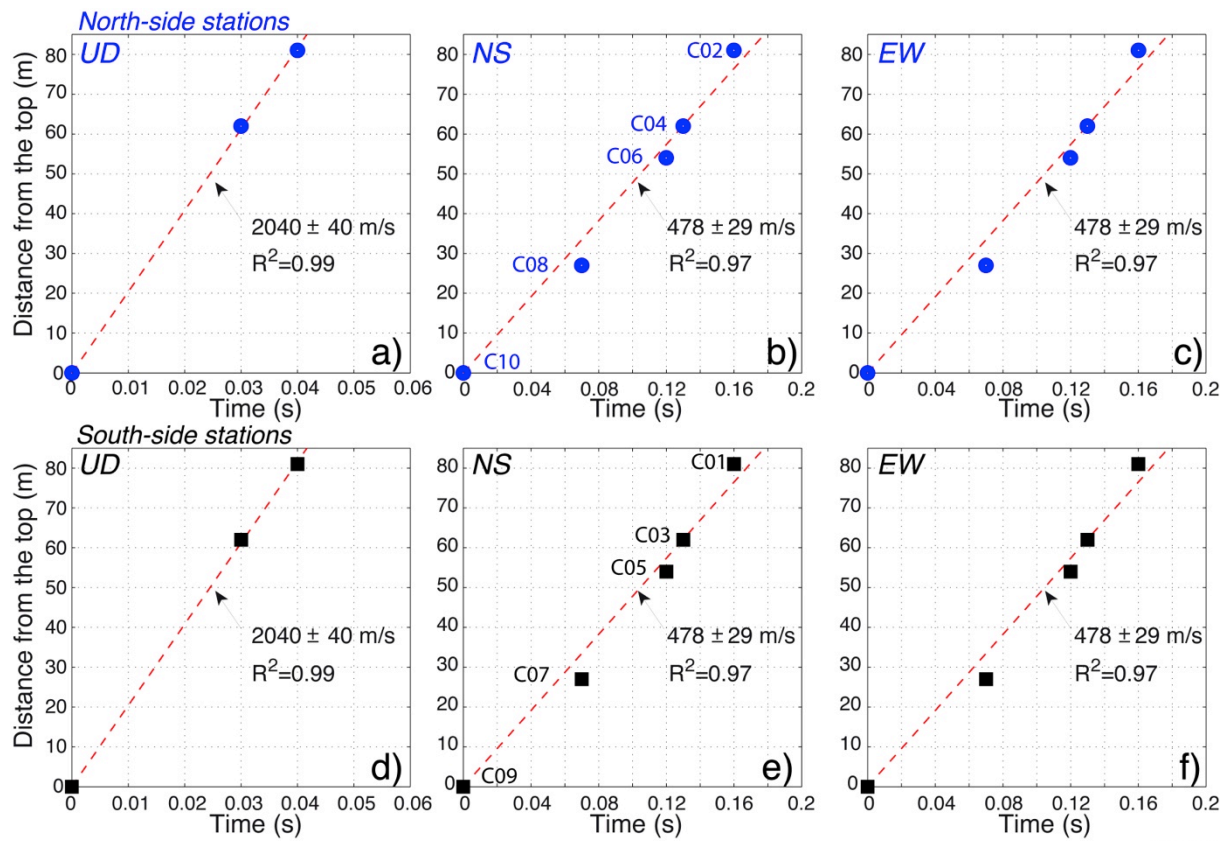


Figure 6. Travel time at different floors versus distance to the roof, considering a,d) the UD component, b,e) NS component and c,f) EW component evaluated from the total IRF stacking (red lines in Figure 7). The velocity is estimated using a least squares fit for the stations located to the North side (upper panel) and to the South side (lower panel). The best fit solution is shown for each component including the error.

5 SOIL-STRUCTURE SYSTEM RESPONSE

As already mentioned, in recent year the dynamic soil-structure interaction has been investigated using earthquakes data [21, 22] or ambient vibration [23] throughout seismic interferometry analysis. In particular, several studies have demonstrated how the change of the modal frequencies may reflect changes in the dynamic properties of the building [24] and in the soil-structure interaction effects [21, 22]. [25]. Luco et al. 1988 proposed to relate the fundamental modal frequency of the building to the interaction soil-structure as follows:

$$\frac{1}{f_{1,sys}^2} \approx \frac{1}{f_H^2} + \frac{1}{f_R^2} + \frac{1}{f_1^2} \quad (2)$$

where $f_{1,sys}$ is the frequency of the first mode (Figure 2c) related to the behavior of the structure interacting with the supporting soil, f_H and f_R are the horizontal and rocking frequencies of a rigid body connected to the deformable soil and f_1 is the fixed-base frequency of a deformable structure.

Quite often, and this was also the case for the Giotto's bell tower, the contribution of the horizontal motion frequency can be neglected, therefore

$$\frac{1}{f_{1,app}^2} \approx \frac{1}{f_R^2} + \frac{1}{f_1^2} \quad (3)$$

In our study, we evaluated what is often called in literature the apparent frequency $f_{l,app}$ [20] as the peak frequency of the transfer function obtained by deconvolution (Equation 1), using as reference, the signal recorded at the stations located on the ground level of Giotto's bell tower (C01 and C02 for the North Side and South Side configuration, respectively). We deconvolved each 120 s ambient vibration and then stack the transfer functions over the entire seismic surveys (Figure 7).

The both seismic configurations North – Side and South-Side stations show the same first apparent frequency of $f_{l,app} = 0.65$ Hz. This value takes into account the rocking contribution of the soil and the flexibility of the structure.

On the other end, the fixed-base frequency of the flexible structure can be estimated by using the basic solution:

$$f_1 = 0.56 \frac{r_i}{h^2} \sqrt{\frac{E}{\rho}} \quad (4)$$

where $r_i = 4.26$ m is the inertia radius of the cross-section, $\sqrt{E/\rho}$ is the compressional P-wave velocity (Figure 6a,b) and h is the elevation of the building, so that equation (4) gives a fixed-base frequency $f_1 = 0.70$ Hz.

Once the first apparent system frequency has been estimated from Fourier analysis, giving a value of 0.623 Hz, and the f_1 has been determined from wave travel time and geometrical characteristics of the structure, the rocking frequency from equation 3 is $f_R = 1.36$ Hz.

These values tell us that in this case the contribution of soil-structure interaction reduces the fixed base frequency by 10%, as it could be expected by considering that the supported soil is characterized by a rather high shear wave velocity of about 380 m/s.

Furthermore, the value of $f_R = 1.36$ Hz can be conveniently used to validate the analytical relationship used to compute the rocking stiffness ($\tilde{K}_r = 2278 \text{ GN} \cdot \text{m} / \text{rad}$) to be used in advanced seismic analyses.

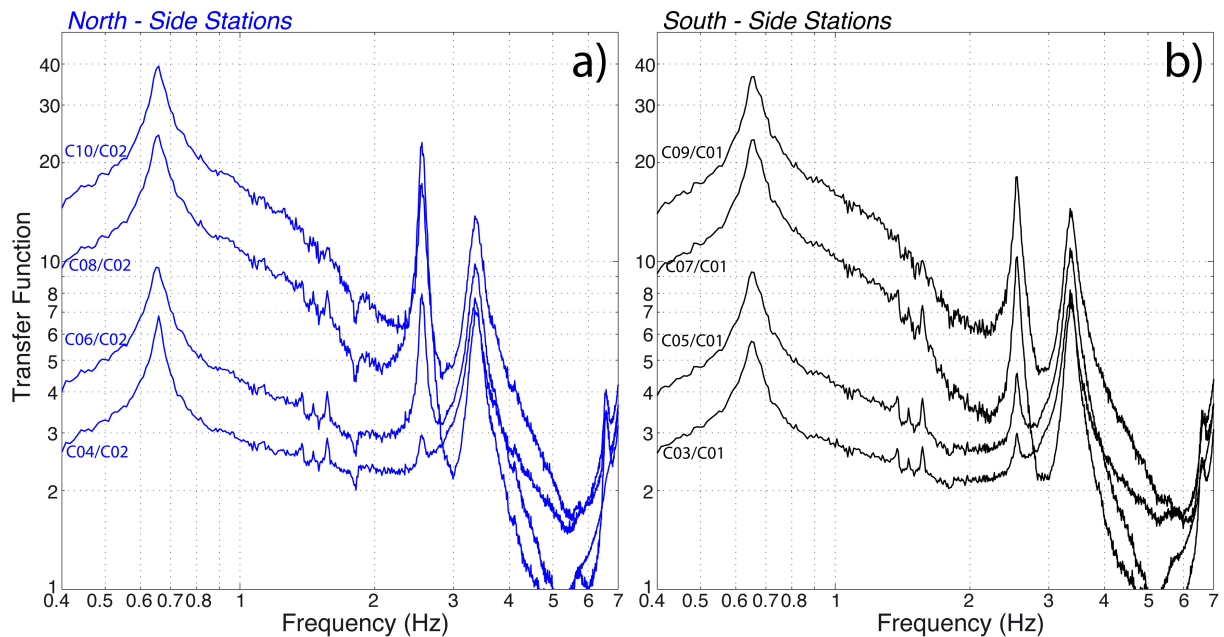


Figure 7. Transfer function between the roof response and the response at ground level. a) North side stations, b) South side stations.

6 FINAL REMARKS

The dynamic response of the Giotto's bell Tower was investigated using ambient noise vibration. Operational modal analysis (OMA) indicates that the first two modes, corresponding to 0.623 Hz and 0.646 Hz respectively, are coupled bending modes. The third (2.543 Hz), fourth (3.081 Hz), fifth (3.156 Hz) and sixth (5.731 Hz), modal shapes are well separated and correspond to torsional and higher bending modes.

The ambient noise was also used to measure the compression wave velocity $V_p = 2040$ m/s propagating along the axis of the bell tower, and this allowed to compute the fixed-base frequency $f_l = 0.70$ Hz.

By using equation (3) the rocking frequency ($f_R = 1.36$ Hz) of the tower, as a rigid body connected to a rocking spring, was isolated and the obtained value indicates the contribution of the soil-structure interaction to the dynamic behavior of the tower. This contribution gives a fundamental frequency that is lower by about 10% if compared with the one corresponding to a base-fixed structure.

However, it must be recognized that these values refer to small amplitude vibration and that the effect of the interaction with the supporting soil may be more pronounced when considering vibrations induced by more severe seismic events.

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