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UNCERTAINTY OF PROPERTIES AND FAILURE LOAD IN COMPOSITE MATERIALS

Piotr Kędziora

Cracow University of Technology
31-155 Kraków, ul. Warszawska 24, Poland
e-mail: piotr.kedziora@mech.pk.edu.pl

Abstract

The impreciseness (uncertainty) of the experimentally obtained data can be described using fuzzy sets theory. It is proposed the fuzzy set method for the correct and concise evaluation of various geometrical and mechanical properties or final failure load. The variability (understood as the fuzziness) of the fiber and matrix properties is described by a membership function. Based on experimental results it is possible to determine the effect of the fuzziness of the mechanical properties. The fuzzy sets theory allows to build the lower and upper bounds of the failure envelopes for the Tsai-Wu criterion. The linear qualifier function is applied to analyze the composite structure subjected to the load.

Keywords: Composite Materials, Fuzzy Set, Fuzzy Logic, Material Uncertainty.

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1 INTRODUCTION

The use of fiber-reinforced composite materials in modern engineering structural design has become a common practice. However, since more design variables typically exist when composite materials are employed and the manufacturing processes for producing composites are more complex, more variability can exist in a design procedures with composites compared to conventional (isotropic) materials [1]. Thus, the variability of properties that occurs in composite structures leads directly to a random field of variables describing constructions. A scatter of properties has a different origin including geometrical and mechanical properties, environmental effects and the influence of the applied technology or existence of the internal defects of flaws, see e.g. papers [2-5]. Therefore, there is a fundamental question: how many and which of the above factors can (or should) be incorporated in the design process and how can we manage to take into account the existing variability of parameters. A broader discussion of those problems can be found in reference [6].

The majority of available references in literature discussing design problems of composite structures is devoted to the analysis conducted in a pure deterministic way. However, the random field of parameters describing composites may be taken into account by the standard use of different variants of the statistical analysis or by the application of non-stochastic methods – a fuzzy set approach.

Referring to composite materials the origin and the source of imprecision (or uncertainty) lies mainly in the lack of information dealing with their microstructure, mechanical properties, behavior and the number of factors responsible for gradual degradation of their properties and final failure. Commonly, the theoretical (deterministic) analysis of composites is based on the homogenization theories that may include an increasing number of different parameters [7, 8]. However, it is unknown in advance what number of parameters is sufficient to describe satisfactorily the problem considered. On the other hand, the material parameters are evaluated in the experimental way being the source of randomness in the traditional (deterministic) analysis or impreciseness or vagueness in the fuzzy set approach. The imprecise, vague, qualitative, linguistic or incomplete information may be present in geometry, material properties, degradation of properties, applied loads or boundary conditions.

Different kinds of methods are investigated for different types of uncertainties. Oskay and Fish [9] calibrated material properties in a deterministic fashion with the aid of genetic algorithms and gradient-based techniques. Jiang et al. [10] proposed the deterministic model to identify the uncertain elastic modulus of braided composites using modal data. Mustafa et al. [11] presented a probabilistic model for estimating the fatigue life of composite laminates using a high fidelity, multi-scale approach called M-LaF (Micromechanics based approach for Fatigue Life Failure). They developed a unified framework for the representation and quantification of uncertainty present in the fiber and matrix properties with the use of the Bayesian inference approach in order to calculate probabilistic composite fatigue failure. Chandrashekhar and Ganguli [12] performed probabilistic analysis using the Monte Carlo Simulation on a refined composite plate finite element model to calculate the statistical properties of the variation in modal parameters of a cantilever composite plate due to structural damage and material uncertainty.

If the system parameters are described in imprecise or linguistic terms the fuzzy theory can be implemented to predict the structural response in the sense of evaluation of its upper and lower bounds, respectively. Various problems connected with fundamentals and concepts used in the fuzzy set theory are presented in a few representative monographs: McNeill and Thro [13], Cox [14], Tsoukalas and Uhrig [15], Dubois and Prade [16], Zadeh et al. [17], Kosko [18].

Fuzzy concepts are used in various areas of mechanics. Moens and Hanss [19] presented an overview of the research activities on non-probabilistic finite element analysis and its application in the representation of parametric uncertainty in applied mechanics. Sodoke et al. [20] used a fuzzy logic model to obtain a simplified view of linguistic variables representing the modulus of elasticity and to reconcile different modules by including the uncertainty inherent to the different measuring techniques. Altmann et al. [21] introduced a fuzzy-probabilistic approach to assess the durability of strain-hardening cement-based composites. Karbhari and Stein [22] described the application of a fuzzy probabilistic approach to reflect the impact of the inherent uncertainty in determining the reinforcing fibers of polymer jackets for the seismic retrofit of columns. Bohlooli et al. [23] presented a suitable model based on fuzzy logic to predict the compressive strength of inorganic polymers with seeded fly ash and rice husk bark ash. Syamsiah et al. [24] proposed the role of input selection using a neuro-fuzzy approach to predict the physical properties of degradable composites, namely the melt flow index and density. Rassbach and Lehnert [25] developed a model based on the mathematical methods of fuzzy-logic which can describe the plasto-mechanical behavior of functionally gradient materials during the deformation process. Muc and Kędziora [26] employed a fuzzy-set based approach in conjunction with a finite element analysis and fracture mechanics. In the numerical analysis four parameters were considered to be fuzzy, namely, three mechanical parameters (Young's moduli, Kirchhoff's modulus) and one geometrical parameter describing the position of the crack center. Kędziora and Muc [27-29] proposed the fuzzy set analysis in order to estimate the uncertainty in the evaluation of critical number of cycles corresponding to the final fatigue damage. Experimental data obtained during fatigue tests conducted for tension and compression were represented by the lower and upper bounds of the stiffness degradation, i.e., as stiffness versus the number of cycles relationships [28, 29].

Therefore, in this way the impreciseness (uncertainty) of the experimentally obtained data can be described using fuzzy sets theory. In this paper, it is proposed the fuzzy set method for the correct and concise evaluation of various mechanical properties or final failure load. The variability (understood as the fuzziness) of the fiber and matrix properties is described by a membership function. Based on experimental results it is possible to determine the effect of the fuzziness of the mechanical properties. The fuzzy sets theory allows to build the lower and upper bounds of the failure envelopes for the Tsai-Wu criterion. The linear qualifier function is applied to analyze the composite structure subjected to the load.

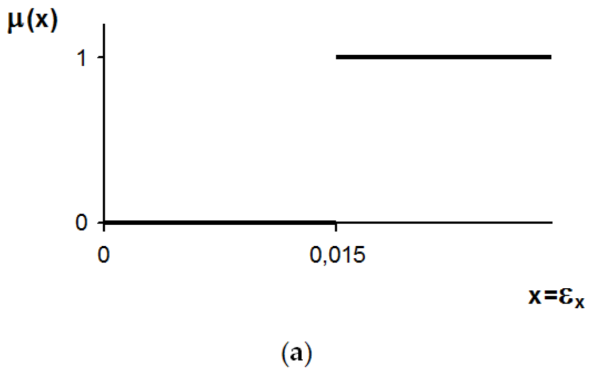
2 FUNDAMENTAL DEFINITIONS

2.1 Representations of Fuzzy Sets

The data "integer less than 10" is the definition of characteristic function Π_A and is represented in the following way:

$$\Pi_A : IN \to \{0,1\}$$
$$\Pi_A(\eta) = \begin{cases} 1, & \text{if less than 10} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

that yields a value 1 for each element of space IN that belongs to set A and a value 0 for each element that does not. The above representation is commonly called a crisp set. However, this concept cannot be used directly as we intend to characterize the typical property for composite materials, e.g. the failure of carbon fiber reinforced polymer under tension occurring as tensile strain ε_x is equal to the ultimate value of 0.015. The characteristic function of this set is depicted in Figure 1a. For the three-dimensional analysis, all the components of the strain tensor can be evaluated in a similar manner. A problem arises if the linguistic term “failure under tension” has to be described. The value of a failure strain in composites depends on various parameters such as the fiber and matrix materials, loading conditions, porosity, environmental effects, etc. It is well-known that from the micromechanical point of view that failure starts from microcracks in the matrix for strain values much lower than the value 0.015. In addition, the value 0.015 is usually an average value characterizing rather a scatter of a random values of macrocracks appearing at the strain level 0.015. Therefore, for some specimens one can observe the final (macroscale) failure as ε_x is equal to 0.0159 or to 0.0141. A possible solution to this problem is to generalize the definition of the characteristic function in a way that it yields values from interval [0, 1] and not just the two values of the {0, 1}. This leads to the notion of a fuzzy set.



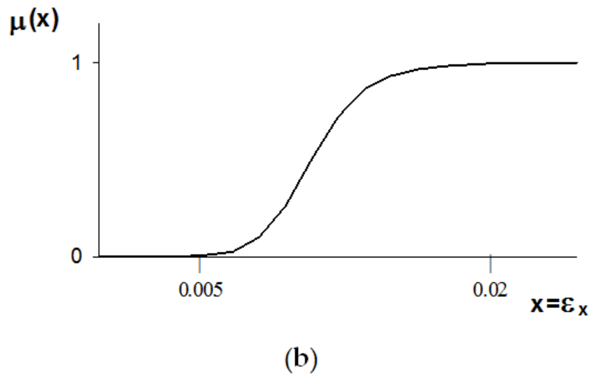


Figure 1: Representation of term “failure under tension”: (a) crisp set; (b) fuzzy set.

Fuzzy set μ of X is a function that maps space X into the unit interval, i.e.:

$$\mu : X \to [0,1] \tag{2}$$

Value μ(x) denotes the membership function of x to fuzzy set μ. Figure 1b shows (subjectively defined) a membership function of fuzzy set μ describing the linguistic meaning of the term “failure under tension”. The use of fuzzy sets to formally represent vague data is often done in an intuitive way because in many applications there is no model that provides a clear interpretation of the membership degrees, though we want or we try to base it on various experimental data.

The application of fuzzy methodologies requires the knowledge of the membership functions of fuzzy quantities. In general, fuzzy numbers are sets that represent numeric quantity. It can be done in a variety of ways. Of course, there are different possibilities to determine and represent membership functions characterizing a fuzzy set. If subspace S contains only a finite number of elements, a fuzzy set μ of X will be defined by specifying for each element $x \in X$ its membership degree μ(x). If the number of elements is very large or a continuum is chosen for X, then μ(x) can be better defined by a function that can use parameters which are adapted to the actual modeling problem. For instance, if we want to represent the term “Young’s modulus is equal to 200 GPa” in the sense of a fuzzy set having a finite amount of experimental data, we can select one of the different representations given in Figure 2.

The determination of the membership functions is difficult. There are different forms of them and in the description of the fuzzy numbers the triangular, trapezoidal, Π, B, Gaussian or Weibull curves are commonly used, whereas in the case of fuzzy qualifiers – the sigmodal or linear curves – see reference [3]. However, the first attempt or trial in building the membership functions can be based on the statistical data – see e.g. the description of failure.

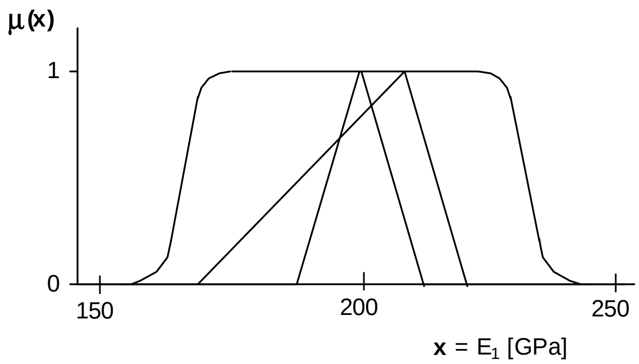


Figure 2: The used descriptions of uncertainty.

One of the possible fuzzy set representations was presented there. Another approach is the so-called horizontal representation of fuzzy sets. This is introduced by using their α-cuts instead of membership functions μ(x) which are called vertical representations.

Let $\mu \in F(x)$ and $\alpha \in [0, 1]$. The set is called the α-cuts of μ:

$$[\mu]_\alpha = \{x \in X | \mu(x) \geq \alpha\} \tag{3}$$

Let μ be the triangular function on IR given in Figure 3. The α-cuts of μ are in this case defined as follows:

$$[\mu]_\alpha = \begin{cases} [a+\alpha \cdot (m-a), b-a \cdot (b-m)] & \text{if } 0 < \alpha \leq 1 \\ IR & \text{if } \alpha = 0 \end{cases} \tag{4}$$

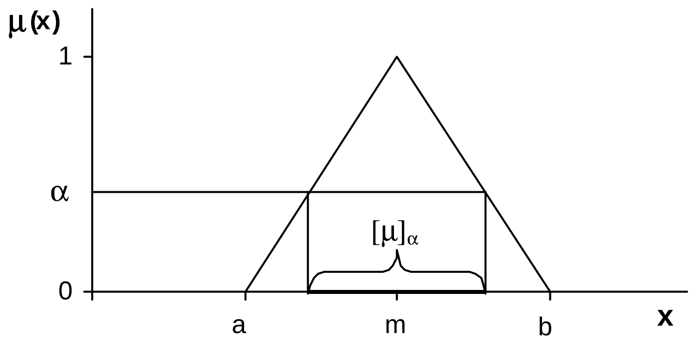


Figure 3: Definition of α-cuts.

There are, generally, two kinds of fuzzy sets:

- numbers that represent an approximate numeric quantity; those values can represent uncertainty (impreciseness, fuzziness) of Young's (see Figure 2) or Kirchhoff's moduli, geometric properties of structures (dimensions, fiber orientations) or allowable strengths/strains in appropriate directions;

- qualifiers that characterize open-ended concepts; these sets provide the framework for describing unbounded concepts (or concepts that are theoretically unbounded), e.g. qualifiers of failure or constraints in various optimization problems – see e.g. Rao [30].

2.2 Vertex Method

Let us introduce N fuzzy parameters describing the material or geometric parameters of the considered composite structure. The membership functions are discretized using several α-cuts – Equation (4). Considering the left and right end points of the α-cut s intervals $[\mu]_\alpha$ (see Figure 3) for all the fuzzy parameters, one can find the total number of combinations $N_{c/\alpha}$ per α-cut in the following form:

$$N_{c/\alpha} = \begin{cases} 2^N & for\ 0 \le \alpha < 1 \\ 1 & \text{for } \alpha = 1 \end{cases} \tag{5}$$

An output response denoted by p is an unknown function of input fuzzy parameters x_i (i=1, 2, …, N), so that:

$$p = f(x_1, ..., x_N) \tag{6}$$

Using the α-cut concept combined with binary representation (5) of fuzzy parameters x_i (i=1,2,…,N) relation (6) can be rewritten in the abbreviated form:

$$p = f(C_{\lambda,j}); \quad \mathrm{j} = 1, 2, ..., \mathrm{N}_{c/\alpha} \tag{7}$$

Since output response p as a function of fuzzy parameters is a fuzzy set, the corresponding interval in p is obtained from relation (6):

$$[\mathrm{p}_\alpha^\mathrm{L}, \mathrm{p}_\alpha^\mathrm{R}] = [\min_{\lambda,\mathrm{j}} \mathrm{f}(\mathrm{C}_{\lambda,\mathrm{j}}), \max_{\lambda,\mathrm{j}} \mathrm{f}(\mathrm{C}_{\lambda,\mathrm{j}}); \quad \lambda \ge \alpha, \ \mathrm{j} = 1, 2, ..., \mathrm{N}_{c/\alpha} \tag{8}$$

As may be seen, relation (8) allows one to obtain a scatter of the output parameters and then to build the appropriate probability distributions and reliability functions by a sweep of α-cut at different possibility levels.

In order to conduct the computations and to evaluate the upper and lower boundaries of output response (8), it is necessary to outline the deterministic method of the definition of function f given in Equation (6). It can be defined in a purely analytical way or alternatively in a purely numerical way. As may be noticed, the vertex method resembles here the Monte Carlo simulation method where the output response also has a deterministic, and therefore unique form.

Function f existing in Equation (6) may describe an arbitrary failure criterion for composites, e.g. buckling, delamination, first-ply-failure (FPF) etc., whereas symbol p denotes the corresponding value of the failure load.

3 FUZZY SET ANALYSIS

The origin and source of imprecision (or uncertainty) can result from the lack of information dealing with their microstructure, mechanical properties, behavior and the number of factors responsible for gradual degradation of their properties and final failure of composite materials.

3.1 Mechanical Properties

Commonly, the theoretical (deterministic) analysis of composites is based on homogenization theories. In the micromechanics analysis the development of the model of repeating unit cells is based on the assumption that the unidirectional composite is represented by fibers which are aligned in the x_1–direction and distributed regularly in the matrix and finally the components form a doubly periodic array in the x_2 and x_3 directions – see Figure 4. In Figure

4 the fibers have a circular cross-section and the same diameter d and are arranged at distances h, l apart. The unit cell is showed in Figure 4b.

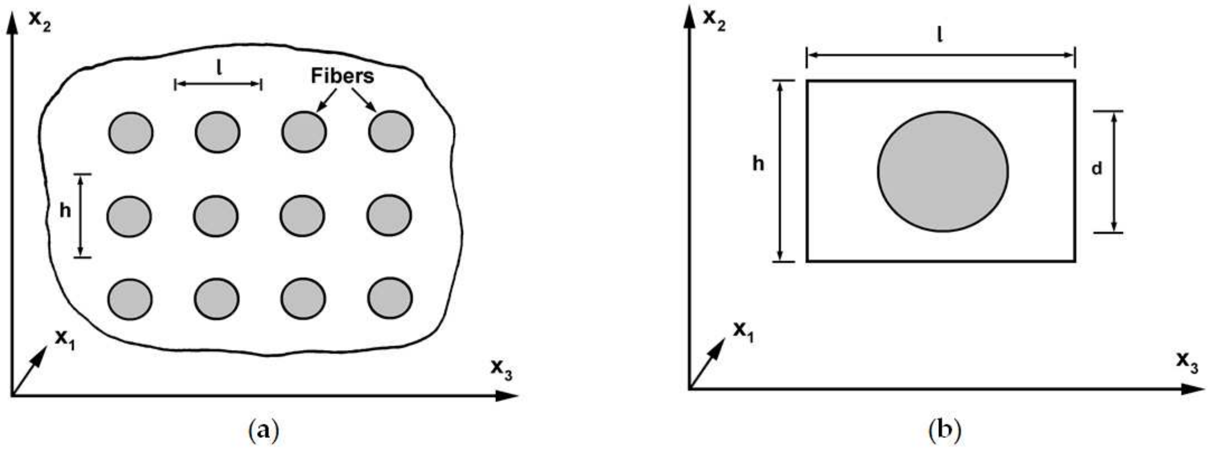


Figure 4: Micromechanics unit cell geometry: (a) periodic array of circular fibers extending in the x_1 direction; (b) representative cell.

At the micromechanics level, the transversely isotropic material is expressed in terms of the five engineering constants. They are functions of the material variables, i.e. of the fiber and the matrix properties E_{f1}, E_{f2}, G_{f12}, ν_{f12}, ν_{f13}, E_m, ν_m, V_f, where f denotes fiber property, m – a matrix property, E – the Young's modulus, G – the Kirchhoff's modulus, ν – Poisson's ratio and V_f is the fiber volume fraction (assuming $V_f + V_m = 1$, where V_m is the matrix volume fraction).

The impreciseness in the modeling is obvious: the ideal forms of the constituents and of representative cells, an influence of the interface is completely eliminated – there are two components in the model fibers and matrices, the matrix and fibers have constant values (there is no defects inside them) etc.

Now, let us assume that the variables characterizing the fiber and matrix properties are fuzzy variables and their fuzziness is described by a triangular membership function – see Figure 3. It is assumed that the variability (understood as the fuzziness) of material parameters is taken to be equal to ±10 %. The nominal (average) values correspond to α=1 (see Figure 3) whereas the parameters of the triangular membership function given by Equation (4) are following :

$$m = \frac{a+b}{2}, \quad a = 0.9 \cdot m \quad b = 1.1 \cdot m \tag{9}$$

Having the Halpin–Tsai model equations of the relation (6) and using the procedure proposed in the Section 2 for a given α-cut one can evaluate the upper (the right end point of the interval (8)) and lower bounds (the left end point of the interval (8)) of the effective longitudinal and transverse Young moduli. The results are evaluated for the unidirectional long glass fibers/epoxy resin, i.e. the nominal properties (denoted by m in relation (10) and Figure 3) E_{f1}=73GPa, E_m=3.5GPa from [31]. The results of computations are plotted in Figure 5. As it may be seen, the nominal curve denoted by the value α=1 (the solid red line in Figure 5) is obtained for nominal values of E_{f1} and E_m. The lower and upper bounds of the curves denoted by the value α=0 plotted in Figure 5 (denoted by the superscripts L and R), are evaluated with the use of Equation (8). As it may be observed the biggest effect of fuzziness is for the maximal values of the fiber volume fraction.

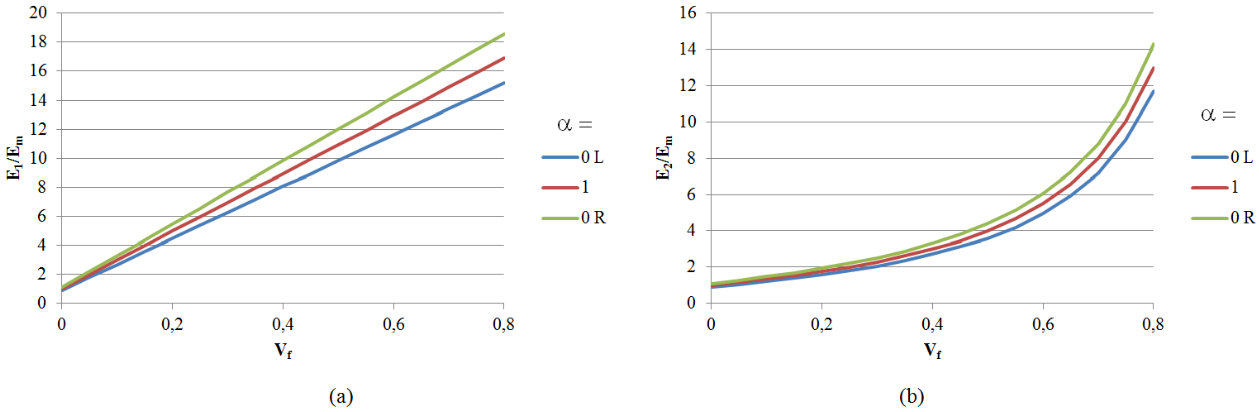


Figure 5: Distributions of the effective longitudinal E_1 (a) and transverse E_2 (b) Young moduli at α= 0 and 1 (L – lower bound, R – upper bound).

3.2 First-ply-failure

One of failure criterion in order to consider these uncertain material properties is applied to the calculation. For example, uncertainties in elastic moduli of laminates are due to several factors, such as e.g. the misalignment of fibers or imperfect bonding between fibers and matrix.

The classical first-ply-failure (FPF) envelopes (e.g. the Tsai-Wu criterion) are described with the aid of variety of material constants characterizing different modes of failures (tension, compression etc.). Those values may be treated as fuzzy ones and in this way the fuzzy set theory allows us to build the lower and upper bounds of the failure envelopes for each fiber orientations of plies in the laminate. Assuming that five strength constants are fuzzy parameters such envelopes have been built and the plot is presented in Figure 6. For compressive deformations the uncertainty of failure locations (at plies having fibers oriented at 0° or 45°) is the characteristic feature of the resulting curves.

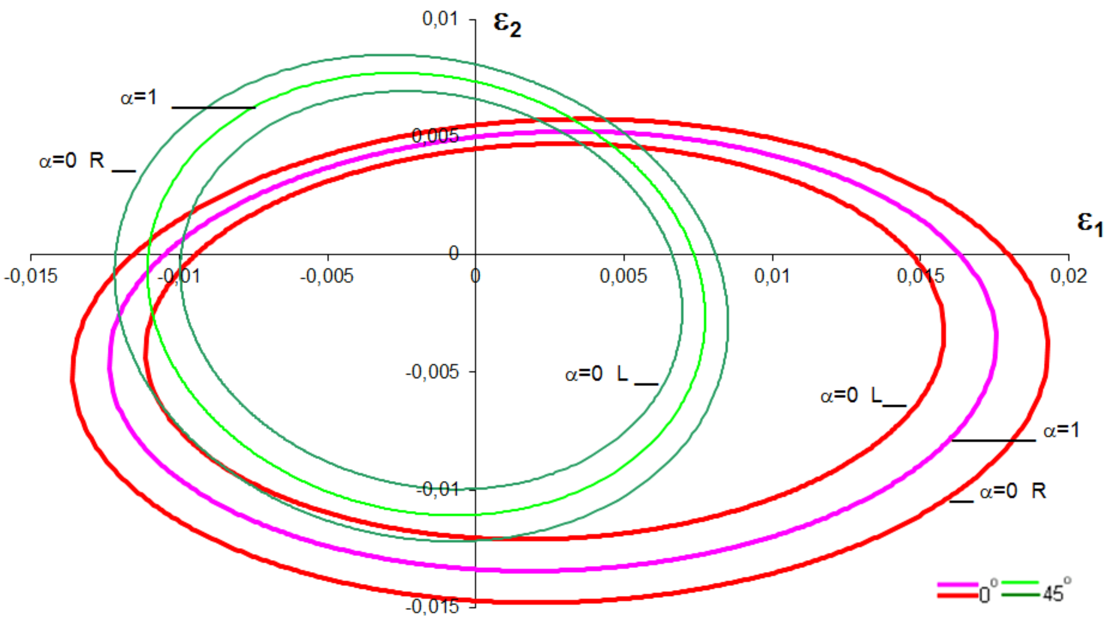


Figure 6: Upper and lower bounds of FPF envelopes – the Tsai-Wu criterion (plies having fibers oriented at 0° or 45°; L – lower bound, R – upper bound).

The results demonstrate evidently that the classical deterministic approach to the FPF analysis can lead to the inconsistency and incorrectness with experimental data. For compression in both directions it is possible to obtain experimentally the FPF initiation at plies oriented both at 0° and 45°. They show simply the limitations of the classical global elastic approach since the matrix cracking associated with the FPF is connected with inelastic deformations (in the global sense), especially for fibers oriented at 45° or with the appearance of intralaminar cracks (a micromechanical approach).

3.3 Fuzzy Constrains

The deformation of composite structure subjected to in-plane and/or bending loads can be considered using fuzzy maximum strain criteria. Fuzzy constrains are defined by maximum strain criteria in the following way:

$$\begin{gathered}\varepsilon_{1c}^{ult}-\Delta\varepsilon_{1c}\le\varepsilon_1^{\pm}(\theta_k)\le\varepsilon_{1t}^{ult}+\Delta\varepsilon_{1t}\\ \varepsilon_{2c}^{ult}-\Delta\varepsilon_{2c}\le\varepsilon_2^{\pm}(\theta_k)\le\varepsilon_{2t}^{ult}+\Delta\varepsilon_{2t}\\ \left|\varepsilon_6^{\pm}(\theta_k)\right|\le\varepsilon_{6t}^{ult}+\Delta\varepsilon_{6t}\end{gathered}\tag{10}$$

where ± denotes upper and lower surface of k-th layer, respectively. t designates tension and c – compression. θ denotes fiber orientation. Ultimate strain strengths (maximum) ε^{ult} are dealt with as fuzzy numbers by using $\Delta\varepsilon$ and β in the following manner:

$$\Delta\varepsilon_{ij}=\beta\cdot\varepsilon_{ij}^{ult}\quad i=1,2,6\quad j=t,c\tag{11}$$

The value of the parameter β denotes the level of fuzziness in the problem. When β is equal to 0% it denotes a crisp optimization problem. The degree of satisfaction of constraint is determined using the membership functions $\mu_\varepsilon(\varepsilon(\theta))$ (see reference [3]) which are represented by linear qualifier function in the form of asymmetrically trapezium. For the positive value of strain $\varepsilon(\theta)$, the membership functions $\mu_\varepsilon(\varepsilon(\theta))$ is given by

$$\mu_{\varepsilon_t}(\varepsilon(\theta_k))=\begin{cases}1 & \varepsilon(\theta_k)<\varepsilon_t^{ult}-\Delta\varepsilon_t\\ \alpha_t & \varepsilon_t^{ult}-\Delta\varepsilon_t\le\varepsilon(\theta_k)\le\varepsilon_t^{ult}+\Delta\varepsilon_t\\ 0 & \varepsilon(\theta_k)>\varepsilon_t^{ult}+\Delta\varepsilon_t\end{cases}\tag{12}$$

where

$$\alpha_t=1-\frac{\varepsilon(\theta_k)-\left(\varepsilon_t^{ult}-\Delta\varepsilon_t\right)}{\varepsilon_t^{ult}+\Delta\varepsilon_t-\left(\varepsilon_t^{ult}-\Delta\varepsilon_t\right)}\in[0,1]\tag{13}$$

and presented in Figure 7a. For the negative value of strain $\varepsilon(\theta)$, the membership functions $\mu_\varepsilon(\varepsilon(\theta))$ is showed in Figure 7b.

The maximum stress criterion has a similar form. Then, in Equations (10)-(13), the notation σ is used in place of ε.

CONCLUSIONS

The present study is a practical tool for engineering activities dealing with evaluating the degradation of real material structure. As it remains an open problem, it is connected with the total number of uncertain parameters that should be considered in order to describe the real behavior of engineering structures with an acceptable accuracy. However, it can be solved for each individual problem only.

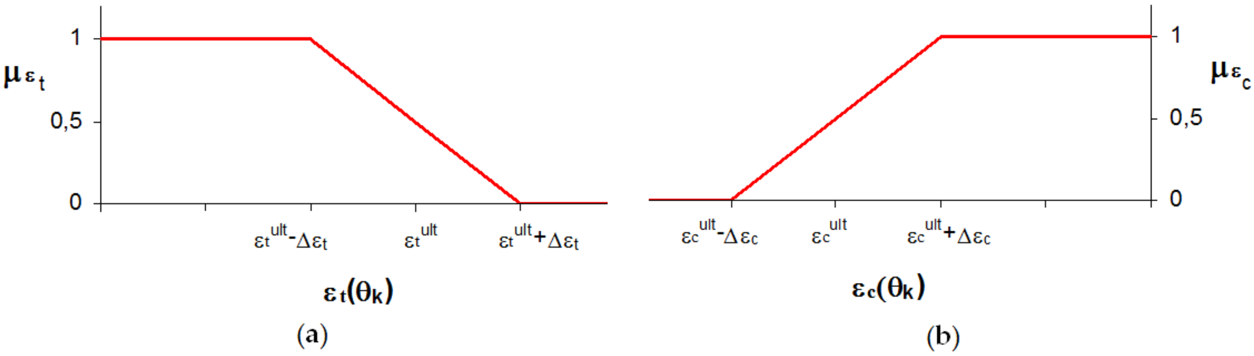


Figure 7: The membership functions for: (a) the positive value of strain; (b) the negative value of strain.

The presented method allows one to build the appropriate membership functions for the evaluated values of the Young moduli.

The fuzzy set theory allows us to build the lower and upper bounds of the failure envelopes for FPF and the linear qualifier function (in the form of asymmetrically trapezium for maximum strains criteria). Fuzziness of failure strain has a great influence on a surface of limit load values. The above conclusions depend on structural geometry and load parameters.

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