

DESIGN SPECTRA FOR THE PRELIMINARY DESIGN OF ELASTIC SEISMIC RETROFIT SOLUTION FROM THE OUTSIDE

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Abstract

The holistic renovation of the existing building stock is now recognized as a priority in order to reach the European targets in terms of sustainability, safety, and resilience. Nevertheless, the average European renovation rate of the building stock is only 1.5%. The Building Performance Institute Europe identified as major barriers to the renovation of the existing buildings the need to relocate the inhabitants, the extended downtime required during the construction works, the high cost of the interventions and the lack of adequate business models fostering the renovation. This is particularly true for buildings located in seismic prone areas, which require extensive structural works to be combined with energy upgrading measures.

To overcome these barriers, especially those connected to the relocation of the building functions, retrofit solutions carried out from the outside of the building are generally preferred. In this work, design spectra for the preliminary design of elastic seismic retrofit solutions carried out from the outside are derived for reinforced concrete (RC) buildings typical of the post-WWII building stock. A parametric evaluation of the retrofitted structure is conducted considering a simplified 2 DOF system, and a set of design spectra are defined in order to simplify the design procedure and to derive the optimal retrofit parameters. Finally, a reference case study representative of a typical RC building is developed to assess and validate the procedure.

Keywords: Renovation, from outside, retrofit, design spectra.

1 INTRODUCTION

It has been recognized that a deep and systematic intervention on the built environment has to be undertaken to reach the ambitious European targets fostering environmental, economic and social sustainability. The actual average European construction rate is only 1.5% [1]; therefore, the sole construction of new high-performance buildings will not entail meeting the targets of the European roadmaps. Sustainability in the construction sector can only be pursued by substantially renovating the existing building stock, which is obsolete, massively energy consuming, and vulnerable to natural and man-induced disasters [2, 3]. Two options may thus be envisioned to account for the multiple deficiencies of the existing building stock: demolition and reconstruction, and an integrated deep renovation fostering safety, resilience and sustainability; the latter solution should always be preferred over demolition and reconstruction [4].

In spite of this severe scenario, nowadays, the average European renovation rate of the reinforced concrete (RC) buildings is very low (1.0% according with BPIE [1]). *The Building Performance Institute Europe* (BPIE) identified, as major barriers to the renovation of the existing buildings, the need to relocate the inhabitants, the extended downtime required during the construction works, the high cost of the interventions and the lack of adequate business models fostering renovation [5, 6]. This is particularly true for buildings located in seismic prone areas, which require extensive structural works to be combined with energy upgrading measures.

To overcome some of these barriers, especially those connected to the relocation of the inhabitants' and the building functions, retrofit solution carried out from the outside of the building are generally preferred.

Starting from the consideration made for bracing system by Ciampi et al. [7], in this work, design spectra for the preliminary design of elastic seismic retrofit solutions carried out from the outside are derived for RC buildings typical of the post-WWII building stock that represents about 60% of the existing buildings. A parametric evaluation of the retrofitted structure is conducted considering a simplified 2 DOF system, and a set of design spectra are defined in order to simplify the design procedure and to derive the optimal retrofit parameters. Finally, a reference case study representative of a typical RC building is developed to assess and validate the procedure by means of non-linear time history analyses.

2 SENSITIVITY ANALYSIS OF THE STRUCTURAL RESPONSE OF THE RETROFITTED SYSTEM: DEFINITION OF THE DESIGN SPECTRA

The interaction between the retrofit (carried out from the outside) and the existing building is here evaluated through a simplified 2 Degrees of Freedom (2DOF) system.

Design spectra are derived for the definition of the stiffness of the retrofit structure required to limit and control the displacement of the existing structure when subjected to the design earthquake. (the objective of the retrofit intervention is to avoid excessive damage and, consequently, the long-term disruption of the building activities, the relocation of its inhabitants, and minimizing the costs after a seismic event.).

2.1 Simplified model of the retrofitted structure

The 2DOF model representative of the retrofitted system is reported in Figure 1, in which the existing building and the retrofit solution responses are described by the degree of freedom u_1 (DOF1) and u_2 (DOF2), respectively.

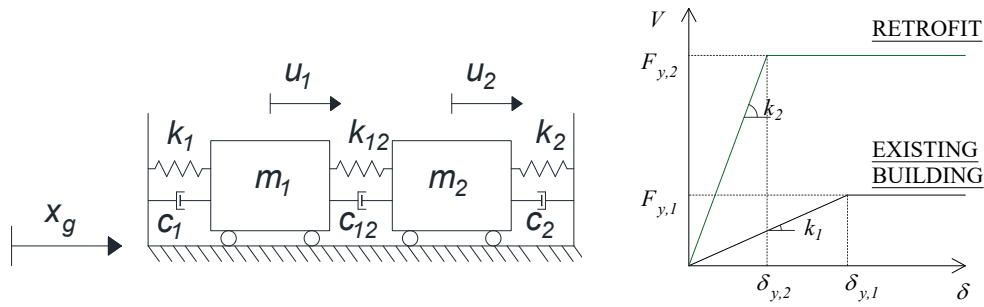


Figure 1 Simplified 2DOF model. a) Simplified 2DOF system; b) Response curve of the retrofitted structure with 2 degrees of freedom (2DOF) working in parallel.

The structural response of the existing building (DOF1) is described by: the fundamental period (T_1), the effective mass (m_1), the initial elastic stiffness (k_1), the damping coefficient (c_1), and the yielding force ($F_{y,1}$).

Given the elastic stiffness (k_1) and the yielding force ($F_{y,1}$), the yielding displacement ($\delta_{y,1}$) can be derived ($\delta_{y,1} = \frac{F_{y,1}}{k_1}$).

For the DOF2, the elastic stiffness (k_2) is defined as a function of k_1 . Feroldi [8] demonstrated that the simplification of the whole system into a 2DOF is acceptable if the ratio between the elastic stiffness of the retrofitting system (k_2) and the stiffness of the existing building (k_1) ranges between 0 and 12. The mass of the retrofit solution (m_2) is assumed in first approximation as equal to 1/10÷1/20 of the mass of the existing building (m_1) [9].

As shown in Figure 1, the two masses are connected through a general link modelling the connection between the existing structure and the exoskeleton system with elastic stiffness (k_{12}), and damping coefficient (c_{12}). The damping coefficient is supposed constant, while the influence of the stiffness is investigated in this work.

The structural response is analyzed with reference to a set of parameters:

- η represents the yielding strength of the existing building, adimensionalized with respect to the mass (m_1) multiplied by the ground acceleration $Sa(T_1)$ as in (1).

$$\eta = \frac{F_{y,1}}{[m_1 \cdot Sa(T_1)]} \quad (1)$$

It is worth noting that, the adimensionalization of the strength parameter η allows to avoid the dependence of the spectra on the values of the maximum ground acceleration (X_g).

- The damage on the existing building is evaluated through the parameter μ that represents the “ductility demand” of the existing building after the retrofit. μ is defined as the ratio between the maximum displacement (δ_{MAX}) experienced by the DOF 1 during a seismic event and the yielding displacement ($\delta_{y,1}$) of the DOF1 (Figure 1b).

$$\mu = \frac{\delta_{MAX}}{\delta_{y,1}} \quad (2)$$

- Another fundamental parameter required to derive the optimal retrofit solution is the stiffness parameter λ which represents the ratio between the elastic stiffness of the retrofit (k_2) and the stiffness of the existing building (k_1) (Figure 1).

$$\lambda = \frac{k_2}{k_1} \quad (3)$$

2.2 Equations of motion

The free-body model of the 2DOF system is represented in Figure 2. By enforcing balance to horizontal translation to DOF1 and DOF2, it yields:

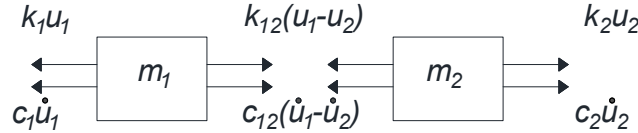


Figure 2 Free-body diagrams of the 2DOF system

From which the equations of motion of the 2DOF can be derived:

$$\begin{cases} m_1(\ddot{x}_G + \ddot{u}_1) + k_1 u_1 + c_1 \dot{u}_1 = k_{12}(u_2 - u_1) + c_{12}(\dot{u}_2 - \dot{u}_1) \\ m_2(\ddot{x}_G + \ddot{u}_2) + k_2 u_2 + c_2 \dot{u}_2 + k_{12}(u_2 - u_1) + c_{12}(\dot{u}_2 - \dot{u}_1) = 0 \end{cases} \quad (4)$$

In matrix form the equations can be re-written as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_{12} & -c_{12} \\ -c_{12} & c_2 + c_{12} \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_{12} & -k_{12} \\ -k_{12} & k_2 + k_{12} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \ddot{x}_g \begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix} \quad (5)$$

and in a compact form:

$$\underline{M} \{\ddot{u}\} + \underline{C} \{\dot{u}\} + \underline{K} \{u\} = \{F\} \quad (6)$$

in which, $\underline{M}$ is the mass matrix, $\underline{K}$ the stiffness matrix, $\underline{C}$ the damping matrix, and the vector $\{F\} = \{M\} \cdot \ddot{x}_g$ represents the seismic action on the simplified system.

2.3 Simplification to a SDOF system

In order to better understand the behavior of the whole elastic system, the frequency response of the two connected masses is investigated by using transfer functions. In the case of MDOF system, the transfer functions can be compacted into a transfer matrix $\underline{T}$ in which each component of the Transfer Matrix ($T(i,j)$) provides information about the response of the system at the DOF i due to a unit force at the DOF j . In order to evaluate the frequency response of the DOF1 the transfer functions of the 2DOF system represented in Figure 1 are developed; the whole procedure is reported in Appendix 1.

In this particular application, among all the transfer functions ($T(i,j)$) of the transfer matrix $\underline{T}(\omega)$, the component $T(1,1)$ is the most significant to analyze, considering that it represents the response of the DOF1 due to a unit force in the DOF1.

The steady-state vibration amplitudes for the 2DOF system by varying the mass (m_2), the stiffness of the DOF2 (k_2), and the stiffness of the connections (k_{12}) were investigated. To generalize the results, the varying parameters were normalized over reference values of m_1 and k_1 . In particular, the properties of the second mass are varied within the following ranges of interest: $m_2 = [1/20, 1/8, 1/4, 1/2, 1]m_1$, and $k_2 = [10, 8, 4, 2, 1]k_1$, where: $m_2 = 1/20m_1$ and $k_2 = 10k_1$ are reasonable values of mass and stiffness of the retrofit system [9], while $m_2 = m_1$ and $k_2 = k_1$ are introduced to emphasize the effect of the DOF2 on the response of the DOF1.

As concern the connections between the two systems, the steady-state vibration amplitude was evaluated by increasing the stiffness of the connection (k_{12}) within the range of interest: $k_{12} = [1, 5, 10, 50]k_1$ in order to evaluate the response of the system when they are considered as rigid. It is important to note that: (a) the damping coefficients are assumed as constant, (b) negative amplitudes corresponding to some masses have been ignored, (c) as expected, when the forcing frequency is close to one of the natural frequencies of the system, resonance phenomenon occurs, (d) for comparable values of mass and stiffness of the 2DOF system, in the point of antiresonance, the amplitude of the vibration is equal to zero.

In Figure 3a, m_2 is assumed to be equal to m_1 , the stiffness k_2 is supposed equal to k_1 , while the stiffness of the connection (k_{12}) increases from k_1 to $50k_1$; in Figure 3b, m_2 is assumed to be equal to m_1 , while the stiffness of the DOF2 increases from k_1 to $10k_1$; in Figure 3c, the stiffness k_2 is supposed equal to k_1 ; m_2 , instead, decreases from m_1 to $0.05m_1$. Some relevant conclusions can be drawn from these results: first of all, when rigid connections are considered, the amplitude of the lowest resonance frequency is generally much greater than the highest frequency modes. For this reason, in this case, it is often sufficient to consider only the lowest frequency mode in the design calculations. The same consideration can be drawn in a damped system in which k_2 is significantly higher than k_1 , and the mass ratio m_2/m_1 is lower than $1/10$. It is worth noting that, to apply this simplification when the connection is not rigid, the hypothesis of equal displacement of the 2DOF system becomes essential. Consequently, this simplification is considered acceptable only when rigid elastic connections are considered.

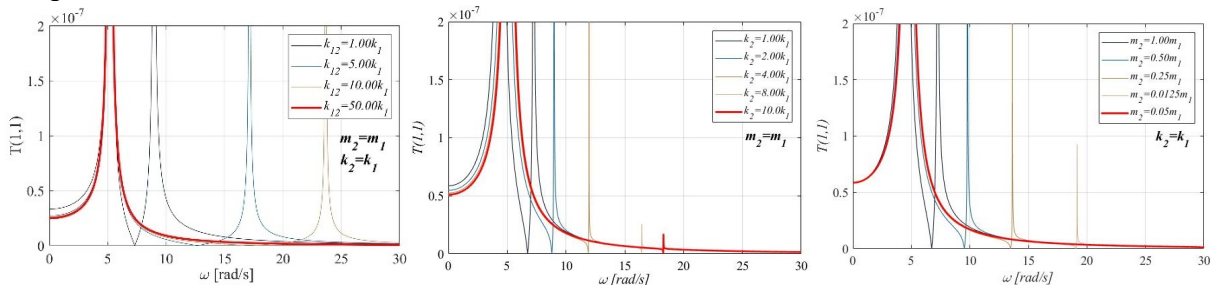


Figure 3 In-frequency response of the 2DOF system for a) $m_2=m_1$, $k_2=k_1$, for varying the retrofit stiffness k_{12} ; b) $m_2=m_1$, for varying the retrofit stiffness k_2 ; c) $k_2=k_1$, for varying the mass (m_2) of the retrofit system.

In this work the connections are considered as rigid, the mass of the retrofit can be considered negligible, while the stiffness is significantly higher than that of the existing building; for these reasons, the system can be idealized as just a Single DOF system (SDOF).

The simplified model is reported in Figure 4a, in which the total mass $m=m_1+m_2$ is considered. It is worth noting that because m_2 can be considered negligible, in many cases m can be considered equal to m_1 .

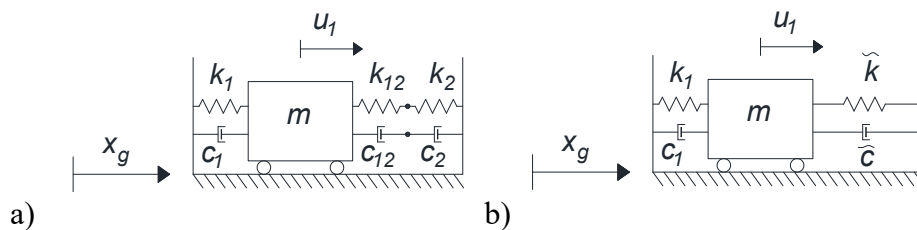


Figure 4 a) Simplified SDOF system; b) Simplified SDOF system with equivalent spring and damping.

In Figure 4b, the equivalent stiffness and damping of the retrofit solution are introduced, where:

and,

$$\tilde{k} = \frac{k_2 k_{12}}{k_2 + k_{12}} \quad (7)$$

$$\tilde{c} = \frac{c_2 c_{12}}{c_2 + c_{12}}$$

If the connections are considered as rigid, the equivalent stiffness (k) can be considered as equal to the stiffness of the retrofit.

It is worth noting that when considering the equivalent system in Figure 4b, the response parameters λ (3) must be re-defined as follows:

$$\tilde{\lambda} = \frac{\tilde{k}}{k_1} \quad (8)$$

where λ represents the ratio between the equivalent elastic stiffnesses of the retrofit (k) and the elastic stiffness of existing building (k_1).

2.4 Parametric analyses on the simplified SDOF system

Adopting this new simplified SDOF system, sensitivity analyses for the evaluation of the retrofit properties were carried out.

In particular, given a target maximum displacement for the existing building -DOF1- (and consequently a target maximum ductility), the spectral displacement ($\bar{S}_d$) can be calculated as show in (9);

$$\bar{S}_d(T) = \delta_{MAX} = \mu \cdot \delta_{y,1} \quad (9)$$

Through the displacement spectra, the elastic period ($\bar{T}$) and the pseudo acceleration ($S_a(\bar{T})$) of the structure can be derived as shown in Figure 5.

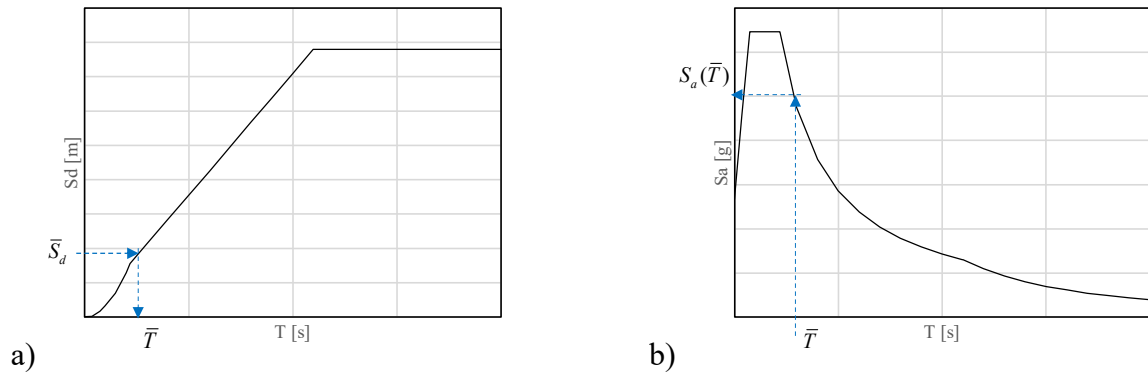


Figure 5 a) design displacement spectrum; b) design acceleration spectrum

The elastic stiffness and the total base shear of the retrofitted system can be calculated as follow:

$$\hat{k} = \frac{4 \cdot \pi \cdot m^*}{\bar{T}^2} \quad (10)$$

$$\hat{V} = m^* \cdot S_a(\bar{T})$$

The total base shear on the retrofitted building ($\hat{V}$) is thus distributed as a function of the elastic stiffnesses of the 2 DOFs while imposing the limit capacity of the DOF1 as equal to $F_{y,1}$ (11).

$$\begin{cases} \text{if } V_1 = \frac{\hat{V}}{\hat{k}} \cdot k_1 \leq F_{y,1} \Rightarrow V_2 = \hat{V} - V_1 \\ \text{if } V_1 = \frac{\hat{V}}{\hat{k}} \cdot k_1 > F_{y,1} \Rightarrow V_2 = \hat{V} - F_{y,1} \end{cases} \quad (11)$$

Given the base shear ($\hat{V}$) and the total displacement (δ_{MAX}) of the retrofitted system, the stiffness of the retrofit solution (k_2) and the stiffness ratio λ can be derived.

$$\begin{aligned} k_2 = \tilde{k} &= \frac{V_2}{\delta_{MAX}} \\ \lambda &= \frac{k_2}{k_1} \end{aligned} \quad (12)$$

In Figure 6, design spectra for the preliminary design of retrofit solution carried out from the outside are reported, where the ductility demand μ is plotted as a function of the stiffness ratio $\tilde{\lambda}$ for given values of the initial period T_I and strength parameter η .

The considered parameters and the range in which they are varied are summarized in Table 1. As concern the DOF1, the inputs parameters were selected to be representative of the ordinary post Second World War RC buildings according to [10] while the equivalent stiffness of the retrofit k is varied in the interval $0 \div 6k_I$, in which $k=0$ represents the As-Is condition (ante retrofit $\lambda=0$), and $k=6k_I$ is a reasonable value of equivalent retrofit stiffness [8]. As for the yielding force, different values of η were considered to represent weak ($\eta=0.30$), medium ($\eta=0.50-0.60$) and strong ($\eta=0.85$) buildings.

Parameter	Symbol	Range	
Elastic period	T_I	0.5-2.5	[s]
Effective mass	m_1	451-800-1000	[kN/g]
Elastic stiffness	k_1	7.5-13-24	[kN/mm]
Strength parameter	H	0.30-0.50-0.60-0.85	[-]

Table 1: Example of the construction of one table.

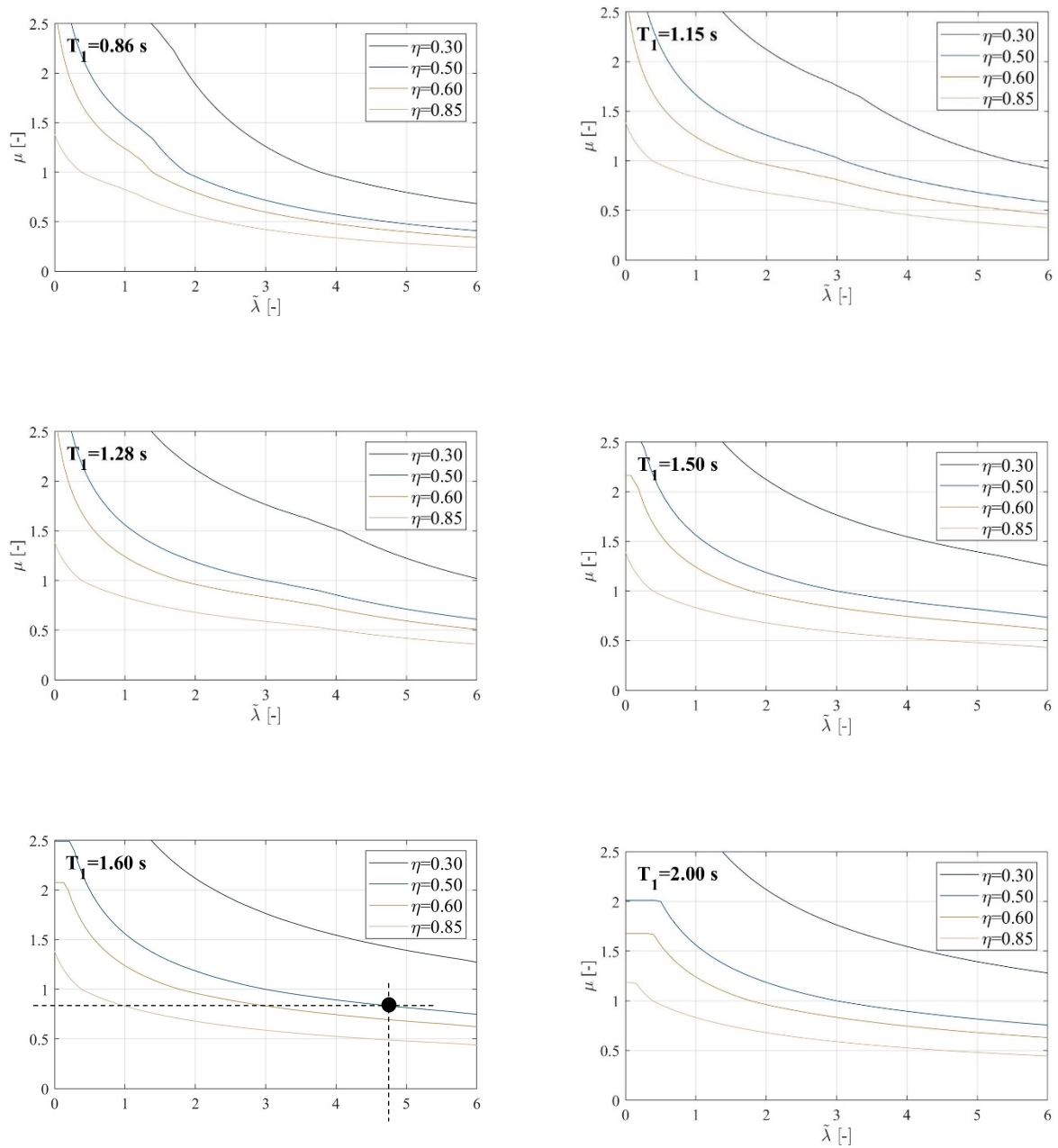


Figure 6 Design spectra

3 APPLICATION TO A REFERENCE CASE

In order to evaluate the effectiveness of the design method, the procedure was applied to a reference building. The reference structure is a post-World War II reinforced concrete building. Main features and structural details of the reference building are described in Feroldi [8] and Passoni [9]. The structure is a five-story rectangular building (24.00mx10.64m) featuring three one-way longitudinal frames and two infilled lateral frames. The inter-story height is 3.15m, and the structural system is made of RC frames in the longest direction designed to withstand static loads only.

Floors are made of a composite RC beam and clay block system featuring a 2.5 cm RC overlay. On the basis of previous studies, it has been assumed that floors can withstand horizontal loads (i.e. they behave like floor diaphragms) by developing an in-plane tied-arch resistant mechanism up to their ultimate capacity [8, 9]. The staircase core is not designed to withstand seismic loads; accordingly, the staircase walls are not considered RC seismic walls, but rather stiff walls with low ductility. Geometry and materials of the main frame are reported in Figure 7.

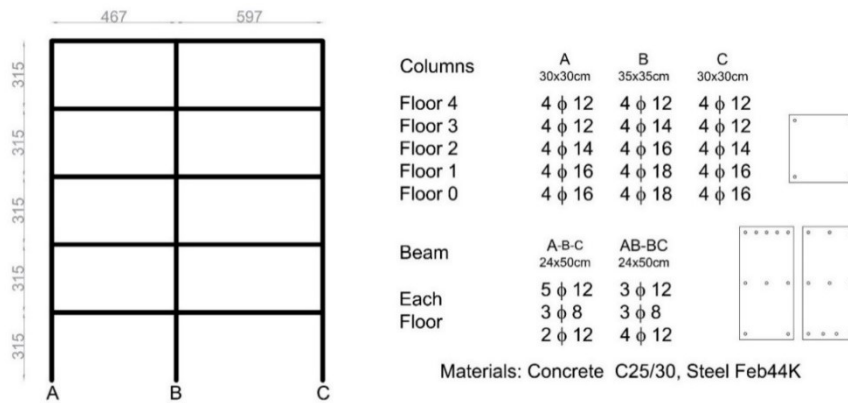


Figure 7 Geometry and materials of the reference building main frame. Characteristics and features are representative of ordinary RC buildings.

3.1 Existing building model

The finite element model was developed with the software MidasGen 2018. In the existing building model, attention has been paid to the correct representation of both structural and non-structural components. Non-linear static analyses were performed in order to evaluate the structural performance of the reference building in the As-Is conditions. Structural elements such as beams, floors, columns and staircase core were modeled like beam elements with lumped plasticity in accordance with the European Building Code [11]. In particular, the beam flexural plastic hinge has been considered by tri-linear Takeda constitutive law [12]. The floors are modeled as rigid diaphragms.

About columns, both shear and bending behavior have been considered by introducing Takeda Tetra linear plastic hinges [12]. In particular, the shear behavior has been assumed to be elastic up to the ultimate capacity of the element and then decays very quickly in order to represent an extremely fragile collapse. Columns are fixed at the base.

Staircase walls are not designed to withstand the horizontal loads and they have been considered stiff walls with low ductility; in the finite element model, the non-linear behavior of staircase walls was modeled with lumped plastic hinges on each floor of the building. It is worth noting that in this case a rotational spring was introduced at the base of each wall in

order to simulate a grade beam. Properties of the rotational hinge were determined by evaluating the maximum bending moment and stiffness of the foundation system [8].

As for the non-structural elements, infilled walls were modeled as two non-linear equivalent trusses converging in the beam-column joints. Cracking and peak forces were evaluated according to Decanini et al. [13], while the selected cracking and peak drifts are in accordance to the traditional values of 0.5% drift for moderate damage and 1.5% drift for the infill collapse [14]. As a result, the equivalent truss dimensions are 1.37m x 0.13m for the shorter and 1.40m x 0.13m for the longer infill walls.

Nonlinear static analyses were performed. The reference building was supposed to be located in L'Aquila, with C soil category and T1 topography. The capacity curve of the reference building and the displacement demands are plotted in Figure 8 and the parameters of the equivalent SDOF system used for the vulnerability analysis are reported in Table 2.

Equivalent SDOF (NTC, 2018)		
m^*	750	[kN/g]
K	11.5	[kN/mm]
F_{yl}	760	[kN]
d_{yl}	66	[mm]
Γ	1.44	[-]
d^{SLV}	260	[mm]
H	0.50	[-]

Table 2: Equivalent SDOF system parameters.

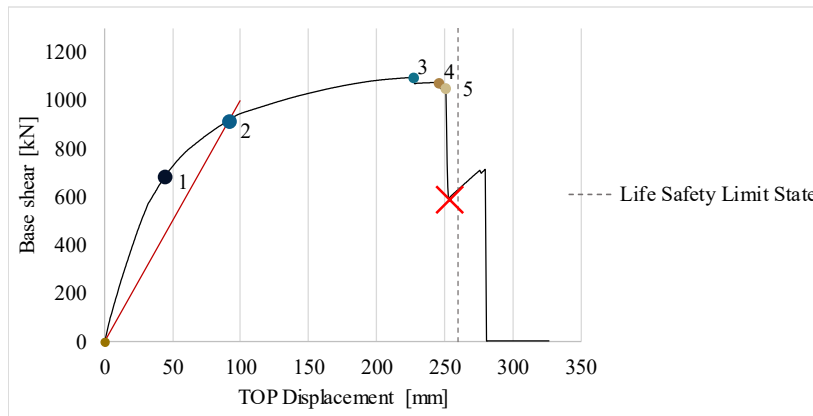


Figure 8 Capacity curve of the existing building: 1. Infill Cracking; 2. Infill Failure; 3. Stair core limit strength; 4. Plastic hinges in the lateral columns; 5. Plastic hinges at the base of all columns. Dot line: the displacement demands at the Life Safety Limit State; Red line: effective stiffness of the existing building in correspondence of the infill failure.

As shown in Figure 8, the existing building does not satisfy L'Aquila displacement demands [15], and, for this reason, the renovation of the existing building is required.

3.2 Retrofit solution: application of the design spectra

For the application of the design spectra, the characteristics of the equivalent SDOF system of the existing building and the target displacement for the retrofit solution are required.

As concern the existing building, the properties of the equivalent SDOF system have already been calculated (Table 1). For the target displacement, a maximum inter-story drift

equal to 0.5% to avoid the failure of the non-structural components at the LSLS was considered. From the spectra, considering the elastic stiffness of the existing building (k_I) equal to 11.50 [kN/mm], in correspondence of a ductility factor (μ) of 0.82 and considering η equal to 0.50, the stiffness ratio between the retrofit structure and the existing building (λ) is equal to 4.80 (Figure 6). In this work a generic elastic retrofit solution was considered to validate the effectiveness of the design spectra; this additional stiffness may be obtained, for example, by means of external shear walls or diagrid exoskeletons.

3.2.1. Non-linear analysis and results: Time history analyses

In order to validate the design procedure for shear-type buildings, 7 nonlinear Time History analyses were carried out.

Accelerograms compatible with the code spectrum were determined by adopting the software Rexel 2.2beta [16]. A maximum scale factor equal to 2 and upper and lower tolerance equal to 10% and 15%, respectively, were imposed. It is worth noting that for the selected accelerograms the lower tolerance limit imposed by the Eurocode [11] is not met. However, such a requirement is not always satisfied in the case of high seismicity areas and, for this reason, a lower tolerance limit was obtained by increasing the Eurocode limit value (10%) by 5% until a compatible set was identified [16] Figure 9.

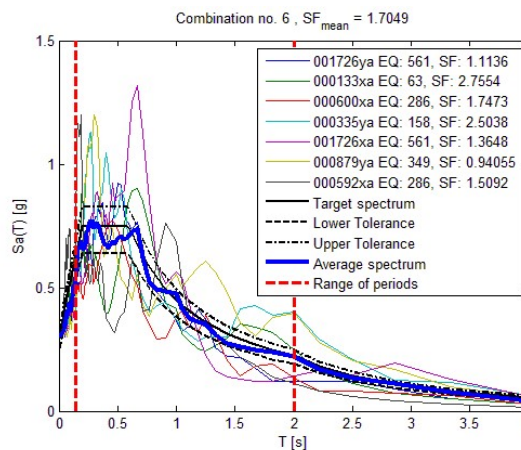


Figure 9 Selected combination of compatible accelerograms used for the time history analyses (Iervolino, et al., 2010).

Time History results, expressed in terms of total base shear, base shear at the base of the retrofit, and top displacement are reported in Table 2.

Accelerograms	$V_{MAX}[kN]$	$V'_{MAX}[kN]$	$D_{TOP}[m]$
000133_xa	6767.623	6211.1	0.061
000335_ya	9236.477	8562.9	0.082
000592_xa	4929.553	4266.6	0.038
000600_xa	8986.49	7863.2	0.081
000879_ya	7501.01	6691.7	0.064
001726_xa	10243.19	9323	0.093
001726_ya	12078.51	10841.8	0.11
Avg.	8534.693	7680.043	0.076

Table 3 Time history analyses results: $V_{MAX}[kN]$: Total base shear; $V'_{MAX}[kN]$: Base shear at the base of the retrofit (no ex Building); $D_{TOP}[m]$: Top displacement.

Results show that both the limit top displacement target of 0.0785 (m) and the maximum inter-story drift target are met (Figure 10b and Table 3 Time history analyses results: $V_{MAX}[kN]$: Total base shear; $V'_{MAX}[kN]$: Base shear at the base of the retrofit (no ex Building); $D_{TOP}[m]$: Top displacement..

Furthermore, as concerns the inter-story shear, the adopted limit value is exceeded and therefore external diaphragms should be introduced, as shown in Figure 10a.

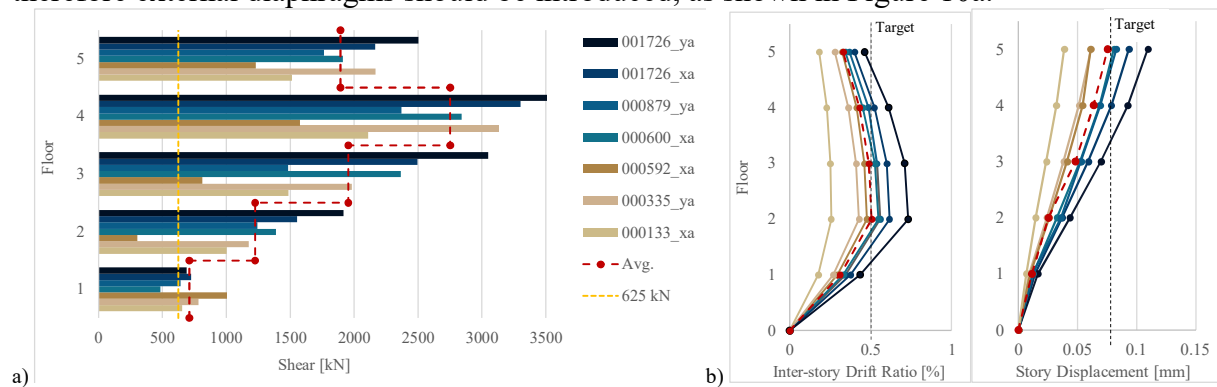


Figure 10 a) Floor shear along the building's height for different accelerograms, b) inter-story drift and story displacement.

4 CONCLUSIONS

This work is part of ongoing research on the holistic renovation of the post-WWII RC buildings. In particular, this paper proposed design spectra for the definition of the minimum stiffness for retrofit solutions carried out from outside the building. The retrofit-existing building interaction has been investigated with a 2 degree of freedom system (2 DOF) and the parametric curves obtained could be adopted to evaluate the minimum stiffness required and to satisfy a fixed target. It is worth noting that other specific considerations depending on the retrofit solution have to be made as, for example, the strength limits of the elements of the retrofit itself. Finally, the effectiveness of the method has been assessed through the application of this procedure to a reference building.

In future research, a much wider range of buildings, with different characteristics and features, will be examined and a sensitivity analysis will be carried out in order to generalize the whole procedure. Moreover, the connection between the retrofit and the existing building will be taken into account.

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APPENDIX 1

Starting from the equations of motion (4), the transfer matrix ($\underline{T}(\omega)$) of the system described in Figure 1 is derived. The solution of the equations of motion, (4), can be expressed as:

$$\begin{cases} \underline{u} = \underline{X} \cdot e^{i\omega t} \\ \dot{\underline{u}} = i\omega t \cdot \underline{X} \cdot e^{i\omega t} \\ \ddot{\underline{u}} = -\omega^2 \cdot \underline{X} \cdot e^{i\omega t} \end{cases} \quad (\text{A1})$$

By substituting (A1) in (6), it yields:

$$[-\omega^2 \underline{M} + i\omega \underline{C} + \underline{K}] \cdot \underline{X} \cdot e^{i\omega t} = \underline{F} \cdot e^{i\omega t} \quad (\text{A2})$$

By defining the Impedance Matrix $\underline{Z}(\omega)$ as:

$$\underline{Z}(\omega) = [-\omega^2 \underline{M} + i\omega \underline{C} + \underline{K}] \quad (\text{A3})$$

and, combining (A1) and (A2), it yields:

$$\underline{Z}(\omega) \cdot \underline{X} = \underline{F} \quad (\text{A4})$$

The transfer matrix is the inverse of the impedance matrix $\underline{Z}(\omega)^{-1} = \underline{T}(\omega)$,

$$\underline{Z}^{-1} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ Z_{21} & Z_{11} \end{bmatrix}}{\det |\underline{Z}|} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \quad (\text{A5})$$

where,

$$\det |\underline{Z}| = Z_{11}Z_{22} - Z_{12}^2$$

The solution can be expressed as:

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (\text{A6})$$

or in the compact form:

$$\underline{X} = [\underline{Z}(\omega)]^{-1} \underline{F} = \underline{T}(\omega) \cdot \underline{F} \quad (\text{A7})$$

where $\underline{T}(\omega)$ is the transfer matrix and represents the behavior of the masses per unit input force as a function of the frequency.

By applying the described procedure to the reference system Figure 1, the frequency response of the system when subjected to a harmonic load can be evaluated. The equations $T(i,j)$ of the transfer function that compose the transfer matrix $\underline{T}(\omega)$ of the 2 DOF system are:

$$T(1,1) = \frac{k_2 + k_{12} + i \cdot (c_2 + c_{12}) \cdot \omega - m_2 \cdot \omega^2}{-(-k_{12} - i \cdot c_{12} \cdot \omega)^2 + (k_1 + k_{12} + i \cdot (c_1 + c_{12}) \cdot \omega - m_1 \omega^2) \cdot (k_2 + k_{12} + i \cdot (c_2 + c_{12}) \cdot \omega - m_2 \omega^2)} \quad (\text{A8})$$

$$T(1,2) = \frac{k_{12} + i \cdot c_{12} \cdot \omega}{-(-k_{12} - i \cdot c_{12} \cdot \omega)^2 + (k_1 + k_{12} + i \cdot (c_1 + c_{12}) \cdot \omega - m_1 \omega^2) \cdot (k_2 + k_{12} + i \cdot (c_2 + c_{12}) \cdot \omega - m_2 \omega^2)} \quad (\text{A9})$$

$$T(2,1) = \frac{k_{12} + i \cdot c_{12} \cdot \omega}{-(-k_{12} - i \cdot c_{12} \cdot \omega)^2 + (k_1 + k_{12} + i \cdot (c_1 + c_{12}) \cdot \omega - m_1 \omega^2) \cdot (k_2 + k_{12} + i \cdot (c_2 + c_{12}) \cdot \omega - m_2 \omega^2)} \quad (\text{A10})$$

$$T(2,2) = \frac{k_1 + k_{12} + i \cdot (c_1 + c_{12}) \cdot \omega - m_1 \cdot \omega^2}{-(-k_{12} - i \cdot c_{12} \cdot \omega)^2 + (k_1 + k_{12} + i \cdot (c_1 + c_{12}) \cdot \omega - m_1 \omega^2) \cdot (k_2 + k_{12} + i \cdot (c_2 + c_{12}) \cdot \omega - m_2 \omega^2)} \quad (\text{A11})$$