

BAYESIAN DAMAGE CHARACTERIZATION BASED ON PROBABILISTIC MODEL OF SCATTERING COEFFICIENTS AND HYBRID WAVE FINITE ELEMENT MODEL SCHEME

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Abstract

Ultrasonic Guided Wave (GW) has been proven to be sensitive to small damage. Motivated by the fact that the quantitative relationship between wave scattering and damage intensity can be described by scattering properties, this study aims at proposing a new probabilistic damage characterization method based on the scattering coefficients in tandem with hybrid wave finite element model (WFEM) scheme. The probabilistic distribution properties of the scattering coefficients estimated using measured ultrasonic guided waves in the frequency domain are inferred based on absolute complex ratio statistics. The theoretical scattering coefficients can be efficiently calculated using WFEM which combines conventional finite element analysis with periodic structure theory. Based on the probabilistic distribution of reflection/transmission coefficients, the likelihood function connecting the theoretical model responses containing the parameters to be updated and the measured responses are formulated within a unified Bayesian system identification framework to account for various uncertainties. The transitional Monte Carlo Markov Chain (TMCMC) is used to sample the posterior probability density function of the updated parameters. A numerical example is utilized to verify the accuracy of the proposed algorithm. Results indicate that the strategies proposed in this study can quantify the uncertainties of damage characterization

Keywords: Wave Finite Elements; Ultrasonic Guided Waves; Damage Identification; Uncertainty Quantification; Bayesian Analysis.

1 INTRODUCTION

Structural Health Monitoring (SHM) involving the observation of a structure using response measurements, the extraction of damage-sensitive features and the analysis of these features to assess structural health condition has attracted widespread attention [1, 2]. Low-frequency damage detection methods utilizing dynamic responses have been applied extensively in engineering structures [3]. However, the damage detection approaches based on the low-frequency characteristics are usually insensitive to small damage.

Nowadays, ultrasonic GW-based SHM methodologies have been widely reported to be sensitive to small damage, convenient and efficient in detecting structural damage [3], such as fatigue cracks in metallic structures, debonding and delamination in composite structures. The scattered waves need to be analyzed using certain damage identification algorithms to extract various characteristics containing essential information about the damage [4, 5]. Over the past decades, tremendous efforts have been directed to extract structural conditions using GWs. In the campaign of structural damage characterization, one critical issue that has been widely accepted is that uncertainties due to endogenous factors should be appropriately considered [6]. Bayesian statistics has been widely considered an excellent candidate for uncertainty quantification in GW-based damage detection [6, 7] since it considers probability as a multi-valued propositional logic for plausible reasoning [8,9].

By making full use of the Bayesian system identification framework for accommodating measurement noise and modeling errors properly, this study aims at formulating a generic methodology for probabilistic damage characterization based on wave scattering characteristics [4, 5]. The scattering coefficients are probabilistically modelled by using absolute complex ratio random variables [10]. A Bayesian scheme makes inferences about the damage characterization parameters directly by processing the statistical information contained in the experimentally measured scattering properties. The TMCMC [11] is finally used to sample the posterior Probability Density Function (PDF) of the updated parameters. The hybrid WFE methodology is hereby employed in order to efficiently simulate wave scattering when ultrasonic GW impinge a damaged segment within a structure of arbitrary layering. Furthermore, a cheap and fast Kriging surrogate model will be employed in tandem with the WFE scheme in order to approximate the output in function of model parameters. The procedure is verified using numerical data in different damage configurations.

2 THEORETICAL BACKGROUND

The piezoelectric transducer excites propagating waves within the structure. The incoming GWs (+) impinge on the damaged structural segment and generate a set of outgoing (-) reflected and transmitted waves. The propagation of waves is often described in terms of “wave modes”. The reflection and transmission coefficients denoted by $\{\mathfrak{R}_k, \mathfrak{T}_k\}$ at ω_k are determined by dividing the frequency spectra of the reflected/transmitted signal by that of the incident wave signal [4]. The reflection/transmission coefficients predicted by the structural damage model are denoted as $\{R_k(\boldsymbol{\theta}), T_k(\boldsymbol{\theta})\}$, defined by a set of damage parameters $\boldsymbol{\theta}$, which are to be identified. Each implemented damage scenario can be FE-modelled and the associated scattering coefficients can be numerically computed.

In the field of wave propagation, the scattering coefficients can be estimated by taking the ratio of the FFT of reflected/transmissive wave and the FFT of the incident wave as:

$$\begin{aligned}\mathfrak{R}_k &= \frac{X_{re}(\omega_k)}{X_{in}(\omega_k)} \\ \mathfrak{T}_k &= \frac{X_{tr}(\omega_k)}{X_{in}(\omega_k)}\end{aligned}\quad (1)$$

The statistical inference can be executed by embedding the “deterministic” structural models within a class of probability models so that the structural models give a predictable (“systematic”) part and the prediction error is modeled as an uncertain (“random”) part [8]. In the context of Bayesian inference with scattering coefficients, the measured outputs and the numerical model outputs are connected as follows [8]:

$$\begin{aligned}\mathfrak{R}_k &= R_k(\theta) + \mu_{re} \\ \mathfrak{T}_k &= T_k(\theta) + \mu_{tr}\end{aligned}\quad (2)$$

In Eq. (2), the error term μ_{re} and μ_{tr} are usually modeled as white noise with constant variances.

The variances of the FFT coefficient of the reflected wave signal and the transmitted wave can be approximated by:

$$\begin{aligned}\sigma_{re}^2 &= \sigma_{in}^2 (R_k^2(\theta) + \gamma_{re}) \\ \sigma_{tr}^2 &= \sigma_{in}^2 (T_k^2(\theta) + \gamma_{tr})\end{aligned}\quad (3)$$

where $\sigma_{in}^2 = \text{var}(X_{in})$, $\sigma_{re}^2 = \text{var}(X_{re})$ and $\sigma_{tr}^2 = \text{var}(X_{tr})$ denote the variations of the incident wave, the reflected wave and the transmitted wave, respectively; γ_{re} and γ_{tr} denote the variances of the prediction errors of reflection and transmission coefficients. $R_k(\theta) = \eta_{re}^{(k)}(\theta)$ and $T_k(\theta) = \eta_{tr}^{(k)}(\theta)$ denote the reflection and transmission coefficients predicted at θ , which are achieved by using the Kriging model [12, 13]. To formulate a Kriging predictor model, it requires initial Design of Experiments (DoE). These samples are frequently referenced as the training set or support points. Appropriate DoE plays a vital role in constructing a high-fidelity Kriging model because DoE influences the creation of the most informative training data. A common choice for the training design is the Latin Hypercube Design (LHD), which guarantees to spread design points evenly across each input parameter dimension. With the training set at hand, one can then calculate the predicted values of the surrogate model at various sample points in the parameter space by performing an “experiment” at each of those samples based on the hybrid WFE scheme [14-16].

The PDF of the scattering coefficients $\mathfrak{R}_k$ and $\mathfrak{T}_k$ are given by [10]

$$\begin{aligned}p_{\mathfrak{R}_k}(r_k | \theta, \gamma_{re}) &= \frac{2r_k (R_k^2(\theta) + \gamma_{re})}{(R_k^2(\theta) + \gamma_{re} + r_k^2)^2} \\ p_{\mathfrak{T}_k}(\varepsilon_k | \theta, \gamma_{tr}) &= \frac{2\varepsilon_k (T_k^2(\theta) + \gamma_{tr})}{(T_k^2(\theta) + \gamma_{tr} + \varepsilon_k^2)^2}\end{aligned}\quad (4)$$

Conditioned on the set of measurements $D = \{\mathfrak{R}_k, \mathfrak{T}_k | k = k_1, \dots, k_2\}$ formed over $\omega \in [k_1 \Delta \omega, k_2 \Delta \omega]$, the likelihood function is given by

$$\begin{aligned}
p(\mathfrak{R}_\varphi | \boldsymbol{\theta}, \gamma_{re}) &= \prod_{k=k_1}^{k_2} p_{\mathfrak{R}_k}(r_k | \boldsymbol{\theta}, \gamma_{re}) = \prod_{k=k_1}^{k_2} \frac{2r_k (R_k^2(\boldsymbol{\theta}) + \gamma_{re})}{(R_k^2(\boldsymbol{\theta}) + \gamma_{re} + r_k^2)^2} \\
p(\mathfrak{J}_\varphi | \boldsymbol{\theta}, \gamma_{tr}) &= \prod_{k=k_1}^{k_2} p_{\mathfrak{J}_k}(\varepsilon_k | \boldsymbol{\theta}, \gamma_{tr}) = \prod_{k=k_1}^{k_2} \frac{2\varepsilon_k (T_k^2(\boldsymbol{\theta}) + \gamma_{tr})}{(T_k^2(\boldsymbol{\theta}) + \gamma_{tr} + \varepsilon_k^2)^2}
\end{aligned} \tag{5}$$

According to the Bayes' theorem, we can condition the prior on the training data and integrate over the prior distribution of the coefficients to obtain the posterior uncertainties of $\lambda = \{\boldsymbol{\theta}, \gamma_{re}, \gamma_{tr}\}$ [8,9]:

$$p(\lambda | \mathfrak{M}, D) = p(\lambda | \mathfrak{M}) \exp(-L(\lambda)) \tag{6}$$

With $L(\lambda)$ denoting the negative-log likelihood function given by

$$L(\lambda) = \sum_{k=k_1}^{k_2} \ln \frac{2r_k (R_k^2(\boldsymbol{\theta}) + \gamma_{re})}{(R_k^2(\boldsymbol{\theta}) + \gamma_{re} + r_k^2)^2} + \sum_{k=k_1}^{k_2} \ln \frac{2\varepsilon_k (T_k^2(\boldsymbol{\theta}) + \gamma_{tr})}{(T_k^2(\boldsymbol{\theta}) + \gamma_{tr} + \varepsilon_k^2)^2} \tag{7}$$

As a result, the posterior distribution $p(\lambda | \mathfrak{M}, D)$ of the damage identification parameters and prediction-error parameters can be achieved using TMCMC algorithm [11].

3 STEP-BY-STEP DESCRIPTION

The procedures of the proposed methodology are outlined below:

- Determine the scattering coefficients for the structure under investigation by GW measurements;
- Construct Kriging surrogate model to numerically compute the relationship between the scattering coefficients and the damage characterization parameters $\boldsymbol{\theta}$;
- Formulate the likelihood function with the scattering coefficient estimates and those predicted by surrogate model in tandem with WFE;
- Calculate the posterior uncertainty of $\boldsymbol{\theta}$ with TMCMC.

4 CASE STUDY

The accuracy of the proposed algorithm is demonstrated by a fundamental spring-mass system shown in Figure 1. The model is parameterized through the stiffness of the spring $k_0 = 10^8 \text{ N/m}$ and the mass of each block $m_0 = 10^{-3} \text{ kg}$. Assume that damage occurs in the 200th spring, and the damage extent is assumed to be 80%, i.e. $k_d = \alpha k_0$ with $\alpha = 0.2$. The system is assumed to be excited by a 9-cycle Hanning-windowed sinusoidal tone burst. The reference pseudo-experimental damage signature is provided by an explicit solution of the full system comprising 400 masses. Contamination is added to the explicit solution by Gaussian noise. Reflection and transmission coefficients for the GW are therefore acquired.

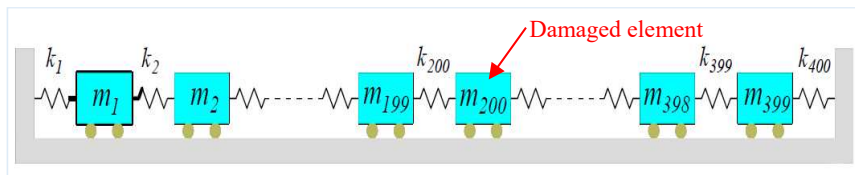


Figure 1: Schematic diagram of the spring-mass system

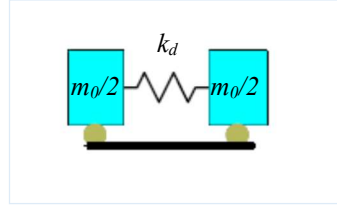


Figure 2: Schematic diagram of the waveguide

The frequency band $f_r = [10, 30]$ kHz is selected for identification, which is symmetrically around the central frequency of the excitation. A thousand training points are generated for the damage identification parameter α using LHD. For each sampling point, the scattering coefficients corresponding to frequencies falling in $f_r = [10, 30]$ kHz are calculated as training outputs using hybrid WFE shown in Figure 2. The training inputs and outputs are then used for constructing Kriging model between scattering coefficients and the damage identification parameter. The parameter vector set to be identified includes $\theta = \{\alpha, \gamma_{re}, \gamma_{tr}\}$. The lower bound and upper bound of the parameters are set to be $\theta_{lower} = \{0.5\alpha, 0.001, 0.001\}$ and $\theta_{upper} = \{1.5\alpha, 1, 1\}$. Then the Bayesian inference is performed by TMCMC, resulting in 10 stages in total. Figure 3 presents the convergence diagram of the TMCMC algorithm at different stages, which demonstrates that the proposed algorithm is rather efficient. The histogram of the stochastic samples of the final stage is shown in Figure 4 accompanied by the kernel density estimation. The MPV of the damage identification parameter is approximately 0.201, which is only 5% away from the actual value. An insignificant c.o.v. (less than 0.3%) is estimated for the extracted damage identification parameter. A relatively large uncertainty of the order of 7% is observed for the prediction errors.

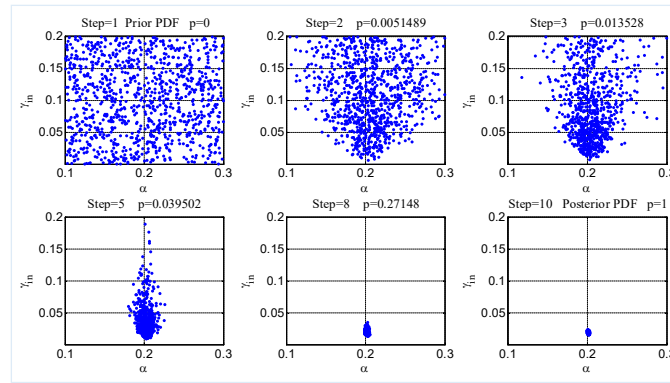


Figure 3: Convergence diagram of TMCMC at different stages of the spring-mass system

The posterior marginal distribution of the identified parameters using TMCMC and Laplace approximation are compared in Figure 4. Results from Figure 4 indicate that discrepancy is found for the PDFs using two different approaches, especially regarding the results of the prediction-error parameters. The mean values of all parameters identified through TMCMC is less than those acquired through Laplace approximation. The results identified through Laplace approximation are much more dependent on the initial guesses. Given that the initial values deviate the true values significantly, the results can be poor, and it can cause significant divergence. Therefore, the TMCMC algorithm has an important advantage over Laplace approximation as it avoids the need for estimating the initial values of the parameters, which is non-trivial in a number of real cases.

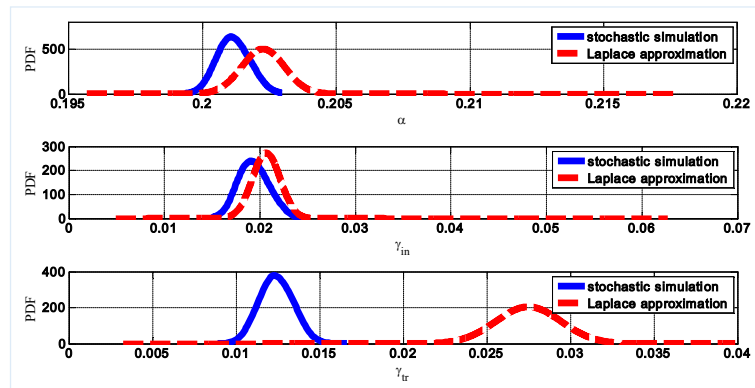


Figure 4: Comparison of the PDFs quantified using TMCMC and Laplace approximation

5 CONCLUSIONS

Ultrasonic GWs have played an important role in modern SHM technologies due to their high sensitivity to small damage. We hereby investigate the possibility of using scattering coefficients for probabilistic damage identification, through the uniqueness of GW interactions with each damage scenario. In the context of damage detection with GWs, modelling error as well as measurement noise will inevitably affects the results. This emphasizes the importance of using a comprehensive statistical framework to account for the uncertainties in the parameters and their propagation when in need for robust predictions consistent with experimental data. By making full use of the Bayesian system identification framework to account for measurement noise and modeling errors, this study formulates a new, generic framework for probabilistic damage identification by integrating a hybrid WFEM scheme employed for scattering coefficient estimates, a Kriging predictor model as well a TMCMC stochastic simulation technique. A numerical study has been used to verify the algorithm properly.

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