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EFFECTS OF UNCERTAINTIES OF SOIL AND PILE MECHANICAL PROPERTIES ON THE DYNAMIC STIFFNESS OF SINGLE PILES IN HOMOGENOUS DEPOSITS

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Abstract

In the soil-structure interaction framework, among the results of the soil-foundation system analysis (kinematic analysis) the dynamic impedance functions represent the force-displacement relationships of the compliant restraint for the superstructure. The intrinsic variability of soil parameters and of design materials represents an uncertainty source which is necessary to include in the definition of the soil-foundation model, in order to predict the superstructure response adequately. The aim of this work is to evaluate effects of uncertainties on soil dynamic and mechanical properties on the impedance matrix of single piles in homogeneous soils, focusing on the sensitivity of the parameters. A Quasi Monte Carlo (QMC) simulation with sampling based on Sobol' sequence is performed starting from the probabilistic distribution of three main parameters characterizing the soil-piles response, i.e. the soil density, the shear waves velocity and the concrete elastic modulus of piles, available from the literature. The analyses are carried out through the numerical model developed by the Authors and a sensitivity study on the optimal number of simulations is performed, comparing the use of Quasi-random sampling with the classical Monte Carlo simulation technique. Moreover, results of sensitivity studies show the influence of variability of each considered mechanical parameter on the global response of the soil-foundation system.

Keywords: Probabilistic Analysis, Pile foundations, Soil-foundation impedance matrix, Soil-Structure Interaction Analysis, Quasi-random Simulation.

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1 INTRODUCTION

It is known from the literature that Soil-Structure Interaction (SSI) has a not negligible impact on the structural response [1, 2]. To properly account for SSI in the structural design, the sub-structure approach can be employed under the assumption of a linear soil-foundation behaviour [3]. The method provides for the analysis of the structural system in two steps. The first one (kinematic interaction analysis) studies the soil-foundation system subjected to the Free-Field Motion (FFM) and provides (i) the dynamic impedance functions, which represent the force-displacement relationships of the compliant restraint for the superstructure, and (ii) the Foundation Input Motion (FIM), i.e. the actual motion experienced by the foundation because of the seismic waves propagating in the soil. The second step (inertial interaction analysis), allows to evaluate the actual superstructure response under the FIM with a compliant base characterized by the soil-foundation impedance matrix.

The soil-foundation geometrical and mechanical parameters largely affect results of the kinematic interaction analysis. Classically, deterministic values are adopted for the main quantities governing the kinematic interaction, basing on design statements, design drawings, field tests or expert engineering judgement. Even if the intrinsic variability of natural and construction materials has been widely studied in the literature, with both focus on the statistical characterization of soil mechanical properties [4, 5] and structural materials [6, 7], few works dealing with probabilistic approach for SSI are available [8-10] and, to the authors' knowledge, none of them address the problem of deep foundations. Since the kinematic interaction may produce not negligible effects on pile foundations [11, 12], a particular interest on the probabilistic characterization of such substructure behaviour arises.

This paper deals with the study of the impedance matrix of a single pile in homogenous soil with a probabilistic approach, suitably accounting for uncertainties related to the mechanical parameters of the soil-foundation system through statistic distributions available in the literature. The effects of the variability of the soil density, the shear wave velocity and the concrete elastic modulus on the dynamic impedance functions are investigated through the analytical model developed by the Authors [13], assuming no correlation between the aleatoric variables. Firstly, sensitivity analyses are performed to validate the sampling method: a comparison between Monte Carlo Simulations (MCSs) [14] and the adopted Sobol' Quasi Random Sampling (QRS) [15], which allows for a significant reduction of the number of samples and consequently of the computational effort, is shown. Then, trends of impedance functions in the frequency range of practical interest for seismic engineering is discussed. Finally, a second sensitivity analysis on the variable parameters highlights the different role of each one on the final output.

2 VARIABILITY OF SOIL AND FOUNDATION MECHANICAL PARAMETERS

In the sub-structure method, the impedance matrix defines the relationship between forces and displacements of the soil-foundation system that can be used in the inertial interaction analysis to simulate the foundation compliance. For a single pile of circular cross-section, the impedance matrix assumes the form

$$\Im(\omega)=\begin{bmatrix} \Im_x & 0 & 0 & 0 & \Im_{x-ry} & 0 \\ & \Im_y & 0 & \Im_{y-rx} & 0 & 0 \\ & & \Im_z & 0 & 0 & 0 \\ & & & \Im_{rx} & 0 & 0 \\ & sym & & & \Im_{ry} & 0 \\ & & & & & \Im_{rz} \end{bmatrix} \tag{1}$$

whose terms are complex-valued and express the harmonic forces to be applied at the pile head, where the degrees of freedom of the system are condensed according to the sub-structuring approach, to produce its uncoupled vibration.

Each term of the matrix depends on the mechanical and geometrical characteristics of the entire system. To investigate the effects of the variability of mechanical parameters on the impedance functions, three fundamental quantities are considered as independent aleatoric variables: the soil density ρ_s, the shear wave velocity V_s and the concrete elastic modulus E_p. The latter is evaluated basing on the variability of the concrete cylindrical compressive strength f_c. Mean values (μ), standard deviations (σ) and Coefficients of Variation (COV) of the variable parameters are chosen to represent a reinforced concrete foundation in homogeneous sand soil [5, 16] starting from values available in the literature [4, 7]. Regarding the other parameters affecting the definition of the impedance functions (i.e. soil and concrete Poisson's modulus, soil damping, concrete density), deterministic values are assumed since available works in the literature [8, 17] reveal that their variability is negligible as much as their effect on the impedance functions. Table 1 shows the statistical distributions and their relevant parameters for the three examined parameters, while Table 2 lists the deterministic values adopted for soil and concrete Poisson's modulus, soil and concrete damping and concrete density.

Uncertainties	Statistical Distribution	μ	σ	COV
Soil density ρ_s [t/m^3]	Normal	17.5	-	10
Shear wave velocity V_s [m/s]	Lognormal	400	σ_{ln} =0.39	-
Concrete elastic modulus E_p [MPa]	Lognormal	3E4	σ_{ln} (f_c) =0.20	-

Table 1: Statistical distributions for ρ_s, V_s, f_c.

Mechanical parameters	value
Soil Poisson's Modulus v_s [-]	0.4
Concrete Poisson's Modulus v_p [-]	0.2
Soil Damping ξ_s [-]	0.05
Concrete Density ρ_p [t/m^3]	2.5

Table 2: Deterministic parameters.

3 CASE STUDY

In this paper, a single floating pile in a homogeneous deposit is considered, with L/d = 16, where L is the total length of the pile and d is its diameter. The analyses are performed through the analytical model proposed by the Authors in [13], in which the problem is formulated in the frequency domain, the pile is assumed to be a linear elastic Euler-Bernoulli beam, the soil is constituted by infinite independent horizontal layers, and the soil-pile interaction is included through Elastodynamic Greens' functions available in the literature. Figure 1 illustrates the case study and summarizes its geometrical and mechanical characteristics.

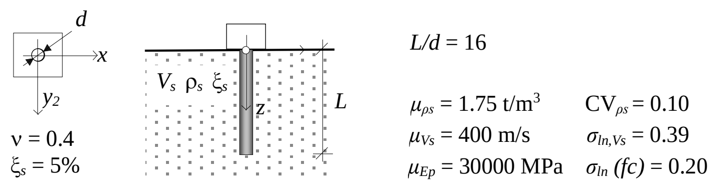

Figure 1: Case study.

4 GENERATION OF SAMPLES

The propagation of the uncertainties through the analytical model starts with a sampling procedure on the independent aleatoric variables according to their probability distributions. It is known that simple Monte Carlo sampling has a low efficiency since it requires a huge number of simulations to achieve a high precision in the results. Many different probabilistic approaches capable of reducing the required number of analyses without a loss in the accuracy of the final output are available in the literature, with many structural and seismic engineering applications (see the use of Latin Hypercube Sampling in [18], Importance Sampling [19] or Subset Simulation in [20, 21]). Among them, Quasi-Random Sampling (QRS) based on Sobol' sequence [15] is adopted in this paper. In the following sections, a brief recall on the QRS is provided, then a sensitivity analysis on the optimal number of simulations required is shown and commented to validate the adopted procedure in comparison with the standard Monte Carlo Simulation (MCS).

4.1 Quasi-Random Sampling (QRS)

Differently from the MCS, where samples are generated from a pseudo-random number source and result randomly distributed, in sense of being completely unpredictable, quasi-random samples derive from a particular algorithm in which the choice of the samples is steered to ensure a uniform cover of the sample space. Hence, quasi-random sampling techniques provide good results, since typical gaps or clusters of the MCS are avoided. Among the various quasi-random sequences available in the literature, Sobol' sequence can be used in sensitivity analysis [22], also for engineering applications [23]. More details about the efficiency of Sobol' sequence can be found in [24].

4.2 Comparison of Sobol' QRS and MCS

A preliminary study on the optimal number of required analyses is developed to validate the adopted sampling procedure, assuming results from the standard MCS as benchmark. The comparison is made on the basis of results of the sensitivity analyses.

Variance-based methods for sensitivity analysis [22] allow to study the influence of the input parameter uncertainties on the uncertainty of the final output, adopting variance as a measure of uncertainty. The first-order index, also called 'importance measure', represents the main effect contribution of each input factor to the variance of the output. Calling X_i the sample vector of the i-th input variable and Y_ω the output vector (one for each ω), the conditional expectation $E(Y_\omega \,|X_i)$ can be calculated by cutting the entire input variable domain into i slices and averaging the values of $(Y_\omega \,|X_i)$ within the same slice X_i. Thus, if the conditional expectation $E(Y_\omega \,|X_i)$ has a large variation across X_i, i.e. the variance of $E(Y_\omega \,|X_i)$, $V[E(Y_\omega \,|X_i)]$, is high, X_i is revealed to be important. The first-order sensitivity index $S_{i,\omega}$ for X_i is obtained dividing $V[E(Y_\omega \,|X_i)]$ for the unconditional variance $V(X_i)$:

$$S_{i,\omega} = \frac{V\left[E\left(Y_\omega \mid X_i\right)\right]}{V\left(X_i\right)} \tag{2}$$

Since the considered variable parameters have been assumed independent, the following condition holds

$$S_\omega = \sum_i S_{i,\omega} = 1 \tag{3}$$

where, for the present applications, i = 1, 2, 3 refers to the aleatoric parameters ρ_s, V_s and E_p. Values of S_ω greater than 1 suggests that a not sufficiently high number of samples have been considered. A comparison between MCS and Sobol' QRS is shown in Table 3 in terms of the mean value $\overline{S}$ of the sum of sensitivity indexes within the investigated frequency range, computed for different total numbers of processed analyses. It is worth noting that QRS performs much better than MCS. A reduction of the number of samples is gradually applied to reach an optimum on the minimum number of analyses to be performed with a maximum loss in precision equal to 1%. As a result, 10000 is chosen as the number of analyses to perform.

	Number of simulations			
	125000 (MCS)	25000 (QRS)	10000 (QRS)	5000 (QRS)
$\overline{S}$	1.164	1.002	1.006	1.012

Table 3: Evaluation of the sum of first-order sensitivity indexes.

5 RESULTS

5.1 Variability of impedance functions

Starting from previous considerations, 10000 analyses were executed on the soil-foundation model described in Chapters 2 and 3. The parameter distributions obtained through Sobol' QRS are illustrated in Figure 3 for the investigated aleatoric variables.

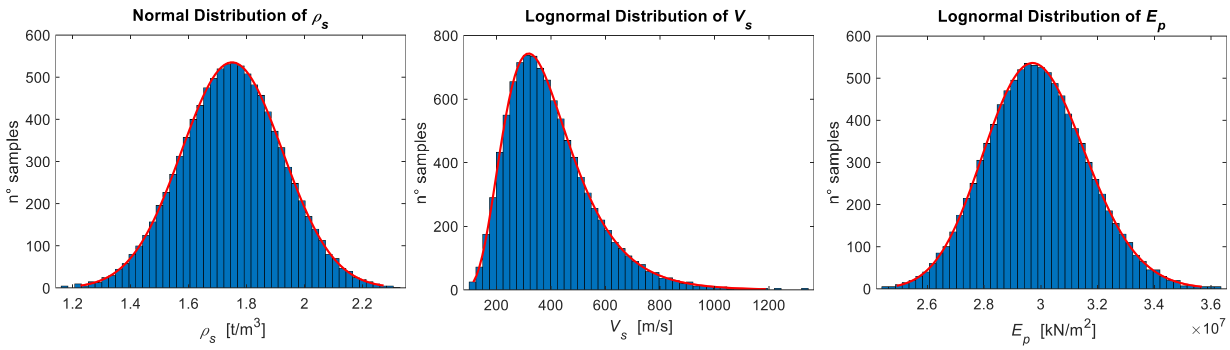

Figure 3: Sample distributions: (a) normal distribution of ρ_s, (b) lognormal distribution of V_s, (c) lognormal distribution of E_p

Boxplots in the following graphs show the statistic distribution of both real and imaginary components of the impedance functions in the frequency range 0-15 Hz, which may be considered of practical interest in earthquake engineering. In detail, the red dashed line represents the median trend of the response, the blue rectangular box encloses the 10th and the 90th percentile of the impedance at each frequency, black vertical dashed lines extend up to the 99th percentile of the output and red crosses represent the "outliers".

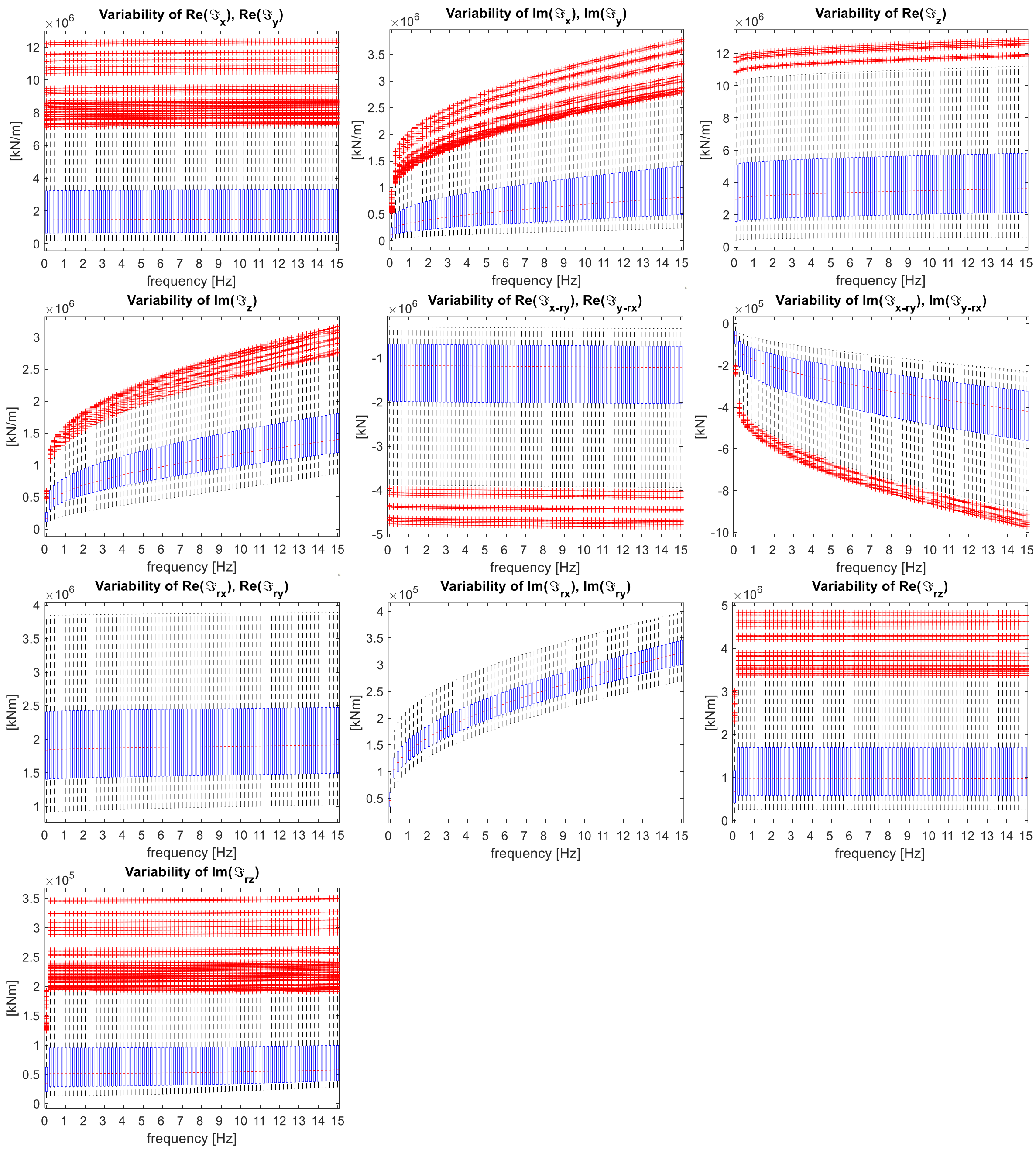

Figure 4: Impedance functions variability on the frequency range 0-15 Hz.

It is worth noting that the median trend well approximates the general one, while the ratio

$$r = \frac{\Im_{90th}}{\Im_{10th}} \tag{4}$$

between impedance values at 10th percentile and those at 90th can vary from 1.25 to 3, reflecting the great variability of the global response. Then, the variability of input parameters plays an important role in the definition of the soil-foundation response under the excitation produced by seismic waves, which should be properly addressed when the kinematic analysis is performed to reliably predict pile stress resultants, impedances and foundation input motion.

5.2 Sensitivity analysis on ρ_s, V_s and E_p

Sensitivity indexes are evaluated for each variable parameter. For sake of brevity, only sensitivity indexes of the real part of rotational impedance function $\Im_{rx}$ are illustrated in Figure 5. The graph shows the higher values on V_s, with S_{Vs} equal to 9 times S_{Ep} and over 10 times $S_{\rho s}$. Furthermore, a constant trend on the frequencies reveals that V_s is the most influential parameter for all the interest frequency domain. Similar results are obtained for other impedances functions. Hence, the response of the overall system is mainly governed by the variability of the shear wave velocity, while the soil density and the concrete elastic modulus have no predominant effect on the response in the frequency domain of interest.

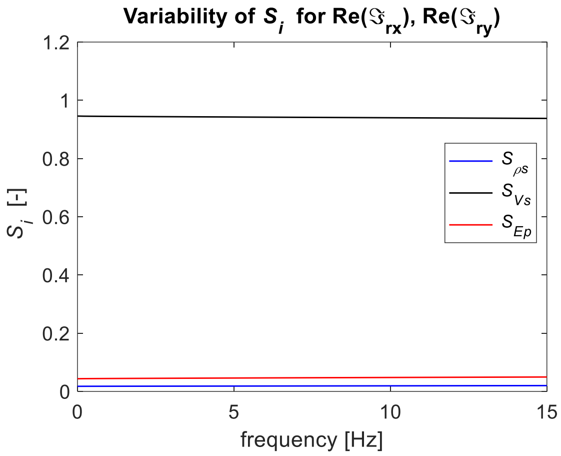

Figure 5: Sensitivity indexes of ρ_s, V_s and E_p; variability on the frequency range 0-15 Hz.

6 CONCLUSIONS

In this paper, first results of a research dealing with the effects of soil and foundation mechanical parameter uncertainties on soil-structure interaction analysis are addressed, focusing on the impedance functions of a single pile in a homogeneous soil deposit. Uncertainties are taken into account through statistical distributions of three fundamental parameters influencing the soil-pile system response, the soil density, the shear wave velocity and the elastic modulus of the pile concrete. The latter are assumed to be mutually independent. A Sobol's sequence Quasi-Random Sampling (QRS) technique is adopted to perform probabilistic analyses. A comparison between QRS and Monte Carlo Simulation (MCS) is discussed with the purpose of evaluating the optimal number of analyses to perform. The results of 10000 QRS analyses are shown and discussed, highlighting the not negligible role of uncertainties of the input parameters on the overall response. It appears that the variability of the input parameters should be included both in the kinematic analysis, where stresses on the foundation, features of the compliant base and foundation input motion are evaluated, and consequently in the inertial one, since the system "filters" the input experienced by the superstructure. Finally, sensitivity indexes of the three independent variables are calculated, addressing the soil shear wave velocity as the most influential parameter for the impedance variability. From the sensitivity analysis results, the primary influence of the shear wave velocity on the overall response allows to say that future applications may be developed considering the variability on V_s only, reducing the total computational effort without a significant loss in the result accuracy. Further studies are needed to investigate the role of uncertainties in the response of other kind of soil-substructure systems (for example, pile-group foundations), on both the impedance matrix and the FIM in order to characterize the kinematic interaction probabilistically.

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