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THE ROLE OF UNCERTAINTY OF MODEL PARAMETERS IN PSHA

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Abstract

Current approaches for the seismic hazard assessment, are mainly based on the classical formulation of the probabilistic seismic hazard analysis, widely known with the acronym PSHA. This procedure is able to compute the annual rate of exceedance of a set of ground motion intensity measures at a site of interest. During years, several efforts were performed for understanding and including the influence of uncertainties underlying the PSHA calculation. PSHA integral is a correct application of the total probability theorem, but it does not account for the uncertainties in model parameters which can be significant, since most of times they are derived from historical data and/or statistical catalogues. For this reason, this work aims to develop a robust semi-analytical formulation, able to assess how uncertainties in model parameters influence the seismic hazard curve's reliability. The proposed mathematical procedure uses the reliability index and its standard deviation for computing a design hazard curve, whose points are characterized by a fixed accepted level of risk. Results show how uncertainties in model parameters affect the hazard curve's dispersion, and how a better parameters' knowledge allows defining lower design values, with the same assumed risk.

Keywords: Instructions, ECCOMAS Thematic Conference, Structural Dynamics, Earthquake Engineering, Proceedings.

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1 INTRODUCTION

Current approaches for the seismic hazard assessment, are mainly based on the classical formulation of the probabilistic seismic hazard analysis, widely known with the acronym PSHA**.** For its nature, PSHA deals with several sources of uncertainties and aims describing effects of stochastic and unpredictable events. For this reason, since its early beginning, several efforts have been made for guaranteeing that all uncertainty sources were correctly accounted for [1]. Usually, scientific literature subdivides uncertainties into two main types: aleatory uncertainty and epistemic uncertainty. The former is the variability naturally inherent to a physical phenomenon, while the latter is due to the limited knowledge about the true model describing the aleatory variability ([2], [3]). Currently, on of the most widely adopted strategy for addressing seismic hazard epistemic uncertainty is the so-called logic-tree approach [4]. In the logic-tree, every node represents a potential source of epistemic uncertainty, and the corresponding outcoming branches represent the possible alternatives. This approach is general and considers both the inter-model uncertainty, due to uncertainty in model parameters, and the inter-model uncertainty, due to the uncertainty among models. Despite its wide adoption in PSHA [5], the use of logic trees is often debated, and there are many potential pitfalls that should be considered [6], mainly due to the fact that two potential interpretation of the logic-tree are possible ([7], [8], [9], [10]). For an extended discussion on the subject the reader is referred to [11] and [12]. In any case, logic tree results strictly depend on how logic tree branches are populated and on the weight assigned on each branch. In this context, the present work wants to contribute to the discussion on the reliability of seismic hazard estimate. In particular, this contribution aims to develop a standardized robust semi-analytical formulation, able to assess the influence of all possible source on variability involved in the seismic hazard computation, by treating them as random variables. The proposed method needs a carefully election of the best functional forms to be adopted in the calculation but does not need the definitions of any branches or weights. The formulation herein presented allows thus computing a given quantile of the seismic hazard demand by treating model parameters as random variables. The required computational time is limited, since no sampling or distribution fitting is required.

2 PROPOSED FORMULATION

The proposed formulation introduces a further development to the classical PSHA formulation, that is able to consider the variability of the hazard curve arising from uncertainties in model parameters ([13], [14]). For this reason, only a brief presentation of the consolidated PSHA procedure will be provided in the following, leaving space to a complete and extensive explanation of the proposed method. Classical PSHA integral aims computing the annual rate of exceedance $\lambda_{\mathrm{IM}\geq \mathrm{im}}$ of a set of ground motion intensity measures IM at a site of interest with the following formula:

$$\lambda_{IM\geq im}=\sum_{i=1}^{n_{sources}} \nu_{m_{min,i}} \int_{m_{min,i}}^{m_{max,i}} \int_{0}^{r_{max,i}} P\left[IM \geq im|m,r\right] f_{M,i}\left(m\right) f_{R,i}\left(r\right) dr dm \tag{1}$$

where $\nu_{m_{min,i}}$ is rate of occurrence of earthquakes greater than a suitable minimum magnitude $m_{min,i}$ of the i^{th} seismogenic sources (SZ), $f_{M,i}\left(m\right)$ is the magnitude distribution for the i^{th} ZS and $f_{R,i}\left(r\right)$ is the distribution of the source i^{th} -to-site distance. The exceedance probability of a threshold *im*, given a magnitude *m* and a distance *r* is provided by $P\left[IM \geq im|m,r\right]$. Once calculating $\lambda_{IM\geq im}$, using the Poissonian distribution it is possible to compute the probability of exceeding each ground motion level *im* in the next *T* years as

$$P\left[IM \geq im,T\right]=1\text{-}e^{-T\cdot\lambda_{IM\geq im}} \tag{2}$$

The exceedance annual rate computed in Eq. (1) depends on $\nu_{m_{min,i}}$ and on parameters which define $f_{M,i}\left(m\right)$, $f_{R,i}\left(r\right)$ and $f_{IM}\left(im\right)$ distributions; in the following, these model parameters are summarized in vector $\mathbf{\Theta}$. As a consequence, probability computed in Eq. (2), called in the following P_f (probability of failure), is function of *im*, *T*, and parameters $\mathbf{\Theta}$:

$$P\left[IM \geq im,T\right]=P_f\left(im,T,\mathbf{\Theta}\right)=1\text{-}e^{-T\cdot\lambda_{IM\geq im}\left(\mathbf{\Theta}\right)} \tag{3}$$

Following Gardoni et al. [15] the solid line of Fig. 1 is a point estimate $\hat{P}_f\left(im,T\right)$ of P_f calculated by computing $P_f\left(im,T,\mathbf{\Theta}\right)$ at a point estimate of $\mathbf{\Theta}$ (i.e., $\mathbf{\Theta}=\hat{\mathbf{\Theta}}$, where $\hat{\mathbf{\Theta}}$ could be the mean or median of $\mathbf{\Theta}$), or the predictive estimate $\tilde{P}_f\left(im,T\right)$ of $P_f\left(im,T,\mathbf{\Theta}\right)$ computed as the expected value of over $\mathbf{\Theta}$ ($\tilde{P}_f\left(im,T\right)=\int P_f\left(im,T,\mathbf{\Theta}\right)f\left(\mathbf{\Theta}\right)d\mathbf{\Theta}$). The risk-based hazard curve $P_{f,d}\left(im,T\right)$ (where *d* stands for design) is characterized by fixed accepted level of risk (Fig. 1); in other words, we want to define a quantile *q* such that the probability of having P_f bigger than $P_{f,d}$ due to uncertainties in model parameters it is equal to *q*; more formally

$$P\left[P_f\left(im,T,\mathbf{\Theta}\right)\geq P_{f,d}\left(im,T\right)\right]=q \tag{4}$$

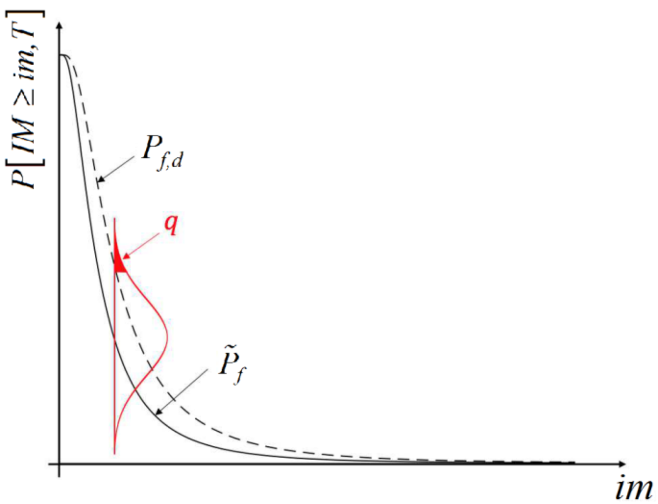

Figure 1: Definition of the $P_f\left(im,T,\mathbf{\Theta}\right)$ distribution, $\tilde{P}_f$, $P_{f,d}$ and *q*.

Since the exact evaluation of the $P_f\left(im,T,\mathbf{\Theta}\right)$ distribution requires nested calculation ([16]), this formulation adopts approximated quantile obtained by a first order analysis ([15]). The design failure probability $P_{f,d}$ can thus be obtained as

$$P_{f,d}\left(im,T\right)=\Phi\left[-\tilde{\beta}\left(im,T\right)+k\cdot\sigma_\beta\left(im,T\right)\right] \tag{5}$$

where $\Phi(\cdot)$ is standard normal cumulative function, $\tilde{\beta}(im,T)$ is the reliability index computed according to the definition of reliability index as $\tilde{\beta}(im,T) = \Phi^{-1}\left[1-\tilde{P}_f(im,T)\right]$ and $k \cdot \sigma_\beta(im,T)$ is the quantile of the distribution reflecting the assumed level of risk. Assuming β normally distributed, *k* is shown to be equal to:

$$k = \Phi^{-1}\left[1-q\right] \tag{6}$$

According to [16] the standard deviation of the reliability index σ_β can be computed by using a first order Taylor series expansion around the mean vector of the model parameters $\mathbf{M}_{\mathbf{\Theta}}$ as

$$\sigma_\beta{}^2(im,T) \approx \nabla_{\mathbf{\Theta}}\beta(im,T)^T \, \mathbf{\Sigma}_{\mathbf{\Theta\Theta}} \nabla_{\mathbf{\Theta}}\beta(im,T) \tag{7}$$

where $\mathbf{\Sigma}_{\mathbf{\Theta\Theta}}$ is the covariance matrix and contains the variances of the model parameters and their possible correlation. The gradient of β $\nabla_{\mathbf{\Theta}}\beta(im,T)$, is computed by applying the chain rule to the definition of reliability index as

$$\nabla_{\mathbf{\Theta}}\beta(im,T) = -\frac{1}{\varphi\left[\tilde{\beta}(im,T)\right]}\nabla_{\mathbf{\Theta}}P_f(im,T) \tag{8}$$

where $\varphi(\cdot)$ is the standard normal probability density function and $\nabla_{\mathbf{\Theta}}P_f(im,T)$ is the gradient column vector of $P_f(im,T,\mathbf{\Theta})$ evaluated in $\mathbf{M}_{\mathbf{\Theta}}$, that can be numerically computed with the definition of derivative as

$$\nabla_{\mathbf{\Theta}}P_f(im,T) = \left[\frac{P_f(im,T,\Theta+\delta\Theta) - P_f(im,T,\Theta)}{\delta\Theta}\right]_{\mathbf{M}_{\mathbf{\Theta}}} \tag{9}$$

3 CASE STUDY

The proposed formulation is applied to a case study, represented by a residential building structure (*T* = 50 years) located in Gemona (13.145, 46.288) and built on a soil-type C [17]. Two seismogenic zones SZs of the Italian source model are considered, SZ 904 and SZ 905 [18]. Main model parameters for each zone are derived from Barani et al. [19] and are reported in Tab. 1

	$m_{max,i}$	$m_{min,i}$	b_i	$\nu_{m_{min,i}}$
i = 1 – SZ 904	5.5	4.3	0.939	0.05
i = 2 – SZ 905	6.6	4.3	0.853	0.316

Table 1: Main parameters of each SZ.

Fig. 2 shows a map representing the site of interest and the considered seismogenic zones. Calculations are performed referring to two *IMs*, i.e. Sa (T = 0 sec.), the so-called Peak Ground Acceleration (PGA), and for Sa (T = 0.1 sec.). In this work, the ground motion prediction equation GMPE proposed by Bindi et al. [20] has been adopted.

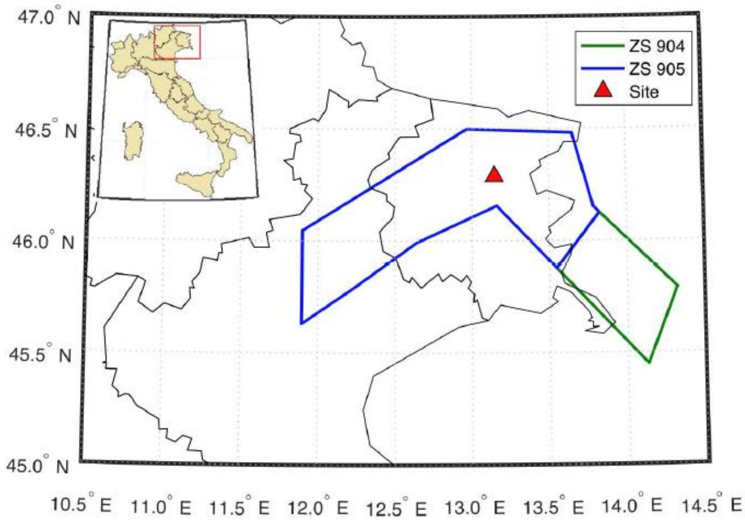

Figure 2: Site of interest and adopted SZs

The parameters that have been assumed as random are the G-R law parameters ($m_{max,i}$, $m_{min,i}$, b_i and $v_{m_{min,i}}$), and the main parameters contained in the adopted GMPE, i.e. e_1 that is the constant term of the regression, F_S and F_{sof} representing respectively the site amplification and the faulting mechanism, and σ_{tot} that is the standard deviation of the predicted *IM* logarithm. The final vector of model parameters $\mathbf{\Theta}$, is thus composed by the following 12 elements

$$\mathbf{\Theta}=\left[m_{max,1},m_{max,2},m_{min};b_1;b_2;v_{m_{min,i}};v_{m_{min,i}};e_1,F_S;F_{sof,1};F_{sof,2};\sigma_{tot}\right] \quad (9)$$

In this way, all the main possible sources of uncertainty, associated to model's parameters involved in the PSHA calculation, are considered. Table 2 lists the mean value and the standard deviation of each parameter, where the mean values are the elements of the vector $\mathbf{M_\Theta}$, while the variances represent the diagonal term of the covariance matrix $\mathbf{\Sigma_{\Theta\Theta}}$. For $m_{max,i}$, m_{min}, b_i, $v_{m_{min,i}}$ and σ_{tot}, a coefficient of variation $\delta=\sigma/\mu$ has been assumed, since no information on the parameters' standard deviation was available.

	$m_{max,1}$	$m_{max,2}$	b_i	b_i	$v_{m_{min,i}}$	$v_{m_{min,i}}$	m_{min}	σ_{tot}
μ	5.5	6.6	0.939	0.853	0.05	0.316	4.3	0.337
σ	0.55	0.66	0.0939	0.0853	0.005	0.0316	0.43	0.0337
δ = σ/μ	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	e_1	F_S	$F_{sof,1}$	$F_{sof,2}$	e_1	F_S	$F_{sof,1}$	$F_{sof,2}$
	Sa (T = 0 sec.)				Sa (T = 0.1 sec.)			
μ	3.672	0.24	-0.0544	0.105	3.796	0.247	-0.0451	0.111
σ	0.316	0.0322	0.0355	0.0296	0.365	0.0357	0.0367	0.0351
δ = σ/μ	0.086	0.134	0.652	0.282	0.096	0.145	0.814	0.316

Table 2: Mean value and standard deviation of the adopted parameters.

Finally, two percentile values have been assumed $q=\{0.05,0.15,0.25\}$ corresponding to $k=\{1.64,1.04,0.67\}$; when q = 0.5, P_f is computed (Eq. (2)), without any further information related to parameter uncertainties (k = 0). When $k\approx\pm1$ the approximate 15% and 85% percentile bounds of P_f are calculated. Fig. 3 shows results of the proposed formulation which allows characterizing P_f uncertainty and its distribution; each design hazard curve is characterized by a constant level of assumed risk q (for completeness also curves corresponding to (1- q) have been plotted). The two different spectral accelerations show a similar behavior; in particular, quantiles of Sa (T = 0.1 sec.) are less close, in absolute terms, to the hazard curve computed without considering parameters uncertainties.

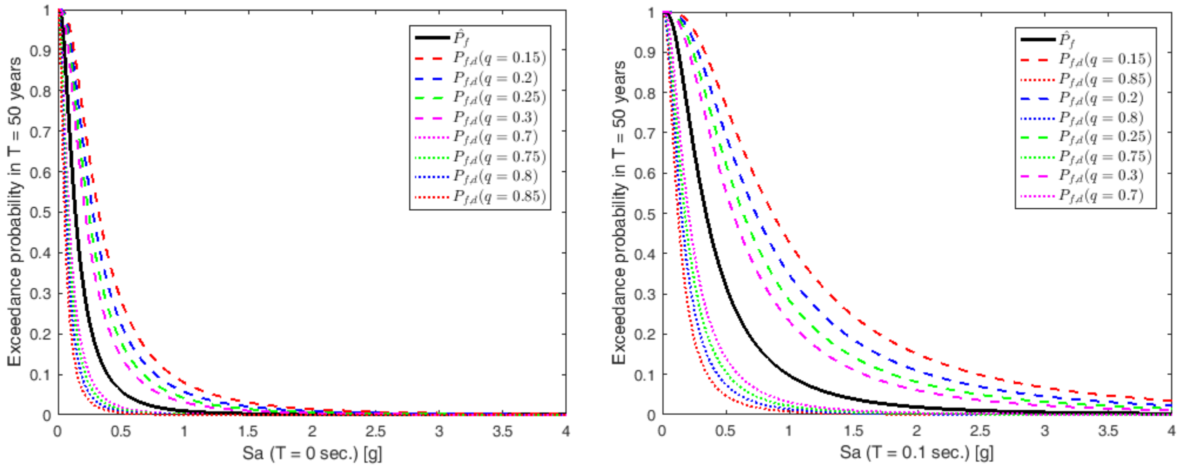

Figure 3: Hazard curve for different values of *q:* a) Sa (T = 0 sec.), b) Sa (T = 0.1 sec.).

For sake of completeness and for showing the influence of the covariance matrix, a smaller fictitious covariance matrix $\mathbf{\Sigma}_{\Theta\Theta}^{\text{small}}$ is assumed with the coefficient of variation of all parameters equal to $\delta^{\text{small}} = \delta / 2$ representing a hypothetical case in which a more accurate knowledge of the distribution parameters is available. Table 3 lists the mean value and the standard deviation of each parameter adopted in this second case.

	$m_{max,1}$	$m_{max,2}$	b_i	b_i	$v_{m_{min,i}}$	$v_{m_{min,i}}$	m_{min}	σ_{tot}
μ	5.5	6.6	0.939	0.853	0.05	0.316	4.3	0.337
σ	0.275	0.330	0.047	0.043	0.003	0.016	0.215	0.017
δ = σ/μ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
	e_1	F_S	$F_{sof,1}$	$F_{sof,2}$	e_1	F_S	$F_{sof,1}$	$F_{sof,2}$
	Sa (T = 0 sec.)				Sa (T = 0.1 sec.)			
μ	3.672	0.24	-0.0544	0.105	3.796	0.247	-0.0451	0.111
σ	0.158	0.016	0.018	0.015	0.182	0.018	0.018	0.018
δ = σ/μ	0.043	0.067	0.326	0.141	0.048	0.072	0.407	0.158

Table 3: Mean value and standard deviation of the adopted parameters.

Fig. 3 shows as that $P_{f,d}$ curves are closer to the one computed without considering parameters' variability. This behavior is clearly due to the lower uncertainty associate to the input parameters. In other words, a better initial knowledge allows considering a lower seismic hazard curve with the same assumed level of risk q.

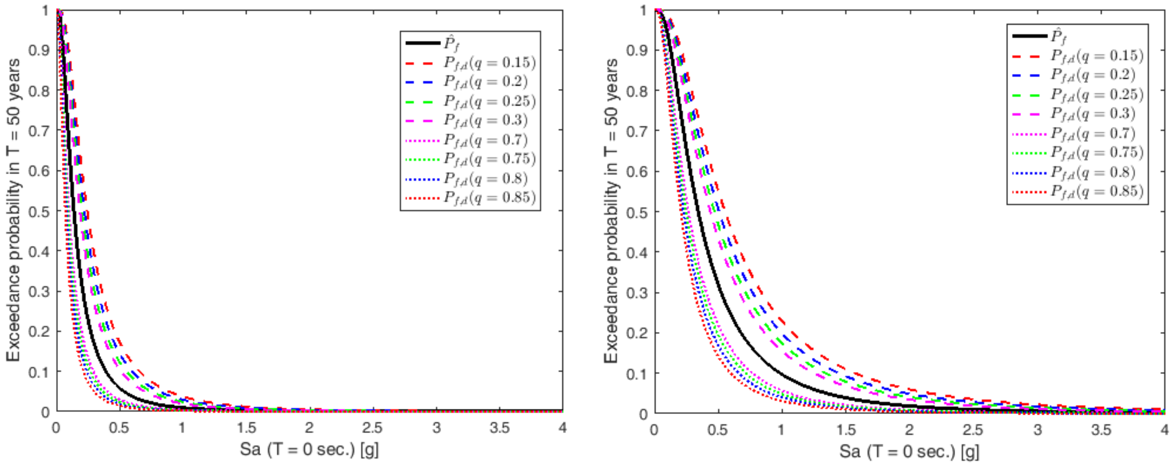

Figure 3: Hazard curve for different values of q computed with a reduced covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\Theta\Theta}}^{\text{small}}$: a) Sa (T = 0 sec.), b) Sa (T = 0.1 sec.)

4 CONCLUSIONS

This work presented a mathematical formulation able to consider the effect of parameters' uncertainty, on the seismic hazard curve. In particular, a procedure for the seismic hazard computation based on a fixed level of risk was proposed. The proposed framework allows computing a specific target quantile of the hazard curve, by mean of an immediate semi-analytical procedure that does not need any sampling or distribution fitting. The formulation is general and flexible and can be easily extended to hazard curve obtained by considering the entire seismic sequence, with foreshocks and aftershocks. Furthermore, the proposed methodology is also an important knowledge tool, since it allows finding the most impacting parameter on final results without running extensive analysis.

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