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Eccomas Proceedia COMPDYN (2021) 2014-2021

ECCOMAS
Proceedia

COMPDYN 2021
8th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Streamed from Athens, Greece, 28 - 30 June 2021

APPLICATION OF WAVELET TRANSFORMS ON ACCELERATION RECORDS: PRACTICAL APPROACH

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Abstract

Building floor accelerations recorded during earthquakes represent valuable data which contribute to the improved understanding of structural dynamic behavior and the state of the art and practice. After recording, all accelerations are considered to be "raw" (or unprocessed, uncorrected, unfiltered), and need to be processed before they can be used. In practice, the most common approach is to apply the conventional high- and low-pass filtering methods. However, some records cannot be corrected in such a manner and remain practically useless. The problems which may occur when conventional filtering methods are applied are: (1) inability to distinguish frequencies of noise and signal, (2) change of noise characteristics in time, and (3) a need to estimate and pre-select the parameters of a chosen filter function. These issues can be avoided if wavelets are used for signal processing. In mathematical terms, wavelets are finite wave-like functions which are able to capture local changes, and are used to transform a signal into a representation which shows its properties in the time-frequency domain. Such transformation is known as a wavelet transform. The continuous wavelet transform (CWT) was applied to recordings obtained from a 3-storey reinforced concrete building shake table test performed at the E-Defense facility in Kobe, Japan. It was demonstrated that the CWT represents a useful tool for the visual interpretation of the energy localization in the time-frequency domain, and that there is a large potential for the CWT application in earthquake engineering.

Keywords: Wavelet, Transform, Acceleration, Noise, Filtering.

ISSN:2623-3347 © 2021 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of COMPDYN 2021.
doi:10.7712/120121.8619.18589

1 INTRODUCTION

Accelerations recorded during earthquakes provide valuable data which can be utilized in various analyses to contribute to the expansion of knowledge and the improvement of the states of the art and practice. After they are recorded, all accelerations are considered to be "raw" (or unprocessed, uncorrected, unfiltered). So, in order to make them applicable, they need to be processed (i.e. "corrected"). In practice, the most common approach is to apply conventional high- and low-pass filtering methods. However, some records cannot be corrected in such a manner, and they practically remain useless.

An overview of the conventional filtering methods' shortcomings can be found elsewhere [1] and will not be discussed here in detail. Nevertheless, the main problems which usually occur are: (1) an inability to distinguish frequencies of noise and signal, (2) the change of noise characteristics in time, and (3) a need to estimate and pre-select the parameters of the chosen filter function. These issues are avoided if wavelets are used for signal processing, and this fact was recognized by researchers and engineers [2, 3]. In mathematical terms, wavelets are wavelike functions that satisfy certain requirements, and they are used to transform a signal into a representation which shows the information of the signal in a more useful form. Such transformation is known as a wavelet transform, which is basically a convolution of the wavelet with the signal [4].

Analysis of a signal by using a wavelet transform starts with the selection of a single wavelet, usually referred to as an "analyzing" or "mother wavelet". Wavelets are manipulated by translation (i.e. movement along the time axis) and dilation (i.e. stretching or compressing), and they are used to decompose the signal into a series of basis functions of the finite length. Understanding the concepts of the scale-varying basis functions is key to understanding wavelets [5]. In a way, roughly speaking, the process can be seen as the "time-frequency" decomposition of the signal, where "modes" represent individual parts of the signal. The essential difference between the wavelet transform and the well-known Fourier analysis is that the former can distinguish localized events at different times at the same frequency [6]. There is a wide selection of wavelets that can be used in practice, and they can roughly be divided into real and complex ones. The choice of wavelet depends on the signal properties and the purpose of analysis. The most commonly used wavelets are Haar, Daubechies, Mexican Hat, Morlet, Meyer, Morse (some of these are shown in Fig. 1), and information about them is widely available in the literature.

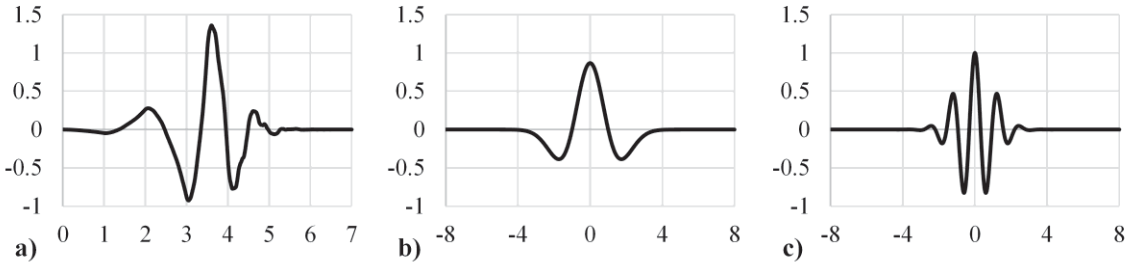

Figure 1: Often used wavelets: (a) Daubechies of order 4, (b) Mexican hat, and (c) Morlet.

This paper provides an insight into the practicality and possible benefits of using wavelets for raw acceleration processing. The paper is written from a practical point of view rather than in a mathematical manner, though the following two sections cover fundamental mathematical terms related to the continuous and discrete wavelet transforms. The benefits of the application of wavelets in the processing of acceleration records will be highlighted to encourage their usage in engineering practice.

2 CONTINUOUS WAVELET TRANSFORM

As previously noted, a function must fulfil certain criteria in order to be classified as a wavelet. These are: (1) its energy is finite, (2) it has a zero mean, and (3) the Fourier transform must be both real and vanish for negative frequencies for complex wavelets [4]. If the mother wavelet is denoted as $\psi(t)$, the continuous wavelet transform (CWT), $T(a,b)$, is defined as:

$$T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \psi^{*}\left(\frac{t-b}{a}\right) f(t)dt \quad (1)$$

where $f(t)$ is a signal function, and a and b denote the dilation and translation parameters, respectively. Both parameters are real numbers, and a is always positive. It should be noted that the asterisk denotes that the complex conjugate of the wavelet is used, which needs to be taken into account only when complex wavelets are applied. The plot of $T(a,b)$ against a and b is commonly referred to as "transform plot".

Integration of the wavelet and signal product described by Eq. (1) represents a convolution. In the sense of its energy, the normalized wavelet function can be written as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad (2)$$

and so Eq. (1) becomes

$$T(a,b) = \int_{-\infty}^{+\infty} \psi^{*}_{a,b} f(t)dt \quad (3)$$

CWTs are operable at every scale, they are performed over a continuous range of a and b, and the wavelet is shifted smoothly over a full domain of the considered signal. There are several advantages in applying the CWT. For example, the movement of wavelet along the signal enables identification of the "coherent structures" within the signal, whereas the Fourier transform does not provide any information on the nature of the signal local features. Also, sudden discontinuities in the signal can be easily identified by the CWT. Furthermore, the selection of $\psi(t)$ can be such that the $T(a,b)$ contains sufficient information for the reconstruction of the signal function $f(t)$, and this is possible by using the inverse wavelet transform. However, it should be mentioned that in the inverse transform the original wavelet function is used, not the conjugate one. Thus, the integration over a range of a scales rather than all a scales makes basic filtering of the original signal possible [4].

Under the assumption that the signal contains a finite energy, the signal's total energy can be obtained by integrating its squared magnitude

$$E = \int_{-\infty}^{+\infty} \left|f(t)\right|^{2} dt \quad (4)$$

It is possible to determine the part of the energy corresponding to the particular scale a and location b as

$$E(a,b) = \left|T(a,b)\right|^{2} \quad (5)$$

Graphical representation of $E(a,b)$ is called "scalogram", and it provides a useful picture of the non-stationary process nature and energy distribution, which is shown in the application example later in the paper. It should be noted that scalograms and transform plots have similar forms, which is a consequence of their definitions.

A known issue with the CWT analysis is the so called "edge" or "boundary" effect. It arises from the fact that the signal data set is finite and that it usually needs to be analyzed as a whole. Practically speaking, wavelets extend beyond the signal edges, and the transform is disrupted. In transform plots and scalograms, the region that is potentially affected by the edge effect is called the "cone of influence". Its extent is proportional to the wavelet width (i.e. it increases linearly with parameter a). However, since logarithmic scale is usually used for a, the cone of influence becomes curved. The extension of the cone depends on the wavelet scale. Unfortunately, the precise way to determine the extent of cone at each scale does not exist, so it is rather a subjective decision. For instance, for Morse wavelets, which are used in the example in this paper, a useful proposal is available in the literature [7].

3 DISCRETE WAVELET TRANSFORM

In order to present the basics of the discrete wavelet transform (DWT), firstly consider Eq. (2), a continuous signal $f(t)$, and the discrete values of parameters a and b, which are linked to each other [4]. Thus, the discretization of the wavelet has the following form:

$$\psi_{m,n}(t) = \frac{1}{\sqrt{a_0^m}} \psi\left(\frac{t - nb_0 a_0^m}{a_0^m}\right) \tag{6}$$

where a_0 is the selected dilation step larger than 1, b_0 is the location parameter larger than 0, and m and n are integers that control the wavelet dilation and translation, respectively.

By considering Eq. (6), the wavelet transform of the signal $f(t)$ becomes

$$T_{m,n} = \int_{-\infty}^{+\infty} \frac{1}{a_0^{m/2}} \psi\left(a_0^{-m} t - nb_0\right) f(t) dt \tag{7}$$

In the case of the DWT, the values of $T_{m,n}$ are known as "wavelet coefficients". In practice, the values of a_0 and b_0 are often chosen to amount to 2 and 1, respectively. These values lead to the so called "dyadic grid" arrangement, a simple and efficient discretization which leads to an orthonormal wavelet basis. By considering $a_0 = 2$ and $b_0 = 1$, Eq. (6) becomes

$$\psi_{m,n}(t) = 2^{-m/2} \psi\left(2^{-m} t - n\right) \tag{8}$$

Moreover, it is common to select the discrete dyadic grid wavelets to be orthonormal (i.e. orthogonal to each other) and normalized so that they have unit energy. In other words, the product of each wavelet with all others in the same dyadic system is zero. This way, the information contained in each coefficient $T_{m,n}$ is not repeated elsewhere, which provides the possibility to fully recover the original signal. If Eq. (8) is considered, the DWT can be expressed as

$$T_{m,n} = \int_{-\infty}^{+\infty} \psi_{m,n}(t) f(t) dt \tag{9}$$

The considered signal can be reconstructed from the inverse DWT:

$$f(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} T_{m,n} \psi_{m,n}(t) \tag{10}$$

Note that for the DWT described by Eq. (9) the transform integral is continuous over the discretized grid. Thus, the summation in Eq. (10) provides the return of the exact signal. It is important to note though that there is a difference between the DWT and the so called discretized CWT, in which the discrete approximation of the transform integral is usually involved, in order to be able to perform its computation. Such computing of the CWT is quite robust, and in practice the fast Fourier transform is used instead.

Finally, the energy of the signal can be determined as

$$E = \int_{-\infty}^{+\infty} \left| f\left(t\right) \right|^2 dt = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left| T_{m,n} \right|^2 \tag{11}$$

4 EXAMPLE OF APPLICATION

In this section an example of application of the CWT is presented, and the application of the DWT may be found in the works of Pan and Kusunoki [2] and Kusunoki et al. [8], who used it to extract the predominant building response for structural health monitoring purposes. The signal that was analyzed is the absolute acceleration at the second floor of a 3-storey reinforced concrete frame building specimen recorded during shake table tests at the E-Defense facility in Kobe, conducted between the 3rd and 7th December 2019. The test was a part of the Tokyo Metropolitan Resilience Project, Subproject C, Theme 2 [9]. Some other recordings from the same building were previously subjected to the CWT by Yeow and Kusunoki [10].

The building properties and the description of ground motions used as input during the test are available in Yeow et al. [11]. For the analysis presented in this paper, the fourth applied motion was considered (denoted as 1.5–2 in [11]). During that input motion the structural response was highly nonlinear. Raw absolute acceleration under consideration is shown in Fig. 2. It was normalized with the respect to its absolute peak value, so it is dimensionless.

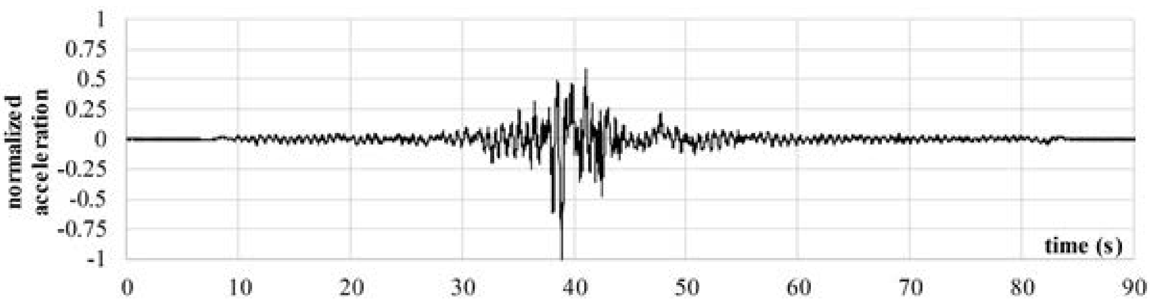

Figure 2: Normalized absolute acceleration recorded at the second floor of the considered building.

The CWT was applied to the recorded signal by using the *cwt* function which is integrated in the Wavelet Toolbox of MATLAB R2019b. The analytic Morse wavelet was adopted as the mother wavelet. The reconstruction of the signal was obtained by using the *icwt* function, which inverts the CWT coefficient matrix, by using the same predefined default parameters of the Morse wavelet as the *cwt* function. In order to perform filtering using the *icwt* function the frequency range of interest needs to be specified. This can be obtained using scalogram plots.

The scalogram obtained for the considered record is shown in Fig. 3. The dashed white line marks the cone of influence described earlier. Horizontal and vertical axes show time and frequency in the logarithmic scale, respectively, and the wavelet transform magnitude scale is shown on the right. The interpretation of the scalogram presented in Fig. 3 is quite straightforward. The parts of it in which the magnitude is sizeable are either related to the response of the structure, or to the noise and certain anomalies (e.g. a loose cable moving during shaking).

In the considered case, the influence of the response in the fundamental mode is localized at approximately 1.5 Hz, whereas the influence of the second mode is obvious at 10 Hz. It can be seen that the scalogram can be used for the visualization of modes.

As noted above, in order to perform filtering, it is necessary to select the frequency range of interest. It is usually done intuitively, which may be seen as a limitation of the CWT approach. Herein, the frequency range of 0.2 to 12 Hz was chosen. In this way, the noise at frequencies above 12 Hz was eliminated. A comparison of the raw and corrected accelerations, normalized to their absolute peak values, is shown in Fig. 4. It is obvious that the spikes are eliminated, and that the original signal is successfully recovered.

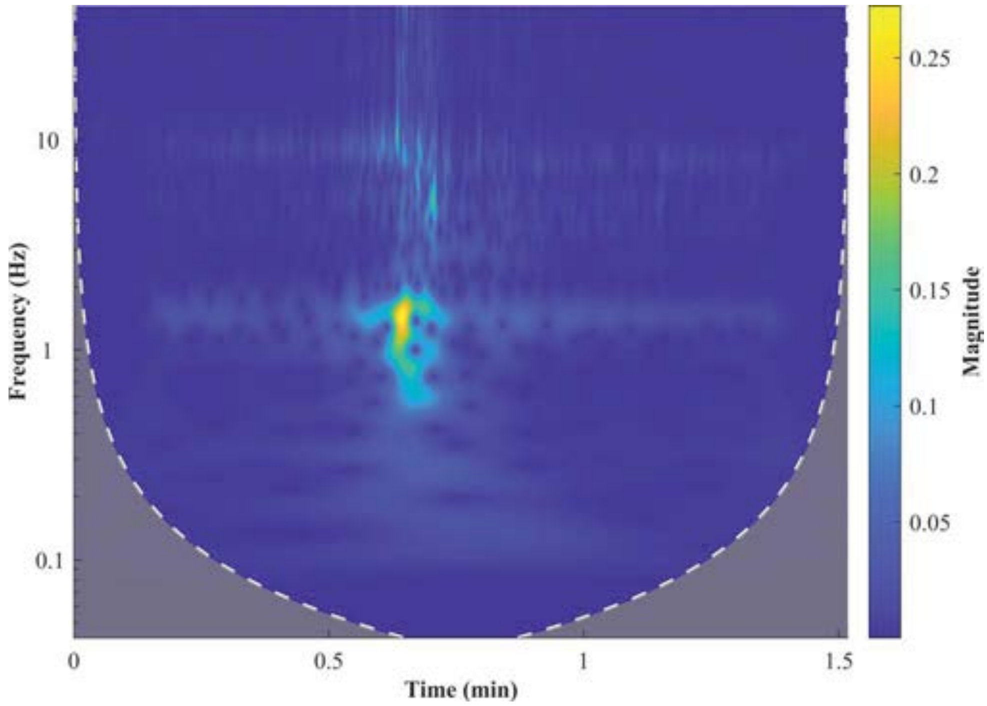

Figure 3: Scalogram of the considered absolute acceleration record.

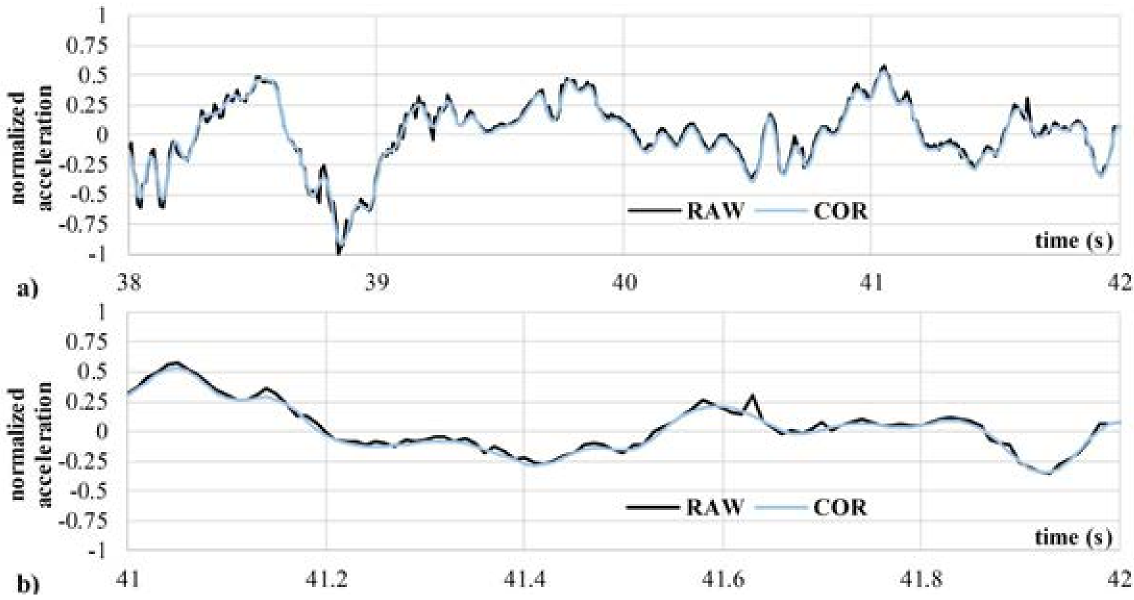

Figure 4: Comparison of the raw and corrected absolute accelerations in (a) the interval of 38 to 42 s, and (b) between 41 and 42 s.

The corrected acceleration can be integrated in order to obtain velocity and displacement, but it usually needs to be baseline-corrected first. This known issue is beyond the scope of the paper, and more details can be found elsewhere [1]. Another issue is that wavelets have zero-mean, meaning that the acceleration signal and its integrals (i.e. velocity and displacement) will always eventually return to zero. This poses an issue with obtaining residual deformations, though such issues already exist when obtaining displacements from acceleration data using conventional means [12]. Alternatives such as using Kalman filters could be an option [13, 14], but this is outside the scope of this research and will not be elaborated upon further here.

While not shown here, the DWT approach has been used in literature to extract out the predominant hysteretic response of buildings based on floor acceleration recordings. This is done by: (i) decomposing the acceleration recordings of each floor using the DWT, (ii) obtaining the relative floor acceleration and displacement response for each decomposition level, (iii) simplifying the multi-degree-of-freedom response into an equivalent single-degree-of-freedom response for each decomposition level, (iv) using the equivalent response of each decomposition level to identify which should be eliminated, and (v) re-computing the equivalent single-degree-of-freedom response using the remaining decomposition levels. Examples of the application are available in literature [2, 3, 8].

5 CONCLUSIONS

Wavelet transforms represent an alternative to the conventional high- and low-pass filtering methods, with whose application it is sometimes not possible to correct the acceleration data recorded during earthquakes. Mathematical background of the continuous and discrete wavelet transforms was briefly provided, along with the example of application of the former on the acceleration recorded during the shake table test. Application of the continuous wavelet transform by using the available commercial software was demonstrated, along with interpretation of the scalogram data and discussion on the pros and cons of the approach. Even though the use of wavelet transforms in earthquake engineering started only relatively recently, their large potential is already recognized.

ACKNOWLEDGMENTS

The present work is partially supported by the Tokyo Metropolitan Resilience Project of the National Research Institute for Earth Science and Disaster Resilience (NIED). The first author is grateful for the opportunity to participate in the Long-term visiting program organized by the Earthquake Research Institute (ERI), The University of Tokyo, where he spent four months working as a Project Researcher.

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