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PRELIMINARY CONSIDERATIONS ON THE LOSS OF SUPPORT FOR PRECAST BEAM ELEMENTS

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Abstract

One of the main objectives to be achieved for the safety of industrial precast buildings with simple friction beam-to-column connections is the reduction of their relative displacement. This goal is generally achieved through a global strengthening of the building or by locally introducing mechanical connecting devices between the elements. However, for low seismicity areas, these interventions may not be required, and it could be interesting to evaluate the level of safety achieved by simple friction; such an information would allow for instance the prioritization of the retrofit interventions.
This paper aims to establish criteria for evaluating the loss of support probability in existing precast buildings in the case of a seismic event. The behavior of the friction connection is evaluated through a simplified model that describes the behavior of a portal frame composed of two columns and one beam. The equations of motion are derived, and a parametric analysis is performed by investigating the influence of the geometry of the structural elements, the nonlinear behavior of the columns, and the friction coefficient used in the column-beam connections for varying horizontal and vertical seismic acceleration. The objective of the sensitivity analysis is to provide some preliminary considerations about the influence of the main parameters that characterize the existing precast structure on the loss of the support probability.

Keywords: Precast buildings; friction connection; beam-to-column connection; simplified models.

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1 INTRODUCTION

The Italian building stock encompasses many precast concrete structures; the main advantage of this construction technology lies on high quality control materials and the short construction time. The typical structural layout of these buildings consists of a single story with a rectangular plan in which the bearing structure is made of prefabricated columns supporting double tapered prestressed beams. A great share of these buildings was designed before the enforcement of modern anti-seismic regulation codes and, therefore, they were not designed to withstand the horizontal loads.

The seismic events that occurred in 2012 in Emilia-Romagna, Veneto and Lombardy, highlighted the main vulnerabilities of existing precast industrial buildings [1, 2, 3, 4, 5, 6, 7, 8, 9]; the main vulnerabilities consist in the breaking of the fork, the overturning of the infill panels, the reaching of the ultimate rotation of the column, the loss of the beam-to-column support and/or the loss of the support of other roofing elements [10, 11, 12, 13, 14, 15, 16, 17]. The loss of the support is due to the absence of adequate structural detailing in the connection between beam and columns.

To date, in Italy, the implementation of mechanical devices between the structural elements of precast buildings (e.g. column and beam at the beam-to-column support) is required for new buildings and new technological solution have been studied in recent years [18]; such a detailing was not common practice in the past. However, for low seismicity areas, the seismic retrofit of these connections may not be necessary, and it could be interesting to evaluate the level of safety achieved by simple friction; such an information would allow for instance the prioritization of the retrofit interventions.

The problem of loss of support in friction beam-column connections has been previously addressed: Magliulo et al. [6] and Belleri et al. [1] evaluated the minimum value of the friction coefficient required to avoid the sliding of the beam under the hypothesis of perfect correlation between the maximum values of the horizontal and vertical components of the seismic inputs; Demartino et al. [19] approached the problem by defining simplified numerical finite element models of the friction beam-column connection, starting from the equations of motion of the friction connection considering a rigid block model having two degrees of freedom. Furthermore, the effect of the seismic-hazard disaggregation was considered. The results showed that the minimum friction coefficient required to prevent the sliding of the beam depends on the period of the structure and the damping coefficients used in the equations.

In this paper, a new analytical model is proposed for the evaluation of the seismic response of a simple portal frame with friction beam-column connections. The portal frame is described by means of a dynamic system with four degrees of freedom (DOF), 3 horizontal and 1vertical. The main novelty introduced by the present work relies on the simplified model proposed and on the evaluation of the influence of various parameters. Section 2 describes the proposed numerical model. In section 3, parametric analyses are carried out to evaluate the influence of the main parameters on the global response of the simplified model. In section 4, the analytical model is compared with finite element nonlinear time history analyses, carried out on a selected reference structure. Finally, a concise discussion of the results obtained in sections 3 and 4 and some considerations on possible improvements of the proposed model are made.

2 SIMPLIFIED SYSTEM DEVELOPMENT

The simplified 4DOF system used to describe the transverse portal-like response of a typical precast industrial building is here introduced and described. Similar models were adopted in previous research work [20, 21, 22, 23, 19].

2.1 Simplified model

The lateral displacement of the 2 columns and the displacement of the beam is described through a system characterized by 3 horizontal DOFs plus 1 vertical DOF (Figure 1b); the simplified system consists of 3 masses connected by springs (k_1, k_{12}, k_{23}, k_3) and viscous dampers (c_1, c_{12}, c_{23}, c_3) and subjected to the ground horizontal and vertical accelerations ($\ddot{X}_g, \ddot{Y}_g$). The chosen degrees of freedom to describe the behavior of the system are the horizontal displacements of the top of the two columns (u_1, u_3), and the horizontal and vertical displacement of the beam (u_2, v_2). The simplified system is drawn in Figure 1b.

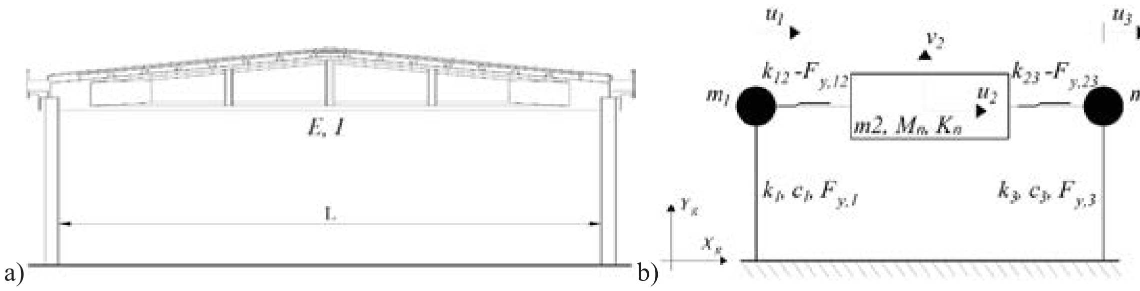


Figure 1: a) layout of a typical portal-like precast industrial building; b) simplified 3DOF system adopted.

The parameters used to describe the considered simplified systems are here presented; the subscripts refer to the element considered (i.e. 1 for the left-side column; 2 for the beam; 3 for the right-side column) or, in the case of connections, the elements that are connected by the link (e.g. subscript-*i* refers to the DOF of the element-*i*, while the subscript-*ij* refers to the link describing the relative displacement between the element-*i* and the element-*j*). The element-1 and the element-3 are described by the masses m_1 and m_3, the elastic stiffnesses k_1 and k_3, the damping coefficients c_1 and c_3, and, to consider the nonlinear behavior of the columns, the yielding forces $F_{y,1}$ and $F_{y,3}$, respectively. The element-2, represented by the mass m_2, is connected to the element-1 and element-3 by means of friction connections. The elastic stiffnesses k_{12} and k_{23} are associated for instance with the neoprene pads. The friction forces are $F_{y,12}$ and $F_{y,13}$.

As for the vertical component of element-2, the generalized mass (M_n) and the generalized stiffness (K_n) are introduced to represent the participating mass and stiffness of the *n*-th vertical mode of the beam. In this work only the first vertical mode is considered; consequently, the subscript -*n* is set to be equal to 1 (M_1, K_1).

In the analyses described in the following section, the main parameter investigated is the relative displacements between the beam and the supporting columns (δ_{12}, δ_{13}). For sake of clarity, δ_{12} can be calculated as the difference between u_2 and u_1; similar considerations can be drawn for δ_{13}.

2.2 Equations of motion and solving method

The free-body model of the 4DOF system is represented in Figure 2.

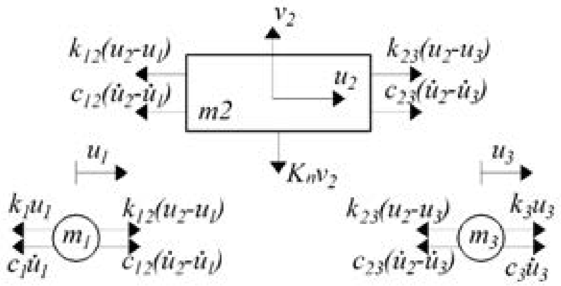


Figure 2: Free-body diagrams of the 3DOF system.

By enforcing balance to horizontal translation of element-1, element-3, it yields:

$$m_1\ddot{u}_1 + k_1 u_1 + c_1\dot{u}_1 - k_{12}(u_2 - u_1) - c_{12}(\dot{u}_2 - \dot{u}_1) = -m_1\ddot{X}_g \tag{1}$$

$$m_3\ddot{u}_3 + k_3 u_3 + c_3\dot{u}_3 + k_{13}(u_3 - u_2) + c_{13}(\dot{u}_3 - \dot{u}_2) = -m_3\ddot{X}_g \tag{2}$$

The horizontal equilibrium on element-2 is

$$m_2\ddot{u}_2 + k_{12}(u_2 - u_1) + c_{12}(\dot{u}_2 - \dot{u}_1) - k_{13}(u_3 - u_2) - c_{13}(\dot{u}_3 - \dot{u}_2) = -m_2\ddot{X}_g \tag{3}$$

The vertical component of element-2, expressed through its time-derivative in principal coordinates [24], is

$$\partial v/_{dt} = [-K_1 v_2 - M_1\ddot{v}_2]\cdot\frac{1}{M_1} \tag{4}$$ [1]

For a simply supported beam [24]

$$M_1 = {}^{m_2}/_2$$
$$K_1 = \frac{\pi^4 EI}{2L^3} \tag{5}$$

Where E and I are the elastic modulus and the moment of inertia of the horizontal beam, respectively, and L is the length of the beam.

From the vertical displacement expressed in principal coordinates, the shear at the beam ends ($V(0,t)$ and $V(L,t)$) can be derived as a function of the time (t).

$$V(0,t) = \left[-EI\cdot\left(\frac{\pi}{L}\right)^3\right]\cdot v(t)$$
$$V(L,t) = -V(0,t) = \left[EI\cdot\left(\frac{\pi}{L}\right)^3\right]\cdot v(t) \tag{6}$$ [2]

It is worth noting that $V(x,t)$ does not account for the mass m_2; the total vertical force on the column -i (N_i) can be calculated by the algebraic sum of $V(x,t)$ and the generalized mass multiplied by the gravity constant ($M_1 g$).

The inelastic behavior of element-1, element-3 and of the link simulating the simple beam-column friction support is modeled by the Bouc-Wen hysteresis law [25]. The nonlinear behavior of the columns is accounted for by substituting $k_i u_i$ (Eq. 1 and Eq. 2) with $P(t)$

$$P(t)_{column} = \alpha\cdot k_i\cdot u_i + (1-\alpha)k_i\cdot\delta_{y,i}\cdot Z(t) \tag{7}$$

where -i is 1 for element-1 and 2 for the element-2, α is the post yielding stiffness ratio, and Z is an internal variable whose behavior is described by its derivative:

[1] $\ddot{v} = \partial^2 \mathrm{v}/_{dt^2}$

[2] $\Phi'' = \partial^2\Phi/_{dx^2}$

$$\frac{dZ}{dt} = \left(\frac{1}{\delta_{y,i}}\right) \cdot \left(\dot{u_i} - \gamma \cdot |\dot{u_i}| \cdot Z(t) \cdot |z(t)|^{n-1} - \upsilon \cdot \dot{u_i} \cdot |z(t)|^{n}\right) \tag{8}$$

n, v, and γ are dimensionless quantities, n governs the smoothness of the curve in the proximity of the yielding point, v and γ control the size and the shape of the hysteretic loop ($|v|+|\gamma|=1$). In this case, the yielding force of the hysteresis model is derived as a function of the columns features and it is introduced through the parameter $F_{y,i} = k_i \cdot \delta_{y,i}$ (Eq. 7).

A similar procedure can be followed for the friction beam-to-column support; in this case, the subscript -ij is set equal to 12 and 23 for the left and right connection, respectively, and the displacements u_{12} and u_{23} refer to the relative displacement between the 2DOFs (u_2 - u_1 and u_2 - u_3). Moreover, to simulate the simple friction support, the yielding forces of the links ($F_{y,12}$ and $F_{y,13}$) are derived according to the Coulomb's Law. Defining a friction coefficient μ to characterize the contact between the column and the beam, the Coulomb's Law define the friction force ($F_{y,ij}$) as the product between μ and the total vertical force acting on the top of the column-i (N_i). Eq. 7 becomes

$$P(t)_{friction} = \alpha \cdot k_{ij} \cdot u_{ij} + (1-\alpha) \cdot (\mu \cdot (\pm V(0,t) + M_1 g)) \cdot Z(t) \tag{9}$$

The equations of motion are solved with the function Ode45 [26], a versatile ordinary differential equation solver that adopts the Runge-Kutta method with a variable time step. The algorithm requires the conversion of the second-order differential equations into an equivalent system of first-order equations.

3 PARAMETRIC ANALYSIS

The input parameters varied in the parametric analyses are summarized in Table 1.

Parameter	Symbol	Value or range	
Behavior factor	q	1, 1.75, 2.5, 3.25, 4.0	[-]
Damping ratio	ζ_i	1,3	[%]
Friction coefficient	μ	0.1337, 0.5	[-]
Vertical component	*V mode*	No, 1st Mode	[-]
Ground Motion	GM	Mirandola, Amatrice	[-]

Table 1: Input parameters varied in the parametric analyses.

For the sensitivity analyses, the masses m_1 and m_3 are assumed equal to half of the self-weight of the single column, the elastic stiffnesses k_1 and k_3 are calculated from the geometry of the columns and account for a 50% reduction of the elastic modulus of the reinforced concrete due to cracking. All these parameters can be derived by the layout and the column cross-section of the reference case described in the next Section. A viscous damping model is considered; the damping coefficients c_1 and c_3 are considered equal and they are calculated in the parametric analyses by varying the damping ratio ζ_i as indicated in Table 1 (with -i equal to 1 or 3), while the damping ratio of the vertical mode of the beam (ζ_2) is set equal to 0.01 [19]. As for the nonlinearity, n, v, and γ are assumed, for the columns, equal to 1, 0.5, 0.5, respectively, while equal to 25, 0.5, 0.5, for the simple friction support, respectively. In both cases the post yielding ratio is set equal to 0.001. The friction coefficient μ is varied among the values 0.1337 to simulate a neoprene-concrete interface [17] and 0.5 to consider a concrete-concrete interface [27]. The stiffness of the Bouc-Wen hysteresis of the friction support is set equal to $k_{12}=k_{23}$=49000 [N/m] [17] for μ=0.1337, while it is assumed equal to 1832461 [N/m] when the friction coefficient is set equal to 0.5. The cases with and without the vertical DOF are both performed to evaluate the influence of the vertical component. The analyses are performed considering as seismic input the main shocks recorded in the L'Aquila station (Italy) during the 2008 seismic events, and in the Mirandola station (Italy) during the 2012 seismic events.

3.1 Parametric analyses and preliminary considerations

The results of the parametric analyses are reported in Figure 3 as a function of q in terms of ratio between the relative displacement ($\delta_{12}= u_2- u_1$) and δ_R, corresponding to the value δ_{12} obtained in the reference case with $\zeta_1= \zeta_3$=3%, μ=0.1337, q=1, and without the vertical component (indicated in the plot with a full black circle). Considering the geometric symmetry of the system, the same results would be obtained plotting δ_{13}.

a) L'Aquila b) Mirandola

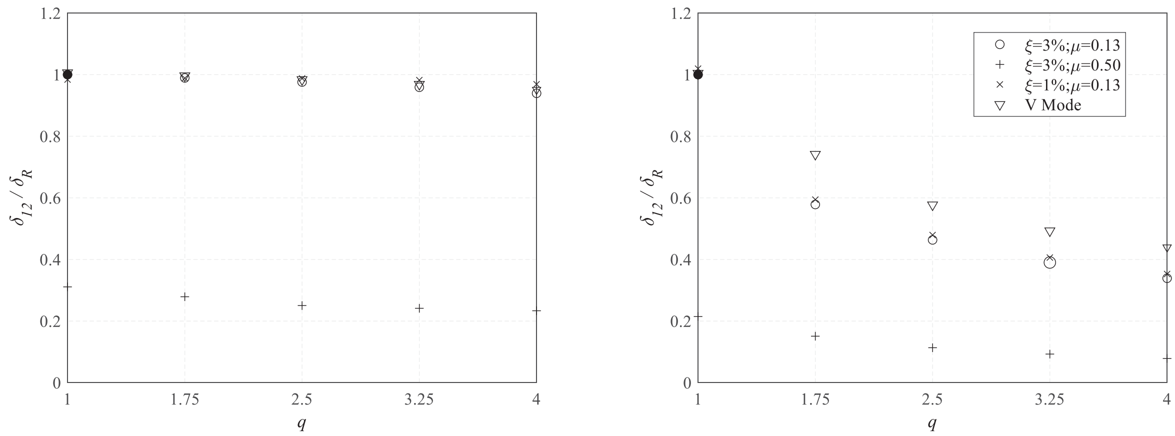


Figure 3: Parametric analyses results for the different ground motions considered. For both the GM the results are plotted as a function of q as the ratio between the relative displacement (δ_{12}) and δ_R corresponding to δ_{12} of the reference case ($\zeta_1= \zeta_3$=3%, μ=0.1337, q=1, No *V Mode*).

Increasing the behavior factor (q) the relative displacement (δ) decreases following an almost linear trend; q affects more the case of Mirandola (with q=4, almost 60% reduction) than the case of L'Aquila (with q=4 almost 10% reduction).

The friction parameter (μ) significantly affects the relative displacement between the beam and the columns; increasing μ such relative displacement decreases.

The relative displacements increase by considering the vertical component; the maximum increasing value is shown in the case of Mirandola (+30%); this is considered a reasonable result observing that Mirandola is the site in which the maximum values of the vertical component were recorded. As for the case of L'Aquila, the increase is lower. Introducing ζ=0.01 leads to variations that range between ±2% with respect to the reference case.

4 REFERENCE CASE

To validate the simplified model presented, time history analyses have been carried out on a reference case study resembling a typical '70s precast industrial building. The building is supposed to be located in Mirandola (Emilia-Romagna, Italy). The building has a rectangular plan and story height (under the beam) equal to 5.35 (m). The bearing structure is made of precast frames designed according to the regulation codes at that time. The main frame of the building is composed of 2 columns and a double-tapered beam with a net span equal to 10.65 (m); accordingly, an I-section beam with a variable height between 0.40 (m) and 1.50 (m) is supposed.

The portal-to-portal distance is about 6.70 (m) and an overall roof weight of 2.40 (kN/m^2) is assumed. The columns have a square cross-section 0.34x0.34 (m^2) with a longitudinal reinforcement ratio equal to 0.8 (%). Furthermore, each column has a RC fork at the top, in which the beam is housed thus creating a simple-friction connection. A C28/35 concrete (28 MPa characteristic cylindrical strength), and Feb38k (380 MPa characteristic yield strength) steel are considered.

4.1 Finite Element Model

The model and the analyses are carried out with the finite element software MidasGen 2020 [28]. Beams and columns are modeled as beam elements; the double-tapered beam is modeled as an elastic element with equivalent rectangular cross-section 0.12x0.85 (m^2). Being simply lying on the columns, it should not be directly damaged by seismic loading. To account for the nonlinear behavior of the columns, Takeda lumped plastic hinges are introduced.

As for the constraints, the columns are fixed at the base while the beam-to-column connection is modeled using a "Friction Pendulum" general link with a very high radius of curvature. The link in the global vertical direction of the beam-column connection is set to behave as a rigid compression-only element.

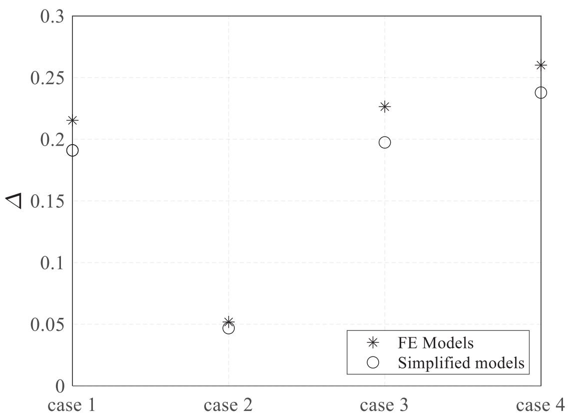
Parametric analyses are carried out for four different cases by varying the parameters ζ, μ, and *V Mode* reported in Table 1; the main shock recorded in the Mirandola station during the 2012 seismic events and a behavior factor (q) equal to 2.5 are assumed. The considered cases are summarized in Table 2.

4.2 Results and discussion

Figure 4 shows the results in terms of the ratio between the relative beam-column displacement and the beam support length indicated by the code ($\Delta = {\delta_{ij}}/{\delta_{REF}}$). In this case, δ_{REF} =11.55cm is considered [29].

According with the preliminary consideration made in Section 3, the results show that the relative displacement decreases when the friction coefficient increases (-16%), Δ increase when the vertical component (1^{st} *V Mode*) is considered; in particular, the relative displacement increases by 17% compared to the reference case. By varying the damping ratio (from ζ_i=3% to ζ_i=1%) an increase of the relative displacement of approximately 5% compared to the reference case is showed.

Second, with the empty circle, the results obtained from the finite element model and the simplified model are compared. From the comparison, it can be observed that the simplified model can accurately predict the relative displacement between the beam and the column at the beam to column support. In this case, the maximum variation is about -10% with respect to the FEM model; the underestimation of the relative displacement could be due to the different models used for describing the beam-column connection in the two models and to the higher number of degrees of freedom in the FE model.



	ζ_i	μ	*V Mode*
case 1	3%	0.13	No
case 2	3%	0.50	No
case 3	1%	0.13	No
case 4	3%	0.13	Yes

Table 2: Considered cases in the parametric analyses.

Figure 4: Parametric analyses results for the different model considered and comparison between the FEM and the simplified model. The results are expressed in terms of ratio between the relative displacement (δ_{ij}) and the length of the support (δ_{REF}).

5 CONCLUSIONS

In this paper, a simplified model to evaluate the behavior of precast portal frames with beam-to-column simple friction connection is presented. The behavior of the portal frame has been investigated through a 4DOF model and parametric analyses have been carried out by varying the main parameters of the frame and of the simple friction connection (q, ζ, μ, *V Mode*).

The results of the parametric analyses showed that the friction coefficient (μ) is the parameter that most significantly affect the relative displacement between the columns and the beam. The vertical component of the ground motion is another parameter that should be accurately considered in the evaluation of the beam- loss of support is; if the vertical component of the ground motion is not considered, there is an underestimation of the relative displacements between the beam and the columns, thus resulting in an unconservative approach. Finally, the more the non-linear behavior of the structure, the less is the relative displacement between the beam and the column; the parameter ζ (i.e. damping ratio) does not significantly affect the relative displacement. The results of the parametric analyses have been validated and assessed through a 2D finite element model.

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