

PARTICLE FILTER-BASED HYBRID DAMAGE PROGNOSIS CONSIDERING BIAS

Tianzhi Li¹, Francesco Cadini¹, and Claudio Sbarufatti¹

¹ Dipartimento di Meccanica, Politecnico di Milano
Milan, via La Masa 1, 20156, Italy

tianzhi.li@polimi.it, francesco.cadini@polimi.it, claudio.sbarufatti@polimi.it

Abstract

Hybrid prognosis combining both the physical knowledge and data-driven techniques has shown great potential for damage prognosis in structural health monitoring (SHM). Current practices consider the physics-based process and data-driven measurement equations to describe the damage evolution and the mapping between the damage state and the SHM signal (or the feature extracted from SHM signal), respectively. However, the bias between the measurements predicted by data-driven equation and the actual SHM measurements, arising from uncertainties like damage geometries and sensor placement or noise, can lead to inaccurate prognosis results. To account for this problem, this paper adopts a methodology typically applied for sensor fault diagnosis, and develops a new hybrid state space model with a bias parameter included into the state vector and the measurement equation. Particle filter (PF) serves as the estimation technique to identify the state and parameters relating to the damage as well as the bias parameter, and RUL can be predicted by the PF estimates and physics-based process equation. The numerical study about the fatigue crack growth shows the new model together with PF can provide satisfactory estimation and prediction results in case of bias in the measurement model.

Keywords: Structural Health monitoring, Fault Prognosis, Hybrid Model, Measurement Equation, Particle Filter.

1 INTRODUCTION

Prognostic is the process of predicting the future state and the remaining useful life (RUL) of a component or system. In the last decades, a great variety of damage prognosis techniques have been developed in SHM depending on the availability of physics knowledge and data. From the perspective of how the prognosis models are formulated, they can be distinguished into physics-based [1-3], data-driven [4-8] and hybrid methods [9-13]. Physics-based methods utilize specific mechanistic knowledge and theories to formulate a pure physics-based model, which describes the structural degradation phenomena as well as the links between the damage states and the SHM measurements. On the other hand, data-driven methods, resorting to data-driven modelling techniques such as neural networks [6] and Markov chains [8], attempt to use amounts of data to build the relationship between the internal degradation behaviour and the external observations. Hybrid methods, taken as a combination of the two above methods, usually consider the physics-based process and data-driven measurement equations to describe the damage evolution and the mapping between the damage state and measurement, respectively.

The right method (or model) should probably be case specific, as each type of method has its pros and cons [14, 15]. However, given the uncertainties arising from the complex structural degradation behaviour, the environment effects and the sensor health conditions [16, 17], both the deterministic physic-based and data-driven models are unable to provide an accurate prognostic result. A well-acknowledged strategy for both the physics-based [1-3] and data-driven methods [6, 8] to improve their prognostic performance is to set the parameters within the corresponding models as unknown components to be updated by a state estimation technique, such as particle filter (PF) given its good performance in nonlinear and non-Gaussian problem. On the other hand, this strategy is not fully exploited in hybrid methods, where the parameters in physics-based process equation are usually taken as unknown variables to be estimated by PF, while the parameters relating to the data-driven measurement equation are not [11, 18, 19].

The relationship between the damage state and the SHM measurement should probably vary in different specimens of same structure [9] due to the uncertainties mentioned above. As a consequence, the bias between the measurements predicted by data-driven measurement equation, often derived from training data, and the measurements from testing specimen is unavoidable. In this context, the measurement equation, that fails to be online updated or take the bias into account, will lead to inaccurate damage estimation [20] and prognosis [9] in case of large-level bias. Only a few studies relating to hybrid prognosis [9, 13] field attempt to solve this bias problem, where an additional measurement system is adopted to collect or calculate the true damage state and to update the measurement equation.

The bias between the predicted measurements and the actual measurements can be regarded as a typical sensor fault, which can be detected, localized and identified by adding a parameter representing a bias in both the process and measurement equations for estimation [21-23]. It has been verified that this bias can have little effect on the estimation of other state components

when it can be accurately estimated by a state estimation technique [22, 23]. However, such idea, that has been validated for the sensor fault diagnosis problem [21-23] and the state estimation considering bias [22, 23], has not been used for hybrid damage prognosis.

This paper develops a new particle filter-based hybrid prognosis framework that combines the methods from sensor fault diagnosis and current hybrid prognosis investigations. The process equation is formulated resorting to the physical law describing damage propagation and a bias parameter, while the measurement equation representing the relationship between the damage state and the measurements is built by a polynomial fitting function and the bias parameter. Then, PF is used to estimate the damage state, damage parameters and the bias parameter. Finally, the future states and, consequently, the RUL can be estimated on the basis of the process equation and the estimated damage state and parameters.

The rest of this paper is organized as follows: Section 2 introduces the traditional and new models as well as the prognosis framework. The numerical validation is discussed in Section 3. Finally, Section 4 concludes this paper with some topics for future work.

2 DAMAGE PROGNOSIS FRAMEWORK

Particle filter-based damage prognosis framework generally has three main steps, namely, (i) formulating a state space model, (ii) estimating the unknown state components using PF and (iii) calculating the future state and RUL by the PF estimates and the physic-based process equation.

2.1 Traditional and new models

Current PF-based prognosis investigations in SHM usually have the hybrid state space model formulated as

$$\begin{cases} \mathbf{z}_k = \begin{bmatrix} \boldsymbol{\theta}_k \\ x_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_{k-1} + \boldsymbol{\omega}_{\theta(k)} \\ f(x_{k-1}, \boldsymbol{\theta}_k, \omega_k) \end{bmatrix} \\ y_k = h(x_k) + v_k \end{cases} \quad (1)$$

where $f(\cdot)$ is the physics-based function describing the evolution of the damage state, and $h(\cdot)$ is a function mapping the relationship between the damage state and the measurement, the subscript k is the time step, \mathbf{z} represents the state vector, y is the output that is given by the measurement system, such as strain [11] and guided wave [9, 10], $\boldsymbol{\theta}$ includes evolution parameters and x is the state relating to the damage (hereafter focusing on crack damage), ω and $\boldsymbol{\omega}_{\theta}$ are the process noises, v is the measurement noise.

The relationship between the damage extent and measurement will inevitably vary in different specimens of same structure [9], due to the uncertainties like damage geometries and sensor error, which means the bias between the measurement predicted by a data-driven measurement equation derived from some specimens and the actual measurements from another one is typically unavoidable. Inaccurate estimation [20] and prognosis [9] may occur in case of large-level bias. Inspired from the sensor fault diagnosis investigations [21-23] and hybrid prognosis studies [9, 10, 13], a novel hybrid state space model is thus developed with the bias parameter b included as

$$\begin{cases} \mathbf{z}_k = \begin{bmatrix} \boldsymbol{\theta}_k \\ x_k \\ b_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_{k-1} + \boldsymbol{\omega}_{\boldsymbol{\theta}(k)} \\ f(x_{k-1}, \boldsymbol{\theta}_k, \omega_k) \\ b_{k-1} + \omega_{b(k)} \end{bmatrix} \\ y_k = h(x_k) + b_k + v_k \end{cases} \quad (2)$$

where ω_b is the process noise for the parameter b . This model can provide satisfactory estimated damage state and parameters, because the bias between the output calculated from the function $h(\cdot)$ and the actual measurement y can be on-line estimated by the parameter b , as will be validated in Section 3.4.

2.2 Particle filter and RUL prediction

Particle filter serves as the state estimation technique in this study, due to its good performance in nonlinear and non-Gaussian problem.

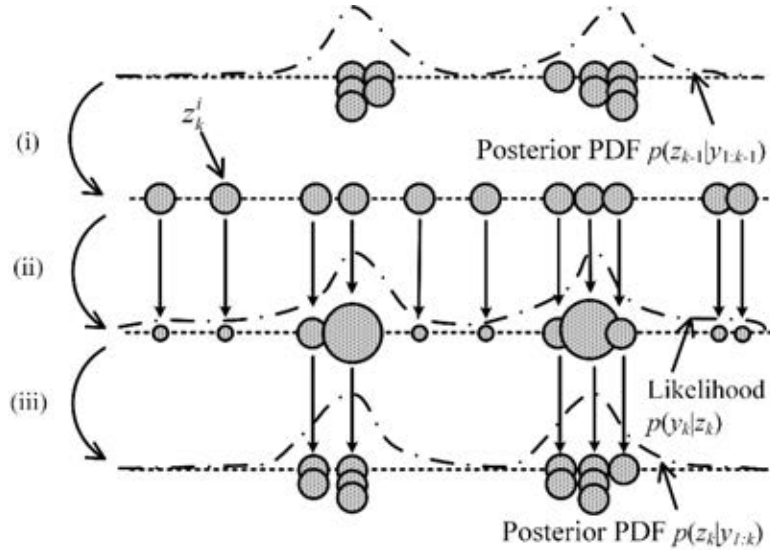


Figure 1 Sampling importance resampling (SIR) particle filter [24]

The sampling importance resampling (SIR) PF [25] with systematic resampling, as presented in Figure 1, is used in this study, and it consists of the three steps, i.e.,

- (i) draw N_p particles $\{\mathbf{z}_k^i: i = 1, 2, \dots, N_p\}$ from the prior probability density function (PDF) $p(\mathbf{z}_k|\mathbf{z}_{k-1})$,
- (ii) calculate the weight w_k^i by the likelihood function $p(y_k|\mathbf{z}_k^i)$ as

$$w_k^i \propto p(y_k|\mathbf{z}_k^i) \quad (3)$$

and assign its normalized form \tilde{w}_k^i to each particle \mathbf{z}_k^i ,

- (iii) Resample for $\{\mathbf{z}_k^i: i = 1, 2, \dots, N\}$ using particle weights.

In addition, the kernel smoothing method [26] is adopted for the parameters $\boldsymbol{\theta}$ due to its good performance in current investigations [3],

$$\boldsymbol{\theta}_k^i = \sqrt{1 - h^2} \boldsymbol{\theta}_{k-1}^i + (1 - \sqrt{1 - h^2}) \hat{\boldsymbol{\theta}}_{k-1} + \boldsymbol{\omega}_{\boldsymbol{\theta}(k)}^i \quad (4)$$

where $\hat{\theta}_{k-1}$ are the means of the samples.

The prediction for future state and RUL is summarized in Table 1. At each time step k , the posterior PDFs of the state and parameters are adopted to calculate the future evolution of the particles. The RUL of each particle can be defined when its future state reaches a predefined threshold.

Initialization: set $\{x_k^{i,0} : i = 1, 2, \dots, N_p\}$ as $\{x_k^i : i = 1, 2, \dots, N_p\}$
For $i = 1 : N_p$
$j = 0$
While $x_k^{i,j} < l_{th}$
Calculate the future state $x_k^{i,j+1}$ by $x_k^{i,j+1} = f(x_k^{i,j}, \theta_k^i)$
$j = j + 1$
End
$RUL_k^i = \Delta N j$
End

Table 1 Calculation of future state and RUL at time step k

3 APPLICATION

3.1 Crack growth and measurement

The crack growth data, as presented in Figure 2(a), are created by Paris' law with parameters as in Table 2.

$$x_k = x_{k-1} + e^{\omega_k} C (F \Delta S \sqrt{\pi x_{k-1}})^m \Delta N \quad (5)$$

where x is the crack length, the subscript k means the k -th time step, ΔN , ΔS and F represent the load cycle increment, the applied fatigue stress range and the crack shape function, respectively, C and m are two empirical values governing damage progression and $\omega \sim \mathcal{N}\left(-\frac{\sigma^2}{2}, \sigma^2\right)$ is the unbiased log-normal process noise with standard deviation σ . The crack length 35 mm is set as the threshold for damage prognosis.

Initial crack length x_0	Parameter C	Parameter m	Noise ω
15 mm	$1.1994 \times 10^{-14} \text{ mm/cycle(MPa}\sqrt{\text{mm}})^{-m}$	3.79	$\mathcal{N}\left(-\frac{0.6^2}{2}, 0.6^2\right)$
Crack shape function F	Applied fatigue stress ΔS	Load cycles ΔN	
1	60 MPa	500	

Table 2 Details for crack growth

Given the difficulty in directly measuring the crack length, current SHM applications usually have advanced measurement system, e.g., guided wave [9, 10], strain gauge [11], adopted for online monitoring to estimate the crack length. In this study, the measurements at different crack

lengths (Figure 2(b)) are created via two steps:

- (i) calculate the measurements without the bias and noise by a polynomial fitting function $h(\cdot)$, representing the relationship between the crack lengths and the SHM measurements

$$h(x_k) = (-0.00001x_k^4 + 0.0025x_k^3 + 0.05x_k^2 + 0.3x_k - 0.4)/40 \quad (6)$$

- (ii) add the varying bias, as given in Eq. (7), and 33dB signal-to-noise ratio white Gaussian noise to the above measurements.

$$b_k = -0.12 \sin\left(\frac{k}{100}\right) + 0.0001k \quad (7)$$

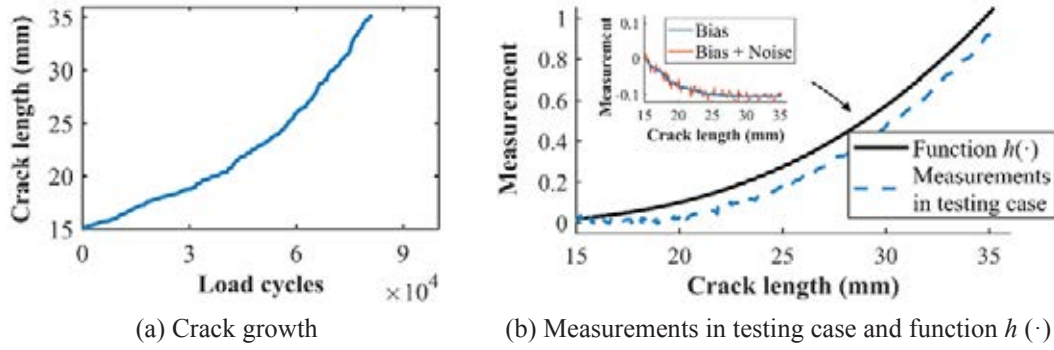


Figure 2 Testing data.

Note: the x -axis for the bias in Figure 2(b) is changed as crack length.

3.2 Formulation of traditional and new models

Given the two parameters C and m will also vary in different specimens, they are taken as unknown variables and added to the state vector to form the traditional augmented state space model as

$$\begin{cases} \mathbf{z}_k = \begin{bmatrix} C_k \\ m_k \\ x_k \end{bmatrix} = \begin{bmatrix} \sqrt{1-h^2}C_{k-1} + (1-\sqrt{1-h^2})\hat{C}_{k-1} + \omega_{1(k)} \\ \sqrt{1-h^2}m_{k-1} + (1-\sqrt{1-h^2})\hat{m}_{k-1} + \omega_{2(k)} \\ x_{k-1} + e^{\omega_k}C_k(F\Delta S\sqrt{\pi x_{k-1}})^{m_k}\Delta N \\ y_k = h(x_k) + v_k \end{bmatrix} \end{cases} \quad (8)$$

where $\omega \sim \mathcal{N}\left(-\frac{\sigma^2}{2}, \sigma^2\right)$ is log-normal process noise with the standard deviation σ [3], while ω_1 and ω_2 are zero-mean Gaussian process noises.

The new hybrid model with a bias parameter included is given as

$$\begin{cases} \mathbf{z}_k = \begin{bmatrix} C_k \\ m_k \\ x_k \\ b_k \end{bmatrix} = \begin{bmatrix} \sqrt{1-h^2}C_{k-1} + (1-\sqrt{1-h^2})\hat{C}_{k-1} + \omega_{1(k)} \\ \sqrt{1-h^2}m_{k-1} + (1-\sqrt{1-h^2})\hat{m}_{k-1} + \omega_{2(k)} \\ x_{k-1} + e^{\omega_k}C_k(F\Delta S\sqrt{\pi x_{k-1}})^{m_k}\Delta N \\ b_{k-1} + \omega_{b(k)} \\ y_k = h(x_k) + b_k + v_k \end{bmatrix} \end{cases} \quad (9)$$

where b is the parameter representing the bias, and ω_b is its process noise.

3.3 Particle filter parameters

All the PF parameters used in this study are reported in Table 3. The strategies about the distributions of initial intervals for C and m and the process noise ω are determined from [3, 11, 12].

Number of particles N_p	STD in likelihood function	h in Kernel smoothing	
2000	0.04	0.1	
Initial distributions for C, m		Initial range for x , mm	Initial value for b
$\begin{bmatrix} C_0 \\ m_0 \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} 1.14e-14 \\ 3.86 \end{bmatrix}, \begin{bmatrix} 1 \times 10^{-30} & -8 \times 10^{-17} \\ -8 \times 10^{-17} & 0.01 \end{bmatrix} \right)$		$x_0 \in (14.5, 15.5)$	$b_0 = 0$
Distributions of process noises $\{\omega, \omega_1, \omega_2, \omega_3, \omega_4\}$ for x, C, m, b			
x, ω	C, ω_1	m, ω_2	b, ω_3
$\mathcal{N} \left(-\frac{0.04^2}{2}, 0.04^2 \right)$	$\mathcal{N}(0, (1 \times 10^{-17})^2)$	$\mathcal{N}(0, 0.003^2)$	$\mathcal{N}(0, 0.008^2)$

Table 3 PF parameters.

3.4 Results from estimation and RUL prediction

Figures 3 (a) and (b) present the estimation results using traditional model. The estimated crack length remains close to the true values until reaching about 17 mm, when the bias grows large enough to hamper damage estimation, as visible in Figure 2. The narrow confidence boundaries after about 6×10^4 load cycles show the existence of a poor particle diversity. In addition, the samples of the parameters C and m also fail to accumulate around the true values. The observation that the bias between the output of the measurement equation and the actual measurement will lead to inaccurate estimation is confirmed in [9, 20]. Figures 3 (c), (d) and (e) present the estimation results using the new model with included bias parameter. The estimated crack length, C and m are noticeably more accurate than those in Figures 3 (a) and (b), as the bias is correctly identified by the PF and is taken into consideration for the estimation for the other state components. Moreover, given the presence of measurement noise and the difficulty in estimating the time-varying parameter, the bias parameters from the 3×10^4 to 7×10^4 load cycles are not accurately estimated, which leads to less accurate estimation for crack length at these load cycles.

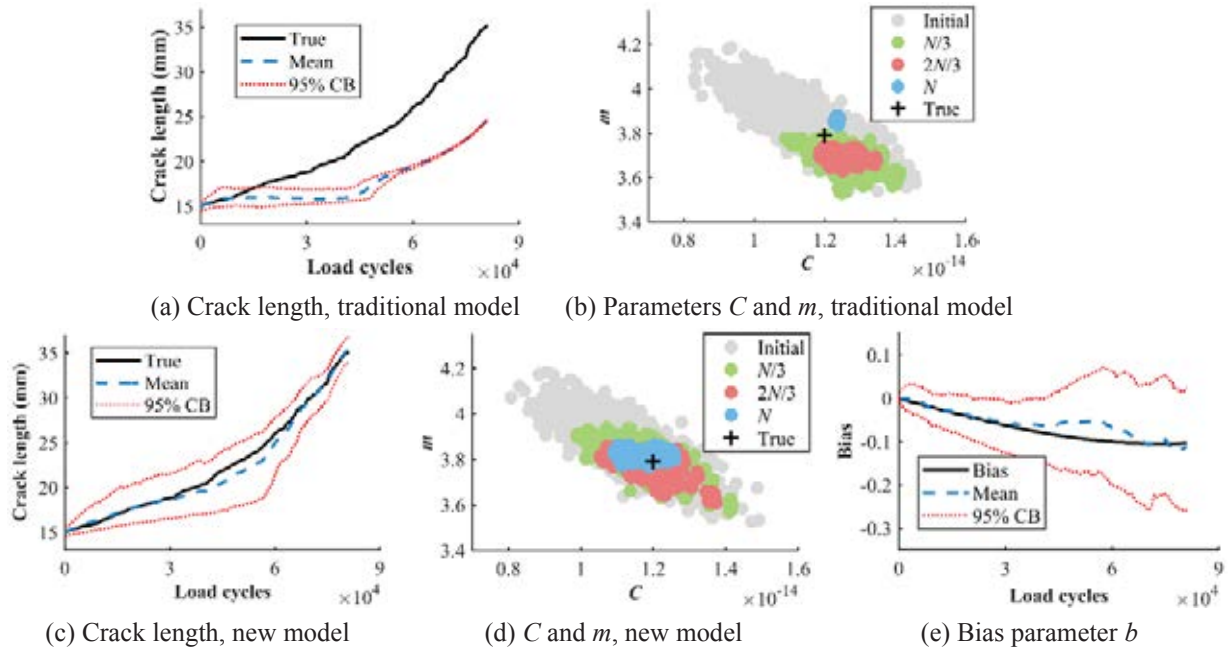
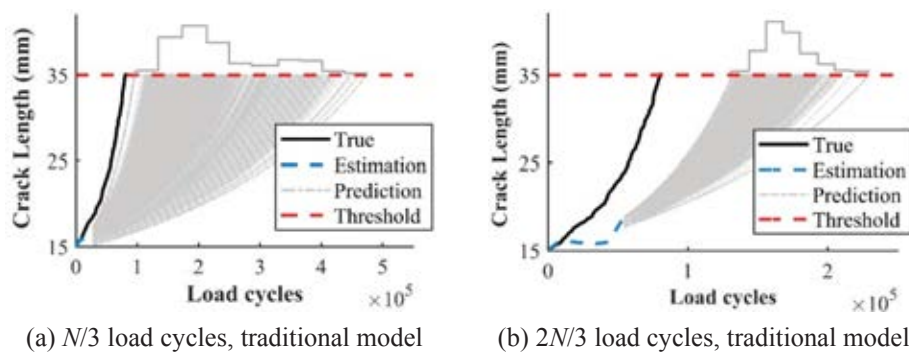


Figure 3 Estimation using traditional and new models

Figures 4 (a)(b) and (c)(d) present the prediction for future states at some selected load cycles using the traditional and new models, respectively. The grey dotted lines are the crack length trajectories predicted by the particles at the selected load cycles, and the grey histogram at the end of life (identified as a critical crack length) is the RUL posterior PDF. Figures 5 (a) and (b) present the RUL predictions using the traditional and new models, respectively, along with their 95% confidence boundary. The predicted states and RULs from the new model always keep close to the true values, while those from the traditional model don't. It can be concluded that the new model can provide more accurate estimation and prognosis results within a PF framework than the traditional model, for which the bias existing between the measurement equation and the actual observation prevents a correct estimation.



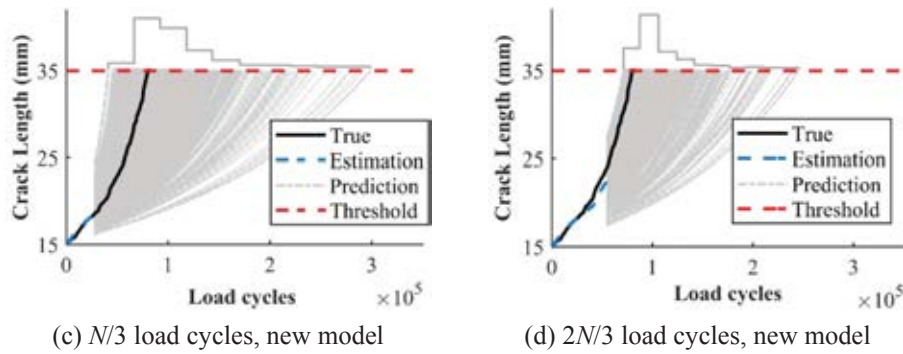


Figure 4 Prediction at two selected load cycles from traditional and new models.

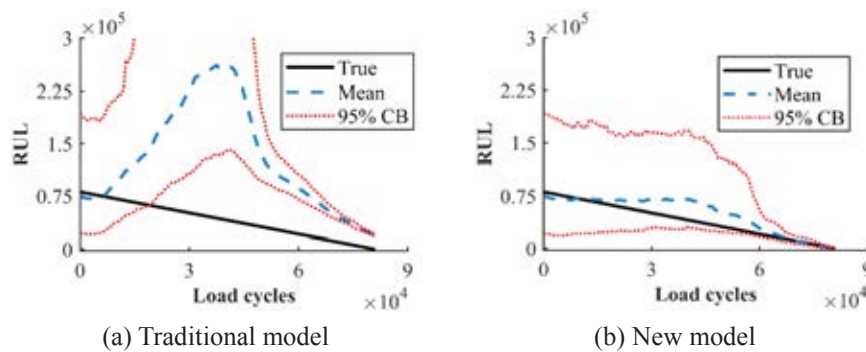


Figure 5 RUL Prediction from traditional and new models.

4 CONCLUSIONS

The bias between the measurements predicted by any data-driven measurement equation and the actual SHM measurements is unavoidable in hybrid prognostic investigations, which might lead to inaccurate prognostic results. Combining the ideas from sensor fault diagnosis and hybrid prognosis, this paper proposes a new hybrid state space model that always includes an adaptive bias parameter in both the state vector and measurement equation. The numerical study proves the new model can provide accurate estimation and prognosis for crack length when the bias can be accurately estimated. Our future work intends to validate this framework by experimental study.

ACKNOWLEDGEMENTS

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 859957.

REFERENCES

- [1] J. Chiachío, M. Chiachío, S. Sankararaman, A. Saxena, K. Goebel, Condition-based prediction of time-dependent reliability in composites, *Reliability Engineering & System Safety*, 142 (2015) 134-147.
- [2] M. Chiachío, J. Chiachío, S. Sankararaman, K. Goebel, J. Andrews, A new algorithm for prognostics using Subset Simulation, *Reliability Engineering & System Safety*, 168 (2017) 189-199.

- [3] M. Corbetta, C. Sbarufatti, M. Giglio, M.D. Todd, Optimization of nonlinear, non-Gaussian Bayesian filtering for diagnosis and prognosis of monotonic degradation processes, *Mechanical Systems and Signal Processing*, 104 (2018) 305-322.
- [4] N. Eleftheroglou, D. Zarouchas, T. Loutas, R. Alderliesten, R. Benedictus, Structural health monitoring data fusion for in-situ life prognosis of composite structures, *Reliability Engineering & System Safety*, 178 (2018) 40-54.
- [5] N. Eleftheroglou, D. Zarouchas, R. Benedictus, An adaptive probabilistic data-driven methodology for prognosis of the fatigue life of composite structures, *Composite Structures*, 245 (2020) 112386.
- [6] F. Cadini, C. Sbarufatti, M. Corbetta, F. Cancelliere, M. Giglio, Particle filtering-based adaptive training of neural networks for real-time structural damage diagnosis and prognosis, *Structural Control and Health Monitoring*, 26 (2019).
- [7] C. Chen, G. Vachtsevanos, M.E. Orchard, Machine remaining useful life prediction: An integrated adaptive neuro-fuzzy and high-order particle filtering approach, *Mechanical Systems and Signal Processing*, 28 (2012) 597-607.
- [8] J. Chiachío, M.L. Jalón, M. Chiachío, A. Kolios, A Markov chains prognostics framework for complex degradation processes, *Reliability Engineering & System Safety*, 195 (2020) 106621.
- [9] J. Chen, S. Yuan, H. Wang, On-line updating Gaussian process measurement model for crack prognosis using the particle filter, *Mechanical Systems and Signal Processing*, 140 (2020) 106646.
- [10] J. Chen, S. Yuan, X. Jin, On-line prognosis of fatigue cracking via a regularized particle filter and guided wave monitoring, *Mechanical Systems and Signal Processing*, 131 (2019) 1-17.
- [11] D. Cristiani, C. Sbarufatti, M. Giglio, Damage diagnosis and prognosis in composite double cantilever beam coupons by particle filtering and surrogate modelling, *Structural Health Monitoring*, (2020) 1475921720960067.
- [12] D. Cristiani, C. Sbarufatti, F. Cadini, M. Giglio, Fatigue damage diagnosis and prognosis of an aeronautical structure based on surrogate modelling and particle filter, *Structural Health Monitoring*, 0 1475921720971551.
- [13] M.E. Orchard, G.J. Vachtsevanos, A particle-filtering approach for on-line fault diagnosis and failure prognosis, *Transactions of the Institute of Measurement and Control*, 31 (2009) 221-246.
- [14] Y. Peng, M. Dong, M.J. Zuo, Current status of machine prognostics in condition-based maintenance: a review, *The International Journal of Advanced Manufacturing Technology*, 50 (2010) 297-313.
- [15] L. Liao, F. Köttig, Review of Hybrid Prognostics Approaches for Remaining Useful Life Prediction of Engineered Systems, and an Application to Battery Life Prediction, *IEEE Transactions on Reliability*, 63 (2014) 191-207.
- [16] I. Lopez, N. Sarigul-Klijn, A review of uncertainty in flight vehicle structural damage monitoring, diagnosis and control: Challenges and opportunities, *Progress in Aerospace*

- Sciences, 46 (2010) 247-273.
- [17] P. Baraldi, F. Mangili, E. Zio, Investigation of uncertainty treatment capability of model-based and data-driven prognostic methods using simulated data, *Reliability Engineering & System Safety*, 112 (2013) 94-108.
 - [18] S. Yuan, J. Chen, W. Yang, L. Qiu, On-line crack prognosis in attachment lug using Lamb wave-deterministic resampling particle filter-based method, *Smart Materials and Structures*, 26 (2017) 085016.
 - [19] S.-L. Ma, S.-F. Jiang, J. Li, Structural damage detection considering sensor performance degradation and measurement noise effect, *Measurement*, 131 (2019) 431-442.
 - [20] F. Cadini, C. Sbarufatti, M. Corbetta, M. Giglio, A particle filter-based model selection algorithm for fatigue damage identification on aeronautical structures, *Structural Control & Health Monitoring*, (2017) e2002.
 - [21] M. Gálvez-Carrillo, M. Kinnaert, Sensor fault detection and isolation in doubly-fed induction generators accounting for parameter variations, *Renewable Energy*, 36 (2011) 1447-1457.
 - [22] A. Giremus, J.-Y. Tournet, V. Calmettes, A particle filtering approach for joint detection/estimation of multipath effects on GPS measurements, *IEEE Transactions on Signal Processing*, 55 (2007) 1275-1285.
 - [23] J.A. Grauer, Real-Time Data-Compatibility Analysis Using Output-Error Parameter Estimation, *Journal of Aircraft*, 52 (2015) 940-947.
 - [24] E.N. Chatzi, A.W. Smyth, The unscented Kalman filter and particle filter methods for nonlinear structural system identification with non-collocated heterogeneous sensing, *Structural Control & Health Monitoring*, 16 (2010) 99-123.
 - [25] M.S. Arulampalam, S. Maskell, N. Gordon, T. Clapp, A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking, *IEEE Transactions on Signal Processing*, 50 (2002) 174-188.
 - [26] J. Liu, M. West, Combined parameter and state estimation in simulation-based filtering, *Sequential Monte Carlo methods in practice*, Springer, 2001, pp. 197-223.