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VULNERABILITY ASSESSMENT OF MONUMENTAL ARTWORKS USING CONTACT TIME-HISTORY ANALYSIS

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Abstract

Strategies for the protection of the Cultural Heritage priceless value are highly demanded, being both earthquake and support movement (and a combination of the two of them) severe threats for their safety and integrity. To this scope, computer-based numerical formulations have been representing a powerful tool for the simulation of expected loss in terms of pieces of art damages, with the scope to save them as much as possible. In this framework, the conference paper is devoted to the vulnerability assessment of monumental artworks using dynamic analysis in a rigid block modelling environment.
The numerical formulation adopts a discrete rigid block-based model, where the mortar joints contribution is accounted for by frictional, no-tension interfaces between the masonry units. The procedure adopted to solve the formulation is based on a dual variational problem where equilibrium, kinematic and failure conditions equations are involved at the same time, and which is solved via mathematical programming.
An application to a system of two blocks vertically stacked, one representing the support and one representing the artefact, subjected to free rocking motion will be proposed in the paper. The main scope is to investigate the seismic response of such a system, which is typical for most of museum's collections (e.g., statues). The computational efficiency and accuracy of the proposed numerical strategy will be discussed.

Keywords: Masonry block structures; Non-linear time history analysis, Free rocking motion, Mathematical programming.

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1 INTRODUCTION

The possibility to develop computational strategy for the investigation of museum collections behaviour against seismic-induced dynamic actions represents a challenging task, playing a crucial role in the artworks integrity protection. The response of blocky structures, such as monumental statues made of stone blocks, against lateral forces is typically characterized by rocking behaviour [1–3].

The development and use of analytical and numerical tools for the investigation of blocky structures and historical masonry monuments subjected to different loading conditions by using both static and dynamic approaches is a widespread topic in literature [4–33]. Various numerical approaches are available in literature for the investigation of the dynamic response of multi-block structures and types undergoing rocking. Among those, the interest in the use of Non-Smooth Contact Dynamics (NSCD) methods has been growing in the recent years, essentially due to the advantageous possibility to account for the blocks interactions through a system made of both equilibrium and complementary conditions. In this framework, NSCD approach represent a valid alternative to common modeling approaches such as finite element method (FEM) and discrete element method (DEM) [34–37].

The use of NSCD technique has been increasing especially in the analysis of the structural response of masonry structures [38]–[40], where fast and accurate algorithms are available in the literature for the numerical solution of the formulations proposed for the mathematical programming problem arising from the conditions governing the contact dynamics [41–44].

In this paper we adopt a simple formulation for dynamic analysis of blocky structures aiming to investigate the rocking behaviour of a system of two blocks, comparable to the configuration of a monumental statue typical of museum's collections standing on its base. The aim of this research is to assess the computational efficiency of the in-house numerical procedure and to investigate to which extent it can be properly adopted in the prediction of the rocking behaviour of museum's artefacts to save them against any possible loss or damage. It is worth noting that the modelling approach for contact interfaces can be regarded as the extension to the dynamic field of a contact point formulation developed for limit analysis of collections of polyhedral rigid blocks.

The paper is organized as follows: the dynamic formulation is described in section 2 in terms of static and kinematic variables as well as the problem formulated to govern the behaviour of the rigid block model. Section 3 deals with the numerical application of the adopted procedure to the case study of a two blocks system subjected to free rocking motion already considered in literature.

2 THE RIGID BLOCK DYNAMIC MODEL

The proposed numerical model is composed by a collection of quadrangular rigid blocks i interacting by means of contact points k located at the vertexes of contact surfaces j (Figure 1a). The behaviour of contact interfaces is governed by a no-tension and associative frictional model under the assumption of infinite compressive strength. The dynamic model is formulated according to the approach proposed in [45, 46] for granular materials.

The contact variables are represented by the internal forces located at contact point k, which are essentially the shear and normal force (Figure 1b), collected in the vector $\boldsymbol{c_k}$.

The relative displacement rates, both tangential Δu_{tk} and normal Δu_{nk}, represent the kinematic variables associated to the contact forces in a virtual work sense. They are finally collected in the vector $\boldsymbol{\Delta u}$ (Figure 1c).

The centroid of the rigid block i is attached by the external loads and the position of the block centroid is collected in the vector $\boldsymbol{x_i}$ as described in equation (1) and Figure 1b.

$$\boldsymbol{x_i} = [x_i \quad z_i \quad \omega_i]^T \tag{1}$$

θ-method is used to discretize the equations of motions, as described in equations (2) and (3) with regard to time $t = t_0 + \Delta t$.

$$\alpha(t) = \frac{v - v_0}{\Delta t} \tag{2}$$

$$v(t) = \frac{1}{\theta}\left[\frac{\Delta \boldsymbol{x}}{\Delta t} - (1-\theta)v_0\right] \tag{3}$$

being $\Delta \boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_0$ the displacement vector, x_0 the known position and v_0 the known velocity at t_0 and $0.5 < \theta \leq 0.5$.

The incremental equation (2) allows to formulate the equation of motion of the collection of rigid blocks using a contact point model as described in the next equation (4):

$$\overline{\boldsymbol{M}}\Delta\boldsymbol{x} + \boldsymbol{A_0}\boldsymbol{c} = \overline{\boldsymbol{f}}_{\boldsymbol{0}} \tag{4}$$

where $\boldsymbol{A_0}$ is the contact equilibrium matrix, $\bar{\boldsymbol{M}} = \frac{1}{\theta \Delta t^2}\boldsymbol{M}$ is related to the mass matrix $\boldsymbol{M}$ which collects the mass and the inertia moment of each block and $\bar{\boldsymbol{f}}_0 = \boldsymbol{f} + \bar{\boldsymbol{M}}v_0\Delta t$.

Ad-hoc expression are formulated to account for non-penetration condition at potential contact point, by using the so-called Signorini unilateral contact condition.

Finally, sliding failure condition at contact interfaces is governed by a Coulomb friction law, expressed by the following equation (5).

$$\pm t_k - \mu n_k \leq 0 \tag{5}$$

being μ the value of friction coefficient.

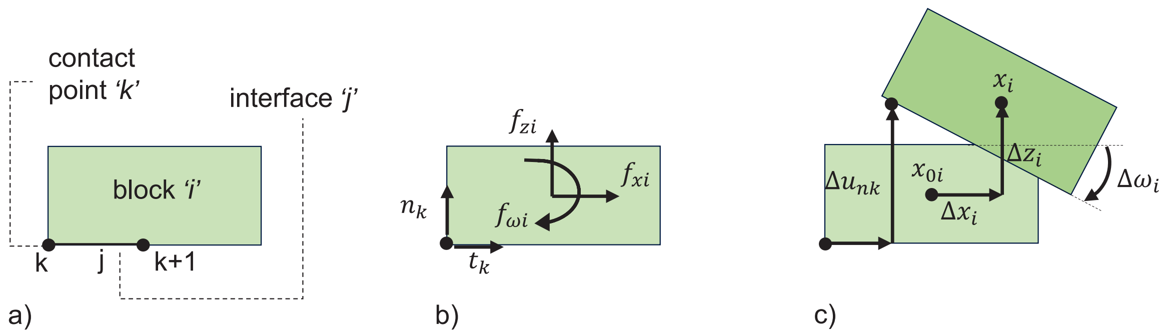

Figure 1. Rigid block dynamic model: (a) Rigid block *i*, interface *j* and contact point *k*; (b) contact forces and (c) kinematic variables at block centroid *i*, and contact point *k*.

2.1 Formulation of the mathematical programming problem

The dynamic problem can be expressed by an equations system which collect the equilibrium equations, the kinematic and the failure conditions in a typical linear complementarity problem (LCP). In this framework, the contact dynamics problem can be posed as described in equation (6), where $\boldsymbol{Y^T}$ is the matrix of failure condition, $\boldsymbol{\lambda}$ is the vector of flow multipliers and $\boldsymbol{g_0}$ is the vector collecting initial gaps at each contact point.

The problem expressed in equation (6) is uncoupled in two dual quadratic programming problems, a force-based and a displacement-based problem, respectively. In this way, the computational costs significantly decrease thanks to the possibility to use efficient solution algorithms.

$$\begin{bmatrix} \overline{\boldsymbol{M}} & \cdot & \boldsymbol{A}_0 \\ \cdot & \cdot & -\boldsymbol{Y}^T \\ -\boldsymbol{A}_0^T & \boldsymbol{Y} & \cdot \end{bmatrix}_{nxn} \begin{bmatrix} \Delta\boldsymbol{x} \\ \boldsymbol{\lambda} \\ \boldsymbol{c} \end{bmatrix}_{nx1} + \begin{bmatrix} \cdot \\ \boldsymbol{y} \\ \cdot \end{bmatrix} = \begin{bmatrix} \overline{\boldsymbol{f}}_0 \\ \cdot \\ -\boldsymbol{g}_0 \end{bmatrix} \tag{6}$$

$$s.t. \quad -\boldsymbol{y} \geq 0 \quad \boldsymbol{\lambda} \geq 0 \quad \boldsymbol{y}^T\boldsymbol{\lambda} = 0$$

It is worth noting that the dynamic problem formulated in the equation (6) is formally equivalent to a classic limit equilibrium problem when the dynamic forces and contact gaps tend to zero. An incremental procedure was implemented to solve the optimization problem assuming kinematic variables derived by Lagrange multipliers and updating the block position and gaps values per each step.

The analyses were carried out a 3.50 GHz Intel Xeon Processor E5-1650 with 16.0 GB of RAM. The value of algorithm parameter θ for time discretization was set equal to 0.7 and time increment was set in the range 0.001-0.002 s.

3 NUMERICAL APPLICATION TO A TWO-BLOCK SYSTEM SUBJECTED TO FREE ROCKING MOTION

The dynamic numerical procedure described in the previous section is here applied to the case study of a two-block system undergoing to free rocking. This case study was previously investigated in [47]. The geometrical properties of the two blocks are showed in Figure 2a, being equivalent to the example performed in [47]. The first block has a slenderness ratio H_1/B_1 equal to 1 whereas the second one has a slenderness ratio H_2/B_2 equal to 2.5. The base widths are B_1 = 1.25 m and B_2 = 1.0 m respectively. As for the material properties, the two blocks are considered homogeneous, with a weight for unit volume $\rho_1 = \rho_2 = 2500$ kg/m^3. The investigated case study is characterized by a value of the starting rotation angles $\theta_1 = \theta_2 =$ 0.15 rad and zero starting angular velocities for both the considered blocks.

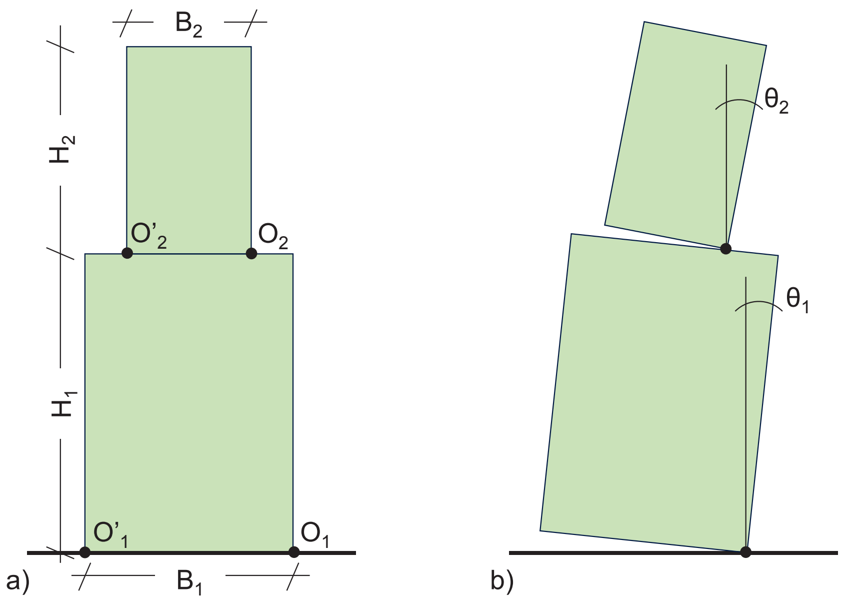

Figure 2. Two-block system: (a) geometrical properties and (b) example of configuration during free rocking motion.

The outcomes of the simulation are reported in Figure 3. According to [47], starting from the initial configuration, the angles of rotation of the two blocks, θ_1 and θ_2, tend to decrease due to the oscillation movement, and this occurs with a faster rate for the block at the base. In the first part of the motion, the two blocks move as one-single block rocking about the point O_1, until the occurrence of an impact at the foundation. After the impact, the two blocks start to independently rock and other impacts occur up to the rest. As for the outcomes obtained with the proposed rigid block dynamic model, the curve showed in Figure 3 deals with the rotation angle θ_2 of the upper block around the point O_2. The results are in a very good agreement with those obtained in [47] until the first impact of the upper block. After this initial stage, the second rotation is somewhat different with respect to the rotation predicted in [47]. Nevertheless, the two curves are very close, and so are the maximum oscillations obtained by the two models. The comparison is, therefore, very satisfying. With regard to the final stage, i.e. after the second impact of the upper block, the results obtained with the proposed rigid block dynamic model tends to predict a faster rocking performance compared with the results obtained in [47]: a higher number of impacts (i.e. change of rotation) is exhibited by the upper blocks. Finally, the two models come to a rest step at about 3.5 seconds analysis.

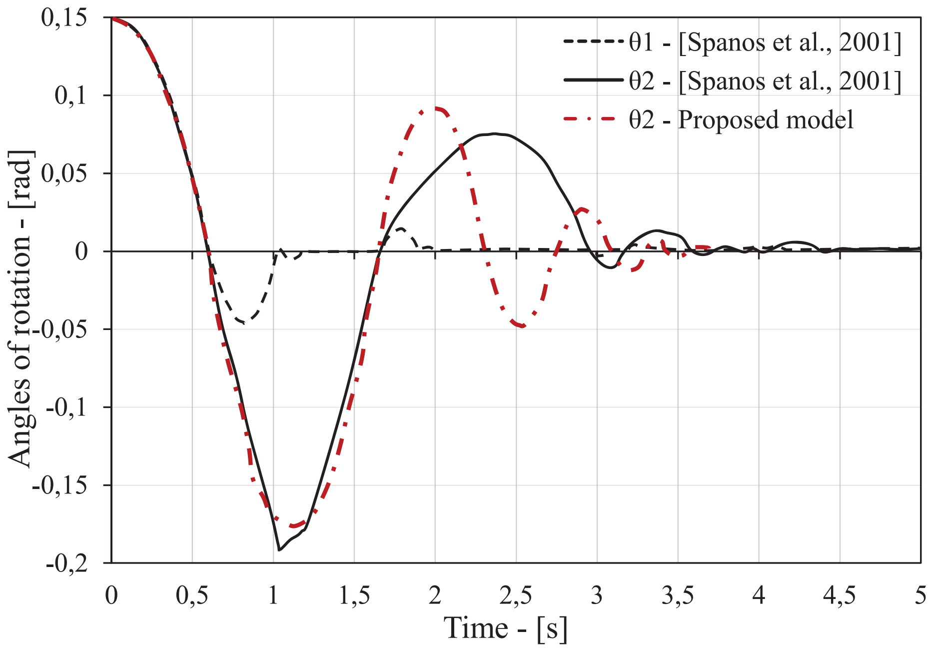

Figure 3. Free vibration response of a two-block system. Comparison with the results obtained in [47].

4 CONCLUSIONS

In this paper an in-house rigid block model for contact time-history analysis was described. The computational strategy is particularly suitable to investigate the response of stacked-blocks objects subjected to ground excitation. From this point of view, the dynamic model could represent a useful numerical tool for the vulnerability assessment of museum's collections affected by seismic-induced action, aiming to the protection of such artworks against earthquakes events. The capabilities of the proposed formulation were tested on a case study of a two-block system experiencing free rocking motion, already investigated in literature. The outcomes showed high potentialities of the adopted strategy in the prediction of the rocking behaviour of blocky objects.

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