

STIFF DYNAMIC ABSORBERS FOR THE VERTICAL SEISMIC PROTECTION OF STRUCTURES

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Abstract

The majority of the existing seismic protection systems and techniques are related to horizontal ground motions, while there are only a few vertical seismic protection systems. The main reason is due to the conflict concerning the demand for isolation stiffness. More specifically, a vertical isolated system must have sufficient vertical rigidity to sustain the weight of the isolated object/system and retain the static vertical deflection in reasonable limits. On the other hand, the isolated system must also have enough flexibility to isolate the accelerations. In order to overcome this difficulty, a novel vertical seismic absorber system is proposed, that combines a Quasi-Zero Stiffness (QZS) design including negative stiffness elements, an enhanced Tuned Mass Damper (TMD) and an inerter. The QZS oscillator manages to maintain the vertical rigidity of the structure, the inerter manages to reduce the frequency of the system, without weakening the structure or increase the seismic load and the TMD is responsible for absorbing the external excitations, significantly increasing the effective damping. This way, the dynamic behavior of the system is improved, in terms of absolute accelerations, and simultaneously, the static settlements are retained at any desired level. The design is based on engineering criteria, and the excitation input is selected according to the seismic design codes.

Keywords: Vertical Seismic Protection, Negative Stiffness, KDamper, Damping, Vibration Absorption.

1. INTRODUCTION

Earthquakes consist one of the major environmental hazards. During 2011 e.g., over 20,000 people died and about a million people lost their homes globally, due to earthquakes and their effects. Moreover, a wide class of ground vibrations generated by heavy and high-speed vehicles such as trains and trucks, construction activities (i.e., dynamic compaction, roadbed compaction, pile driving, blasting) or blasts, present a frequency spectrum in the order of a few Hz. Together with other sources inside the building, (e.g., impact noise between floors), noise pollution is their major effect.

The conventional design approach to address such loads consists of stiff and highly damped structures. Apart from technical (and financial) constraints in their realization, (e.g., current unavailability of large sized dampers, cases of heavy structures, etc.), such designs may result to reverse effects concerning certain aspects of their desired dynamic behavior. For example, stiff and highly damped structures can enhance the transfer of base acceleration excitations to the main structure. This may result to damaging effects in the interior of the structure relating e.g., to the secondary loads (machines, sensitive equipment, etc.) in the case of earthquakes.

Isolation appears to be a promising alternative, as it is based on the concept of altering the dynamics of the system and to reduce the vibration, rather than increasing the resistance capacity of the structure. The concept of isolation relies on a flexible layer between the structure and its base. This way, the fundamental natural period of the base isolated system is significantly longer, and thus, the structure can absorb less energy from the broad base excitation frequency spectrum. However, in order to isolate the building from its base, large displacements are required due to the fundamental dynamic behavior of the system, which can be even in the range of tenths of centimeters for certain earthquakes. These displacements can be hardly acceptable for numerous reasons, such as structural pounding, sensitivity in horizontal loads, proper connection for utilities, and more, rendering this concept inadequate for retrofitting.

The above comments have motivated the development of alternative vibration control strategies, which among others, include: (i) Tuned Mass Dampers (TMDs), (ii) Inerters, (iii) Negative Stiffness Devices (NSD) and “Quazi Zero Stiffness” (QZS) oscillators, and (iv) Negative Stiffness driven Absorbers/KDampers. (Figure 1)

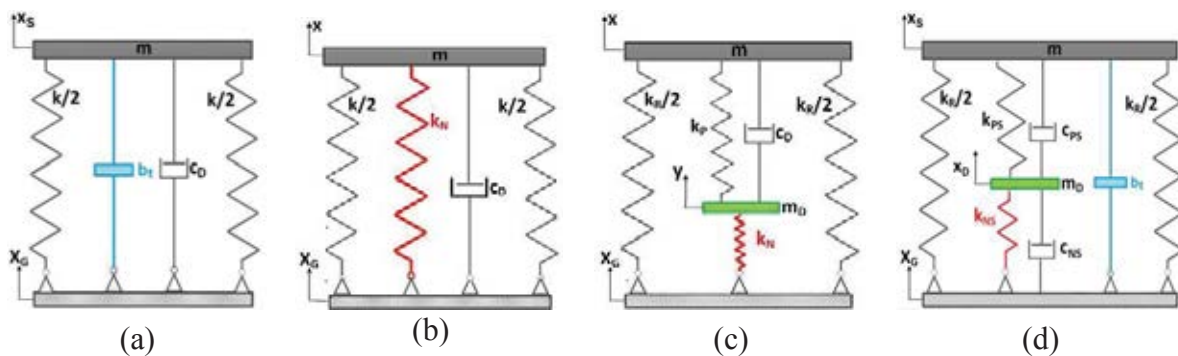


Figure 1 (a) Inerters; (b) Negative Stiffness Absorbers/QZS oscillators; (c) KDamper concept; (d) Proposed Stiff Dynamic Absorbers

The Tuned Mass Damper (TMD) has a long history, with applications in several types of engineering structures, such as high-rise buildings and skyscrapers [1,2,3], bridges [4], wind and wave excitation in wind turbines [5], etc. In addition, the TMD can be used as a possible supplement to the conventional base isolation approaches, implemented within the bases of structures, aiming to reduce the displacement demand [6–8]. The main disadvantages of

vibration absorption techniques that use a TMD-related approach are associated with the requirement for large additional masses in order for the TMD to be effective, and with the fact that a slight alteration of its parameters can alter the TMD tuning and consequently reduce the system's performance significantly [9].

In an attempt to reduce the requirements for heavy oscillating masses under the TMD concept, the inerter has been introduced in the early 2000s [10,11]. The inerter is a two-terminal element (Figure 1.a), which has the property of generating at its ends a force that is proportional to the relative acceleration of its terminals. The main advantage of the inerter lies in the fact that it does not need to have large mass in order to achieve the same inertia effect as the additional mass required under the TMD approach. The essential limitation of the inerter is the complex and elaborate mechanical design configurations needed for its implementation, and the reduction of the effective damping of the system due to the increase of the effective mass. Moreover, the major disadvantage of the TMD in base isolation applications, is that enhances the transfer of base acceleration excitations to the main structure, even more significantly than a damper of a high value. As mentioned, this can result to damaging effects in the interior of the structure.

A parallel direction to the various TMD approaches is the concept of introducing negative stiffness (NS) elements for vibration absorption (Figure 1.b). The use of negative stiffness elements (or “anti-springs”) for vibration isolation was first introduced in the pioneering works of Molyneaux [12], and of Platus [13]. The central concept of these approaches is the significant reduction of the isolator's stiffness, which, consequently, leads to the decrease of the natural frequency of the system even at almost zero levels. Towards this direction, the so called “Quasi-Zero Stiffness” (QZS) oscillators have been proposed [14]. An initial comprehensive review of such designs can be found in [15]. Nagarajaiah et. al [16] introduced a new structural modification approach for the seismic protection of structures using an adaptive negative stiffness device that resulted to the decrease of the dynamic forces imposed on the structure. The QZS oscillators are designs of an essentially non-linear stiffness, combining a positive spring in parallel with a non-linear spring, which exhibits a negative stiffness region under nominal load. In this way, QZS oscillators combine a high static stiffness, capable of maintaining the structural load, with a low dynamic stiffness, enabling a low isolation frequency. However, QZS oscillators offer a low damping capacity. In addition, even small load disturbances around the static equilibrium point of the QZS designs can significantly affect their response.

Extending the concept of the traditional Tuned Mass Dampers, a novel passive vibration absorption and damping concept (the KDamper) is introduced [17]. Compared to the TMD, the KDamper uses an additional negative stiffness spring (Figure 1.c). connecting the additional mass to the ground. This negative stiffness spring generates a force in phase with the acceleration response, acting essentially as an inertial force of the additional mass. In this way, the performance of the KDamper can be drastically increased, just by increasing the value of the negative stiffness spring and without the need to increase the value of the additional mass. Thus, the KDamper has been shown to present extraordinary damping/absorption properties, with relatively small (even minimal) values of the additional mass.

Although the KDamper incorporates a negative stiffness element, it is designed to be both statically and dynamically stable. Thus, the KDamper can be designed to present the same overall (static) stiffness as a traditional reference original oscillator, while simultaneously offering drastically increased damping properties. Since the presence of a negative stiffness spring in series with a positive stiffness spring can result to a negative stiffness value, the static behavior of the KDamper can be controlled to be similar to that of a QZS oscillator, ensuring both high static stiffness and low dynamic stiffness.

A number of KDamper designs [18] and extensions [19] have been applied as seismic absorbers in the bases of structures. Their overall static stiffness is allowed to vary, ranging from low values that are similar to that of seismic isolation systems, up to values corresponding to a significantly higher stiffness. In the former case, the KDamper presents a behavior similar to that of a base isolated system, combined however with a drastically increased damping. In the latter case, the KDamper has shown the ability to drastically reduce the structure base displacements relative to the ground in the range of few centimeters, while maintaining the absolute structure accelerations within reasonable limits. This aforementioned result implies that the KDamper can be used as an alternative method to conventional base isolation systems, enabling its implementation (under certain conditions) even for retrofitting existing structures.

Quite recently, the extended KDamper has been used in parallel to an inerter (Figure 1.d). [20] for vertical seismic protection. Under a proper optimization procedure, this combination has shown to result to configurations offering a stiff vertical static stiffness (and thus an acceptable vertical static displacement of the structure) with an acceptable level of reduction of the vertical accelerations induced to the structure. This concept is the underlying notion and configuration of the Stiff Dynamic Absorbers.

The first key difference between the proposed design and the one proposed in [20], is that the static loading is now borne only by the external k_R (Figure 1.d) spring element of the SDA, and not by the overall static stiffness of the assembly. The rest of the spring elements k_N , k_P (Figure 1.d) need to participate only in the dynamic response of the excited system, similarly to the concept of the QZSs.

The second key difference is the fact that a new more general objective function is introduced for the optimization procedure. It is thus shown that a unique optimal SDA design can be used, not only for the vertical seismic protection of the structures, but also for a range of important vertical dynamic absorption applications, including low frequency machine mounts and impact noise insulation.

2. STIFF BASE ABSORBER WITH EXTENDED KDAMPER AND INERTER

2.1. Conventional SDoF damping system

The fundamental aim of seismic isolation is to decrease the natural frequency of the system, f , below the dominant energy-containing frequencies of ground motion excitation, thus allowing reduced peak accelerations of the structure. A major constraint with this approach lies in the weakened static loading capacity of the structure as a result decreasing its overall stiffness, k , leading to large static displacements. Denoting with the X_{VSD} vertical static deflection of the conventional spring-mass system in Figure 2:

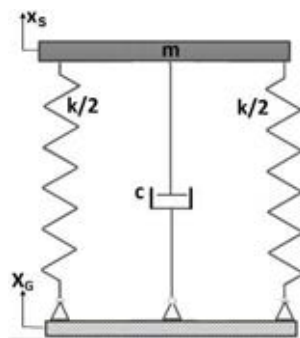


Figure 2. Conventional vibration damper

$$X_{VSD} = \frac{mg}{k} = \frac{g}{(2\pi f)^2} \quad (1)$$

The equation of motion of this system is:

$$m\ddot{u}_S + c\dot{u}_S + ku_S = -m_S\ddot{X}_G, \quad (2)$$

where $u_S = X_S - X_G$. Assuming a harmonic base excitation in the form of $\ddot{X}_G(t) = A_G e^{j\omega t}$, the steady-state response will be:

$$u_S(t) = \tilde{U}_S e^{j\omega t} \quad (3)$$

where \tilde{U}_S denotes the complex amplitude. The equation of motion of the SDoF system can be written in the following form:

$$-\omega^2 m \tilde{U}_S + j\omega c \tilde{U}_S + k \tilde{U}_S = -m A_G \quad (4)$$

The Transfer Functions of the system are calculated as:

$$\tilde{H}_{US} = \tilde{U}_S / A_G = -\tilde{H}^{-1} m \quad (5)$$

where:

$$\tilde{H} = [-\omega^2 m + j\omega c + k] \quad (6)$$

$$\tilde{H}_{AS} = A_S / A_G = -\omega^2 \tilde{X}_S / A_G = 1 - \omega^2 \tilde{H}_{US} \quad (7)$$

The natural frequency and the damping ratio of the SDoF isolated system are:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (8)$$

$$\zeta = c / (2\sqrt{mk}) \quad (9)$$

2.2. Proposed extension of KDamper equipped with inerter

This paper proposes a seismic protection system based on the combination of a Tuned Mass Damper (TMD) and a Quasi-Zero Stiffness (QZS) design introducing negative stiffness elements, accompanied by an inerter (Figure 1.d). The first component of the proposed system (combination of a TMD with a QZS design) is essentially based on the KDamper concept [17], whereby the introduction of the negative stiffness element reduces transmissibility of the system by lowering the natural frequency while increasing the effective damping of a conventional TMD. The introduction of an inerter between the structure and the ground results to an increased effective mass of the system without the need for additional weight, further improving its dynamic behavior. The proposed design is based on the one proposed by [20], with several differentiations. The positive stiffness element k_P connects the structure to the additional mass m_D , which in turn is connected to the ground through the NS element k_N , as per the initial KDamper design [17]. The k_R spring element connects the structure directly to the ground, accompanied by the inerter (b_R).

The overall stiffness of the system can be calculated as:

$$k_{tot} = k_R + \frac{k_P k_N}{k_P + k_N} \quad (10)$$

A key difference between the proposed design and the one proposed by [20], 2020 is that the static loading is now borne by the k_R element only (which is equal to the stiffness of the initial structure, k_S) and not the overall k_{tot} , while the k_N , k_P elements participate only in the dynamic response of the excited system. As the static capacity of the system is fully maintained, the system can be designed for larger absolute magnitudes of the NS element k_N , while the static deflection X_{VSD} remains at acceptable levels. This enables better seismic isolation and reduced transmissibility of the system, while maintaining its static rigidity.

Denoting the relative displacement between the structure and the ground as $u_s = X_S - X_G$, the equations of motion for the two masses are:

$$(m_S + b_R)\ddot{u}_S + c_P(\dot{u}_S - \dot{u}_D) + k_R u_S + k_P(u_S - u_D) = -m_S \ddot{X}_G \quad (11)$$

$$m_D \ddot{u}_D + c_P(\dot{u}_D - \dot{u}_S) + c_N \dot{u}_D + k_N u_D + k_P(u_D - u_S) = -m_D \ddot{X}_G \quad (12)$$

The steady state responses of the system are:

$$u_S(t) = \tilde{U}_S e^{j\omega t} \quad (13)$$

$$u_D(t) = \tilde{U}_D e^{j\omega t} \quad (14)$$

where \tilde{U}_S, \tilde{U}_D , denote the response complex amplitudes. The abovementioned equations of motion of the system hence become:

$$-\omega^2(m_S + b_R)\tilde{U}_S + j\omega c_P(\tilde{U}_S - \tilde{U}_D) + k_R \tilde{U}_S + k_P(\tilde{U}_S - \tilde{U}_D) = -m_S A_G \quad (15)$$

$$-\omega^2 m_D \tilde{U}_D + j\omega c_P(\tilde{U}_D - \tilde{U}_S) + j\omega c_N \tilde{U}_D + k_N \tilde{U}_D + k_P(\tilde{U}_D - \tilde{U}_S) = -m_D A_G \quad (16)$$

The resulting transfer functions (TFs) can be derived:

$$\begin{bmatrix} \tilde{H}_{US} \\ \tilde{H}_{UD} \end{bmatrix} = \begin{bmatrix} \tilde{U}_S/A_G \\ \tilde{U}_D/A_G \end{bmatrix} = H^{-1} \begin{bmatrix} m_S \\ m_D \end{bmatrix} \quad (17)$$

where:

$\tilde{H} =$

$$\begin{bmatrix} -\omega^2(m_S + b_R) + j\omega c_P + (k_R + k_P) & -(j\omega c_P + k_P) \\ -(j\omega c_P + k_P) & -\omega^2 m_D + j\omega(c_P + c_N) + (k_N + k_P) \end{bmatrix} \quad (18)$$

$$\tilde{H}_{AS} = A_S/A_G = 1 - \omega^2 \tilde{H}_{US} \quad (19)$$

$$\tilde{H}_{AD} = A_D/A_G = 1 - \omega^2 \tilde{H}_{UD} \quad (20)$$

The following quantities represent the area under the transfer functions of the system (scaled by the factor $d\omega$) and are indicators of the transmissibility of the system within the frequency range of interest (ω_1 to ω_2).

$$Q_{US} = \frac{1}{d\omega} \int_{\omega_1}^{\omega_2} |H(\omega)_{US}|^2 \quad (21)$$

$$Q_{UD} = \frac{1}{d\omega} \int_{\omega_1}^{\omega_2} |H(\omega)_{UD}|^2 \quad (22)$$

$$Q_{AS} = \frac{1}{d\omega} \int_{\omega_1}^{\omega_2} |H(\omega)_{AS}|^2 \quad (23)$$

Denoting by S_A the PSD of the ground motion acceleration, the response power spectral densities can be calculated:

$$S_{US}=|H_{US}|^2 S_A \quad (24)$$

$$S_{UD}=|H_{UD}|^2 S_A \quad (25)$$

$$S_{AS}=|H_{AS}|^2 S_A \quad (26)$$

The root mean square (RMS) values of the responses are finally defined as the root under the area of the PSD curve, which can be used as indicators of the actual energy content of the response:

$$R_{US}=\sqrt{\int_{-\infty}^{+\infty} S_{US}(\omega)d\omega} \quad (27)$$

$$R_{UD}=\sqrt{\int_{-\infty}^{+\infty} S_{UD}(\omega)d\omega} \quad (28)$$

$$R_{AS}=\sqrt{\int_{-\infty}^{+\infty} S_{AS}(\omega)d\omega} \quad (29)$$

Next, the following quantities concerning the system configuration are introduced:

$$m_D=\mu_D m_S \quad (30)$$

$$b_R=\mu_R m_S \quad (31)$$

$$\zeta_N= c_N/(2\sqrt{m_S k_R}) \quad (32)$$

$$\zeta_P= c_P/(2\sqrt{m_S k_R}) \quad (33)$$

3. OPTIMISATION METHOD FOR THE CONFIGURATION OF THE VERTICAL EXTENDED KDAMPER & INERTER SYSTEM

3.1. Objective function

The quantity Q_{AS} as defined in equation (23) is selected as the primary objective function for the reference optimization of the structure under the proposed approach. Q_{AS} represents the area under the acceleration transfer function of the system and is a measure of the transmissibility of the system. Hence, this quantity can be a good indicator of the portion of energy that is transferred to the structure as absolute acceleration, and consequently for the magnitude of the expected force reactions borne by the structure during the earthquake. It is noted that this approach does not require the use of a specific excitation power spectrum or time history excitation, leading to an optimized design that depends solely on the characteristics of the structure and can be used across a range of low-frequency excitations.

As a secondary approach, and for the purposes of comparison and verification of the proposed method, the quantity R_{AS} as defined in equation (29) is used as an alternative objective function. R_{AS} is an indicator of the actual energy content of the response to a specific excitation PSD. Section 4.2 explores the extent to which the response results derived from solving the problem with either of the two objective functions are equivalent.

3.2. Fixed parameters

The following are considered as fixed parameters of the system with regards of the optimization problem:

- **X_{VSD} :** The static deflection of the structure depends on the initial stiffness of the structure and its mass, or, in other words, its natural frequency, f_0 :

$$X_{VSD} = \frac{m_S g}{k_S} \quad (34)$$

As mentioned in section 2.2, a highlight in the proposed system is the fact that the static vertical deflections are determined by the initial vertical stiffness elements of the structure, hence:

$$k_R = k_S \quad (35)$$

This way, the static vertical deflections of the system are maintained at the desired level, while the negative stiffness element participates only in the dynamic response of the system. The static settlement of the rigid mass X_{VSD} (or, alternatively the initial stiffness of the structure (k_S) or its initial natural frequency (f_0)) is therefore the first fixed design variable of the system. X_{VSD} is taken equal to 2cm in the reference problem. The impact of X_{VSD} is also explored in section 4.2 where results of the optimization are shown for the range 1-3cm.

- **m_S, m_D :** The reference structure mass m_S is taken 1000kg. The small oscillating mass m_D is determined through (30) where μ_D is taken equal to 5%.
- **$\varepsilon_N, \varepsilon_P$ and ε_R :** Variations from the design values of the various stiffness can occur due to many factors such as temperature variations, manufacturing tolerances, or nonlinear behavior. The values of $\varepsilon_N, \varepsilon_P$ and ε_R are taken equal to 10%, 5% and 5% respectively.

3.3. Optimized parameters

The four parameters that are optimized under the proposed method are:

- **k_N :** The value of the NS element. The NS element maximum (absolute) value is equal to -100 N/m per kg of structure mass, as presented in [17]; the minimum value is equal to -800 N/m per kg
- **c_N and c_P :** The damping factors related to the stiffness elements k_P, k_N . In the reference problem, the upper limit of the damping ratios ζ_N, ζ_P is set 1.0%.
- **μ_R :** The inertance ratio of the main structure inerter, b_R . The upper limit of μ_R is set to 2.0.

3.4. Dependent variables

Positive stiffness element K_p derived from the static stability limit case:

To ensure that potential loss of the static stability is prevented, the possible variations of k_N, k_P, k_R should be taken into consideration in the design and optimization of the system. The stiffness parameters of the system may present significant fluctuations since almost all NS designs result from unstable non-linear configurations [21]. Consequently, an increase of the absolute value of k_N and/or a decrease of the values of k_P and k_R by a factor $\varepsilon_N, \varepsilon_P$ and

ε_R respectively, may result in the system being unstable. In the limit case that the total overall stiffness k_{tot} as calculated by equation (10) is equal to zero, the system is unstable:

$$k_{tot}=0 \Leftrightarrow (1 - \varepsilon_R)k_R + \frac{(1 - \varepsilon_P)k_P(1 + \varepsilon_N)k_N}{(1 - \varepsilon_P)k_P + (1 + \varepsilon_N)k_N} = 0 \quad (36)$$

$$\Rightarrow k_P = - \frac{(1 - \varepsilon_R)(1 + \varepsilon_N)}{(1 - \varepsilon_P) [(1 - \varepsilon_R) + (1 + \varepsilon_N)(k_N/k_R)]} k_N \quad (37)$$

Equation (36) shows the dependance of k_P on k_N , ε_N , ε_P and ε_R . The values of ε_N , ε_P and ε_R are fixed design parameters (paragraph 3.2), and k_N is obtained through the optimization (paragraph 3.3); hence the value of k_P can be derived from the above.

4. OPTIMIZATION RESULTS AND ANALYSIS

4.1. Reference optimization problem and results

The extended KDamper & inerter system is configured to control vibrations of a rigid mass of $m_S=1000$ kg. The optimal parameters are selected according to the optimization procedure described in section 3, using the interior-point optimization method. The results are presented for different values of the fixed parameter X_{VSD} within the allowable range of [1 3] (cm). The full set of parameters is presented in Table 1. The transfer functions H_{AS} , H_{US} , and the response PSDs S_{AS} , S_{US} are presented in Figure 3 and Figure 4 for different values of the design parameter X_{VSD} in order to evaluate the sensitivity of the TFs and response PSDs to the parameter X_{VSD} . The acceleration response spectrum utilized in this paper is the one used in [20] and it is based on a database of 50 artificial accelerograms, generated to match the EC8 spectrum with characteristics: Type 1 with spectral acceleration $\text{avg}(g)=0.9*0.36$.

It can be observed from Figure 3 that the maximum values of acceleration H_{AS} and S_{AS} are not substantially affected by the X_{VSD} . The relative displacement H_{US} is affected in the low-frequency range, while the S_{US} is less affected.

	μ_D	μ_b	k_N (kN/m)	k_R (kN/m)	k_P (kN/m)	ζ_N (%)	c_N (kNs/m)	ζ_P (%)	c_P (kNs/m)
$X_{VSD}=1\text{cm}$	0.05	0.60	-164.31	981.00	236.03	1.00	0.626	1.00	0.626
$X_{VSD}=2\text{cm}$	0.05	0.43	-86.45	490.50	125.76	1.00	0.442	1.00	0.442
$X_{VSD}=3\text{cm}$	0.05	0.36	-59.10	327.00	86.55	1.00	0.362	1.00	0.362

Table 1: Full set of the system parameters in the range of the design parameter $X_{VSD}=[1\ 3]$ (cm).

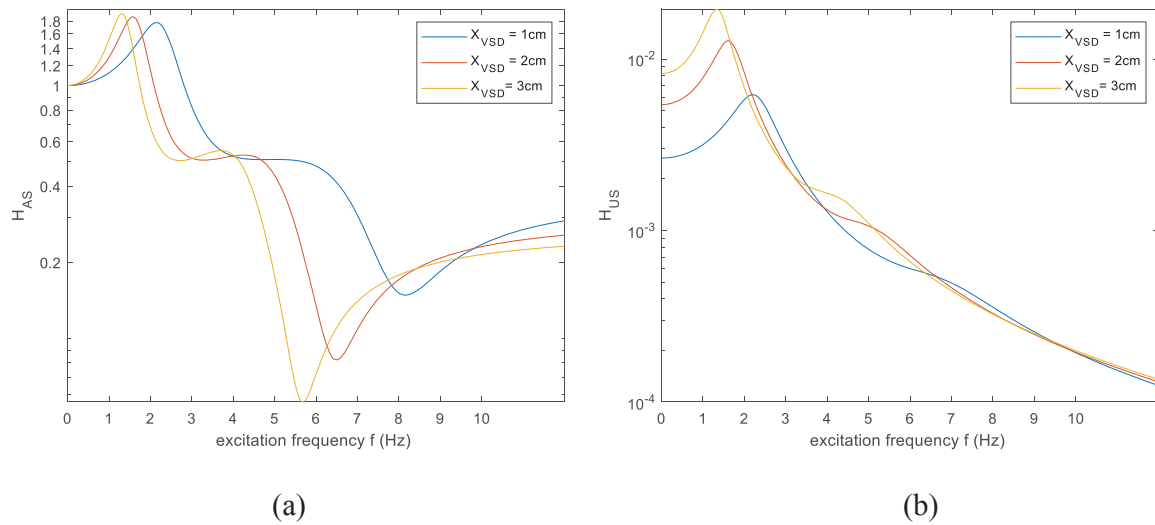


Figure 3. Transfer Functions of the optimized system with X_{VSD} varying in the range [1 3] (cm). (a) Structure absolute acceleration H_{AS} ; (b) structure relative displacement H_{US} .

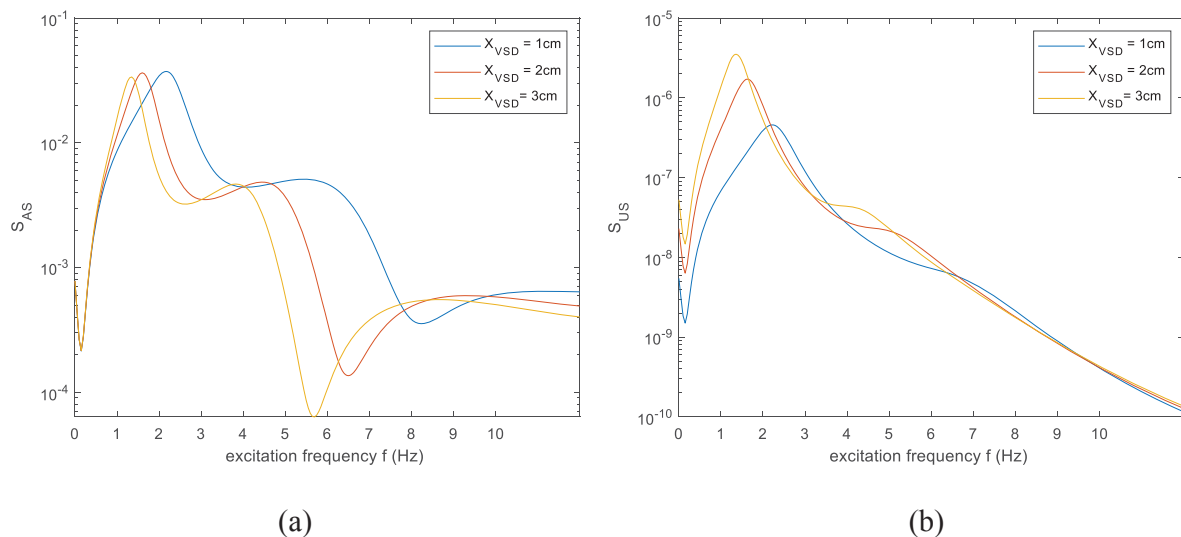


Figure 4. Response PSDs considering the ground motion acceleration excitation PSD, S_A , (a) Structure absolute acceleration S_{AS} ; and (b) relative displacement S_{US} .

The following section explores the impact of the selection of objective function to the optimization problem. This is evaluated in terms of response to excitation PSD and time history response to the sample of 50 artificial accelerograms. In order to better observe the effectiveness of the proposed configuration, in the next sections the responses are compared with a highly damped ($\zeta=15\%$) conventional damper (CD) as described in paragraph 2.1.

4.2. Effect of objective function (Q_{AS} , R_{AS})

As discussed in paragraph 3, the optimization approach considers the area under the acceleration transfer function of the system (scaled by a factor $d\omega$) as the primary objective function for this problem, Q_{AS} as defined in equation (23). One of the motivations of this paper is to explore its equivalency vs. the more conventional approach of using a measure of the actual content of the response (such as R_{AS} defined in equation (29)) as the objective function.

The results are shown for a ground motion acceleration excitation PSD and subsequently verified through time history analyses. An acceleration response indicator r_{AS} is defined as:

$$r_{AS,CD} = \frac{R_{AS,CD}}{\sqrt{\int_{-\infty}^{+\infty} S_{AS}(\omega) d\omega (= PGA)}} \quad (38)$$

$$r_{AS,KD,Opt} = \frac{R_{AS,KD,Opt}}{\sqrt{\int_{-\infty}^{+\infty} S_{AS}(\omega) d\omega (= PGA)}} \quad (39)$$

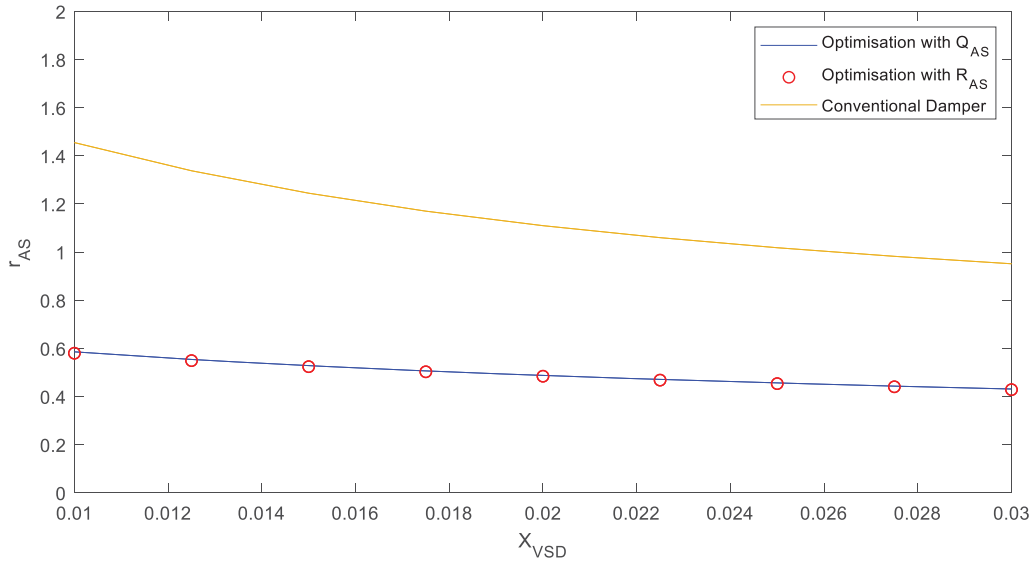


Figure 5. Variation of the acceleration response indicator r_{AS} over the X_{VSD} , of the conventional damping system and proposed configuration, for the two objective functions examined.

Figure 5 above presents the response indicators with respect to varying X_{VSD} in the cases of: (i) the Ext. KDamper & inerter system optimized via Q_{AS} as the objective function (blue line), (ii) the Ext. KDamper & inerter system optimized via R_{AS} as the objective function (red circles), (iii) the conventional damper (yellow line). It is evident that the two optimization methods coincide.

The PSD response results are subsequently verified with time history analyses from the 50 accelerograms. Below are the cumulative results showing the peak values of the quantities of interest (structure absolute acceleration a_S , structure relative displacement u_S , and oscillating mass relative displacement u_D) for X_{VSD} in the range of [1 3] (cm). Blue color corresponds to the reference optimization of the extended KDamper & inerter system, as described in Table 1. Red color shows the optimization using the same fixed parameters and constraints, but using R_{AS} as the objective function (instead of Q_{AS}). Finally, the orange bar represents the conventional damping system. Figure 6(a) shows the peak a_S averaged over the 50 accelerograms, while Figure 6(b)-(d) show the overall maxima of the peak quantities. It can be verified that the two objective functions yield very similar results across all quantities of interest. Moreover, it is observed that the proposed optimized system can reduce the structure peak acceleration up to more than 50% when compared to a conventional damping system, while maintaining the displacement within a satisfactory range.

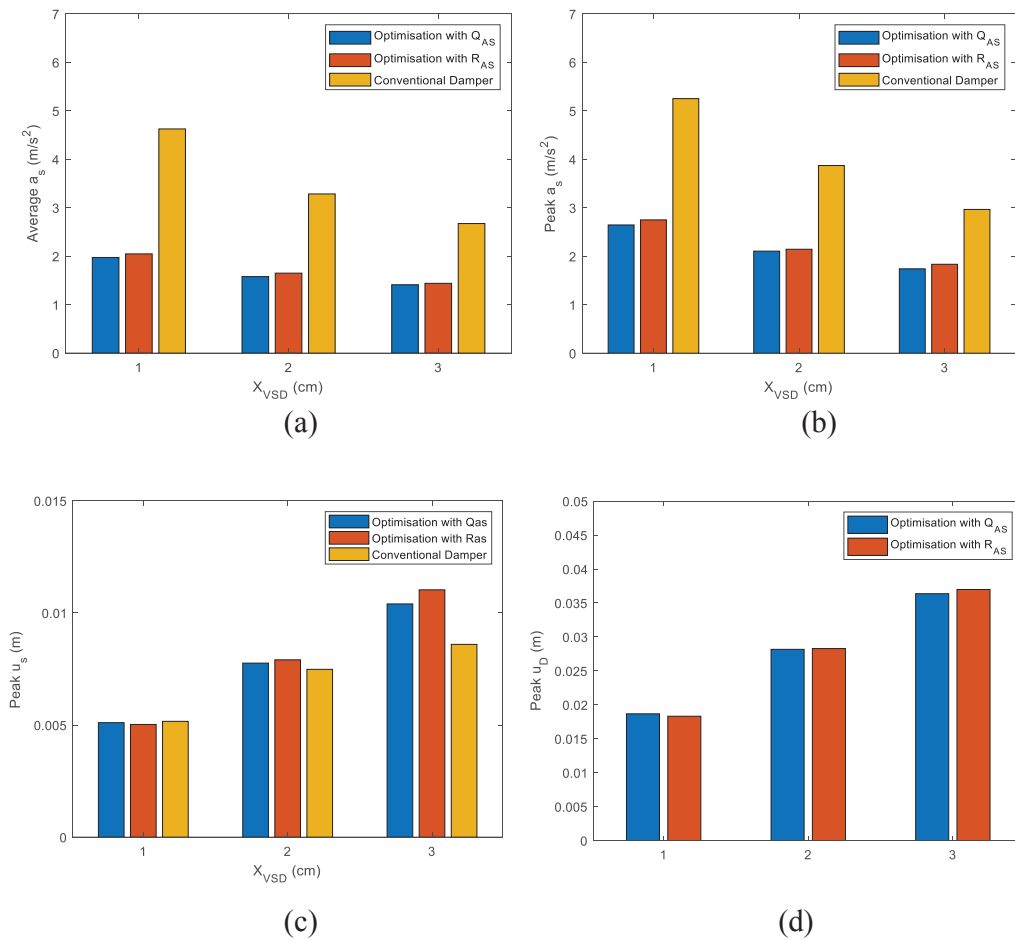


Figure 6. Results from the time history responses over 50 artificial accelerograms with the system optimized for the two different objective functions, and for the conventional damper. (a) Peak absolute acceleration averaged over the 50 accelerograms; (b) Overall maximum of the peak absolute acceleration; (c) Overall maximum of the peak relative structure displacement absolute accelerations; (d) Overall maximum of the peak relative oscillating mass displacement.

5. CONCLUSIONS

This paper proposes a stiff dynamic absorber, combining a negative stiffness element, a tuned-mass damper and an inerter. The system is configured under a proper optimization procedure, and the results are subsequently verified with an excitation PSD, generated from a database of artificial accelerograms designed to match the EC8 vertical spectrum as well as with time history results from the individual accelerograms.

Results indicate that the proposed system can drastically reduce structure accelerations (>50% reduction compared to a conventional damping system), while the vertical displacement remains at satisfactory levels. The proposed system thus demonstrates an enhanced dynamic behavior while maintaining suitable vertical stiffness. A key highlight of the proposed system in relation to previous works is the fact that the initial static stiffness is retained, since the negative stiffness elements participate only in the dynamic response.

Moreover, a new optimization approach is utilized through the introduction of a more general objective function which depends solely on the characteristics of the structure and not the excitation parameters. The suitability of this method is tested using response spectra and time history analyses for the abovementioned excitations, and verified through a comparison with previous optimization strategies that depend on a specific spectral excitation. The comparison

results confirm that a unique optimal stiff dynamic absorber design can be used across a wide range of important vertical dynamic absorption applications, as well as structure seismic protection.

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