

A NOVEL METHOD FOR THE GENERATION OF FULLY NON-STATIONARY SPECTRUM COMPATIBLE ARTIFICIAL ACCELEROGRAMS

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Abstract

The increasing availability of strong motion records makes the use of real accelerograms an attractive option for a proper definition of the input motions in the dynamic analysis of structural and geotechnical systems. Despite different real accelerograms selection procedures have been proposed in the literature, there are situations in which it may be impossible to select a suitable code-required number of accelerograms without applying large scale factors which distort the actual characteristics of selected motions leading to unrealistic input. In these situations, the use of artificial accelerograms could represent a possible alternative.

A novel procedure for generating fully non-stationary spectrum compatible artificial accelerograms, is presented in this paper.

Depending on the aim to be achieved, it is possible to obtain the spectrum-compatibility between the generated samples and the target action in terms of response spectrum or Fourier spectrum, using two different corrective Power Spectral density function terms.

A numerical application shows the validity of the generation procedure and the accuracy of the model in reproducing realizations with statistical characteristics similar to those of the target motion.

Keywords: Artificial accelerograms, Fully non-stationary stochastic process, Real ground motion records, Spectrum-Compatibility.

1 INTRODUCTION

The worldwide increasing availability of strong motion records makes the use of actual acceleration time-histories an attractive option to properly define the input motions for dynamic analysis of both structural and geotechnical (*S&G*) systems [1,2].

The characteristics of the expected ground motion at a given site are strongly affected by the possible coupling between the frequency content of the input ground motion and the frequencies of vibration of the soil deposit which, in turn, depend on the non-linear behaviour exhibited by soils when subjected to cyclic and dynamic loadings. Consequently, soil mechanical properties considerably influence the site response to the motion imposed by an earthquake at the bedrock level and, thus, should be properly accounted for in the definition of the interval of periods of interest for the accelerogram selection [3].

In this vein and with reference to one dimensional site response analysis problems, a method for the selection of proper sets of input ground motions accounting for most of these relevant issues has been proposed in Ref. [4]; Refs[5] and [6] showed that the soil heterogeneity in terms of shear wave velocity profile and the soil non-linear behaviour under cyclic loading significantly affect: *i*) the interval of vibration periods relevant for the accelerogram selection; *ii*) the characteristics of the selected input motions. However, despite the availability of earthquake records, frequently it is not possible to find sets of time histories that satisfy the compatibility criteria between the selected records and a target code-prescribed response spectrum, without applying large scale factors to each record of the set [7]. For this reason, different numerical procedures capable to generate artificial accelerograms with energy and frequency content consistent with those of actual acceleration records have been proposed in literature.

In [8], the authors proposed a deterministic modification method based on the harmonic wavelet transform, aimed at matching the elastic response spectrum of a given accelerogram to a target code-prescribed elastic response spectrum, while in [9], a novel formulation based on the wavelet transform has been used for the stochastic generation of fully non-stationary spectrum-compatible accelerograms. In [10] the evolutionary power spectra of seismic accelerograms, estimated using the method of separation, have been used to produce artificial accelerograms compatible with given recordings while, an iterative procedure based on the spectral representation of stochastic processes in which the ground motion is constituted by two waveforms, a real record and an artificial accelerogram, has been proposed in [11].

A new method for generating fully non-stationary spectrum compatible artificial accelerograms, that requires only the knowledge of the numbers of peaks, zero level up-crossings and the value of the total energy of the target signal, is presented in this paper.

Depending on the aim to be achieved, it is possible to obtain the spectrum-compatibility between the generated samples and the target action in terms of response spectrum or Fourier spectrum, using two different corrective Power Spectral density function (*PSD*) terms.

The proposed procedure requires the following steps: *i*) subdivide a selected target accelerogram in many contiguous time intervals in which appropriate unimodal Power Spectral density functions and polynomial form of modulating function have been chosen to obtain the same energy and frequency content of real accelerograms expected at the site of interest, applying the stochastic generation method recently proposed in [12]; *ii*) evaluate the mean elastic response spectrum and the Fourier spectrum of a set of generated fully non-stationary accelerograms samples; *iii*) satisfy the compatibility of so determined spectra to target ones by means of an iterative procedure.

A numerical application shows the validity of the spectrum-compatible generation procedure and the accuracy of the proposed model in reproducing realizations with statistical characteristics similar to those of the target motion both in time and frequency domain.

2 EVOLUTIONARY POWER SPECTRAL DENSITY FUNCTION METHOD FOR MODELLING SEISMIC ACTION

The procedure for generating a set of *fully non-stationary* artificial accelerograms compatible with a target spectrum and having the same characteristics of an earthquake induced ground motion \ddot{U}_g , consist of the following four steps:

1) divide the time duration $0 \div T_D$ of the analysed accelerogram \ddot{U}_g , in n contiguous time intervals of amplitude $\Delta T_k = t_k - t_{k-1}$ ($k=1, 2, \dots, n$), in each of which a uniformly modulated process is introduced as the product of a deterministic modulating function, $a(t)$, times a stationary zero-mean Gaussian sub-process $X_k(t)$, whose power spectral density (PSD) function $G_{X_k}(\omega)$ is filtered by two Butterworth filters:

$$G_{X_k}(\omega) = \beta_k \left(\frac{\omega^2}{\omega^2 + \omega_{H,k}^2} \right) \left(\frac{\omega_{L,k}^4}{\omega^4 + \omega_{L,k}^4} \right) \frac{\rho_k}{\pi} \left(\frac{1}{\rho_k^2 + (\omega + \Omega_k)^2} + \frac{1}{\rho_k^2 + (\omega - \Omega_k)^2} \right); \quad k=1, \dots, n \quad (1)$$

where β_k is a coefficient, given in closed form solution in [12], that makes unitary the variance of each sub-process $X_k(t)$ while the predominant circular frequency Ω_k and the frequency bandwidth ρ_k can be evaluated by the following expressions:

$$\Omega_k \cong \frac{2\pi N_{0,k}^+}{\Delta T_k}; \quad \rho_k \cong \frac{\pi N_{0,k}^+}{2\Delta T_k} \left[\pi - 2 \frac{N_{0,k}^+}{P_k} \right] \quad (2)$$

being P_k and $N_{0,k}^+$ respectively the number of *maxima* and of *zero-level up-crossings* of each k -th part in which the target accelerogram is subdivided;

2) evaluate the modulating function $a(t)$ by least-square fitting the *cumulative expected energy function* of the stochastic process to the cumulative energy function $E_{\ddot{U}_g}(t)$ of the target accelerogram subdivided in three-time intervals, according to [12]:

$$a(t) = \sum_{j=1}^2 \bar{a}_j(t) \mathcal{W}(t_{j-1}, t_j) + a(t_2) \exp \left[\frac{t-t_2}{T_D-t_2} \ln \left(\frac{|\ddot{U}_g(T_D)|}{a(t_2)} \right) \right] \mathcal{W}(t_2, t_3). \quad (3)$$

being $\mathcal{W}(t_{j-1}, t_j) = \mathcal{U}(t-t_j) - \mathcal{U}(t-t_{j-1})$ the window function and $\mathcal{U}(t)$ the unit step function;

3) generate the i^{th} *fully non-stationary* sample of the stochastic process $F_0^{(i)}(t)$ via the formula:

$$F_0^{(i)}(t) = a(t) \sqrt{2\Delta\omega} \left[\sum_{k=1}^n \sum_{r=1}^{m_N} \mathcal{W}(t_{k-1}, t_k) \sin \left(r \Delta\omega t + \theta_r^{(i)} \right) \sqrt{G_{X_k}(r \Delta\omega)} \right] \quad (4)$$

$\theta_r^{(i)}$ being the random phase angles, uniformly distributed over the interval $[0, 2\pi]$ and m_N being the number of parts in which the k -th PSD function $G_{X_k}(\omega)$ is discretized with a $\Delta\omega$ frequency sampling interval;

4) obtain the spectrum-compatibility reducing the gap between the mean spectrum of the generated samples $\bar{S}^{(j-1)}(\omega, \zeta_0)$ and the target one $S^{(T)}(\omega, \zeta_0)$, through the introduction of a corrective iterative *PSD* function $\bar{G}_{X_k}^{(j)}(\omega)$:

$$\bar{G}_{X_k}^{(j)}(\omega) = \bar{G}_{X_k}^{(j-1)}(\omega) \frac{S^{(T)}(\omega, \zeta_0)^2}{\bar{S}^{(j-1)}(\omega, \zeta_0)^2} \quad (5)$$

being $\bar{G}_{X_k}^{(0)}(\omega) = 1$ [14] and ζ_0 the viscous damping.

According to the formulation described in [13], the generic spectrum-compatible sample can be generated as:

$$\bar{F}_0^{(i)}(t) = a(t) \sqrt{2\Delta\omega} \left[\sum_{k=1}^n \sum_{r=1}^{m_N} \mathbb{W}(t_{k-1}, t_k) \sin(r \Delta\omega t + \theta_r^{(i)}) \sqrt{\bar{G}_{X_k}^{(j)}(r \Delta\omega) G_{X_k}(r \Delta\omega)} \right]. \quad (6)$$

2.1 Spectrum-compatibility

Depending on the purposes of the accelerogram generation procedure, the spectrum compatibility can be checked against the following targets:

- i) the pseudo acceleration response spectrum $S_R^{(T)}(\omega, \zeta_0)$ of the target accelerogram;
- ii) the Fourier spectrum $S_F^{(T)}(\omega)$ of the target accelerogram;
- iii) the code-prescribed elastic response spectrum $S_C^{(T)}(\omega, \zeta_0)$.

To achieve the Fourier-spectrum compatibility $S^{(T)}(\omega, \zeta_0) = S_F^{(T)}(\omega)$, the corrective iterative *PSD* function $\bar{G}_{X_k}^{(j)}(\omega)$ term can be particularized as follows:

$$\bar{G}_{X_k}^{(j)}(\omega) = \bar{G}_{X_k}^{(j-1)}(\omega) \frac{S_V^{(T)}(\omega, 0)^2}{\bar{S}_V^{(j-1)}(\omega, 0)^2} \quad (7)$$

being $S_V^{(T)}(\omega, 0)$ and $\bar{S}_V^{(j-1)}(\omega, 0)$ the velocity response spectra for $\zeta_0 = 0$.

3 NUMERICAL APPLICATION

In this section, in order to verify the accuracy of the proposed method, two different sets of one hundred artificial accelerograms spectrum-compatible with the pseudo acceleration response spectrum and the Fourier spectrum of the target accelerogram, have been generated.

3.1 Target accelerogram

The North-South component of the time history recorded at Vasquez Rocks Park ($R_{JB} = 23.1$ km [15]) during the $M_w = 6.7$ 1994 Northridge earthquake has been used as target accelerogram.

The selected accelerogram, having an overall duration $T_D = 36.6$ s and a sampling interval $\Delta t = 0.02$ s, has been recorded by a station with an average shear wave velocity in the upper 30 m equal to $V_{s,30} = 996$ m/s.

The target ground motion is characterized by a peak ground acceleration $PGA = 0.132$ g, a total intensity equal to $I_0 = 1.9$ m²/s³, a significant strong motion duration (i.e. interval of time elapsed between the 5% and 95% of I_0) $SMD = 7.3$ s and a total number of zero-level up-crossings and of peaks equal to $N_0^+ = 196$ and $P_0 = 212$, respectively.

As highlighted in [12], to obtain accurate results, the analysed accelerogram must be subdivided into n time intervals, each of which containing a number of *zero level up crossings* at least equal to one; consequently, in this application, the target accelerogram is subdivided in 73-time intervals with a constant time step of 0.5 s.

Further details about the parameters used to characterize the modulating function $a(t)$ and the *PSD* function $G_{X_k}(\omega)$ associated with each time interval, have been detailed in [13].

3.2 Spectrum compatibility

According to the procedure described in Section 2, a set of one hundred artificial accelerograms has been generated by using Eq. (4).

Then, to satisfy the spectrum compatibility in terms of pseudo acceleration response spectrum (*RSC*) or Fourier Spectrum (*FSC*), the iterative procedure has been applied four times and two different sets of one hundred accelerograms, have been obtained by Eq. (6).

Figure 1 shows a comparison between the time history of the target accelerogram and the i -th generated sample $\bar{F}_0^{(i)}(t)$, after 4 iterations, using the pseudo acceleration response spectrum compatibility (*RSC*) model (a) and the Fourier spectrum-compatibility (*FSC*) model (b). In both cases the variation in amplitude of the generated samples appears to be preserved in the time domain.

In Figure 2, the mean value of the cumulative energy function $I_0(t)$ and the cumulative *zero level up crossing* function $N_0^+(t)$ of the generated samples applying the pseudo acceleration response spectrum compatibility (*RSC*) model or the Fourier spectrum compatibility (*FSC*) model, after 4-th iterations, have been compared with the trend of the target functions.

In Figure 3 and 4 the average acceleration response spectrum $Sa(T)$ and the average Fourier spectrum module $|\mathcal{F}[\ddot{U}_g(t)]|$ of the two sets of artificial accelerograms, generated after 4 iterations by the *RSC* and *FSC* model, are compared with the corresponding target spectrum, respectively.

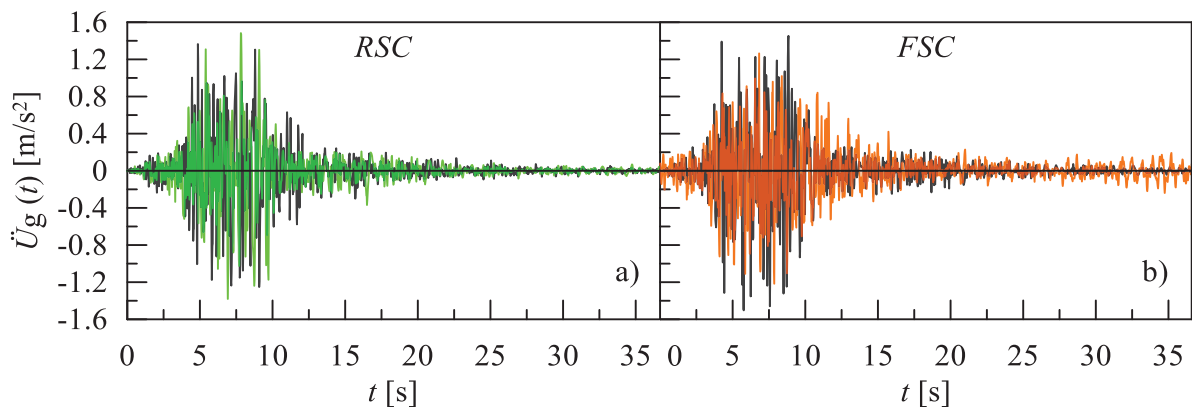


Figure 1: Comparison among the selected accelerogram (black line) and the corresponding i -th generated sample by the proposed fully non-stationary spectrum-compatible model, after 4 iterations: a) pseudo acceleration response spectrum-compatibility (*RSC*) model, b) Fourier spectrum-compatibility (*FSC*) model.

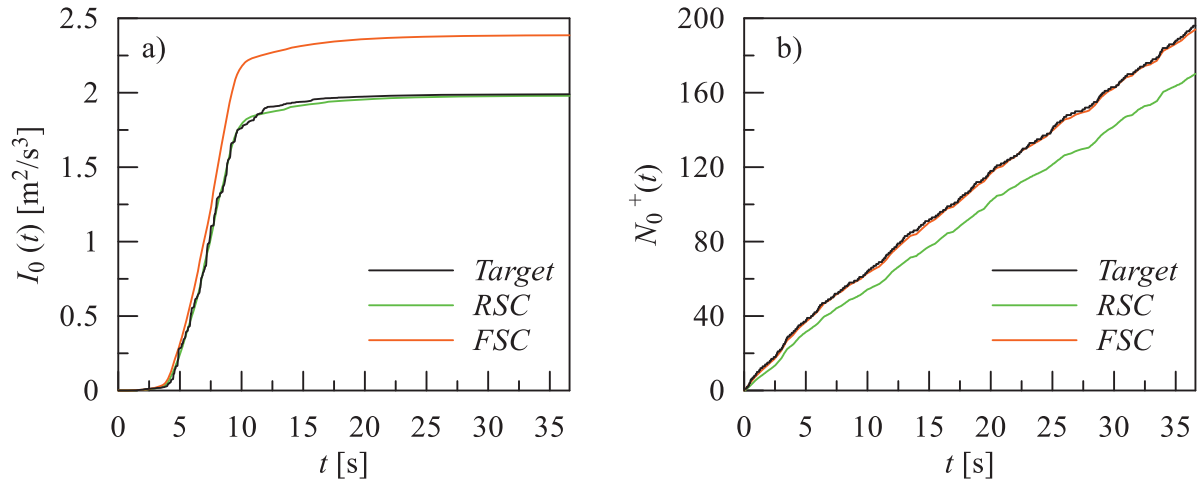


Figure 2: Comparison among the cumulative energy function (a) or cumulative zero level up crossing function (b) between the *target* accelerogram and the corresponding mean cumulative functions of the 100 samples, evaluated using RSC model and the FSC model, after 4-th iterations.

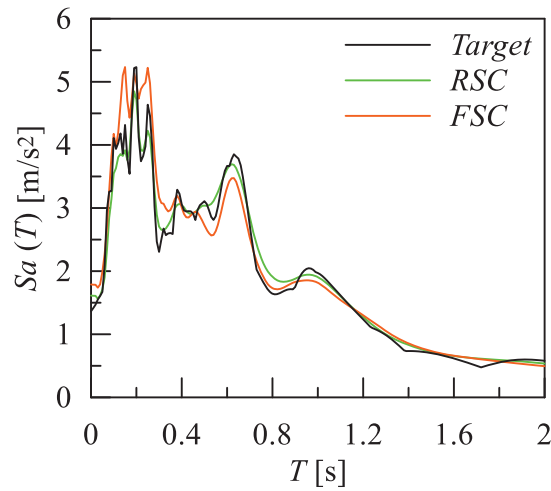


Figure 3: Comparison between the pseudo acceleration response spectrum and the mean response spectra of the 100 samples, evaluated using the RSC model and FSC model after 4-th iterations.

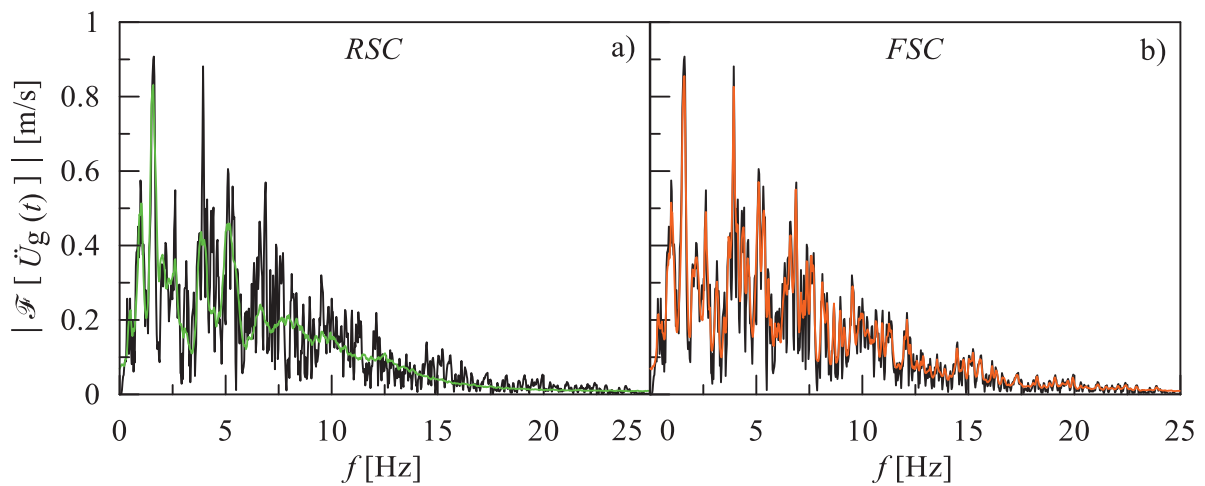


Figure 4: Comparison among the Fourier spectrum module of the selected accelerogram and the mean Fourier spectrum module of the generated sample, evaluated after 4 iterations by the: a) RSC model, b) FSC model.

From the observation of the numerical results obtained in the time and frequency domain, it emerges that:

- the use of the pseudo acceleration response spectrum-compatibility (*RSC*) model leads to outcomes statistically closer to those of the target one in terms of cumulative energy function $I_0(t)$ and pseudo-acceleration response spectrum $Sa(T)$ respect to the trends obtained by the application of the *FSC* model;
- the application of the Fourier-compatibility (*FSC*) model allows to obtain samples having an average *zero level up crossing function* $N_0^+(t)$ and Fourier spectrum closer to target one trend respect to those evaluated though the *RSC* model.

4 CONCLUSIONS

In this paper, a new procedure for generating fully non-stationary artificial spectrum-compatible artificial accelerograms, has been presented.

Firstly, the target accelerogram is subdivided in many contiguous time intervals in which the modulating and the unimodal Power Spectral density functions have been evaluated to obtain an average trend of the cumulative energy function $I_0(t)$ and *zero-level up crossing function* $N_0^+(t)$ very close to the ones of the target accelerogram.

Then, depending on the type of spectrum compatibility to be achieved (response spectrum compatibility or Fourier spectrum compatibility), an iterative procedure based on the use of different corrective iterative *PSD* function terms, has been implemented.

The numerical results show that using the acceleration response spectrum compatible (*RSC*) model, the generated samples have been characterized by an average cumulative energy function and pseudo-acceleration response spectrum very close to those of the target one, while the use of the Fourier spectrum model (*FRC*) leads to samples having an average *zero level up crossing function* and Fourier spectrum in good agreement with the target one.

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