

A DEGRADING BOUC-WEN DATA-DRIVEN MODEL FOR THE CYCLIC BEHAVIOR OF MASONRY INFILLED RC FRAMES

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Abstract. *Mechanics-based macro-models are often used to simulate the cyclic response of infilled reinforced concrete (RC) frames. However, these approaches are affected by uncertainties regarding damage and failure mechanisms. Therefore, this contribution proposes a new smooth data-driven model for the hysteresis of infilled RC frames. The infill panel is modeled through a damage-based Bouc-Wen element, which accounts for both pinching and deterioration of the mechanical characteristics. The parameters of the model are calibrated from an experimental data set of cyclic responses of RC infilled frames. Analytical correlations between parameters and geometric and mechanical characteristics of the infilled frame are derived. Blind validation tests are carried out in order to demonstrate the effectiveness of the proposed model.*

Keywords: Infilled RC frames, Hysteresis, Macro-model, Bouc-Wen, OpenSees.

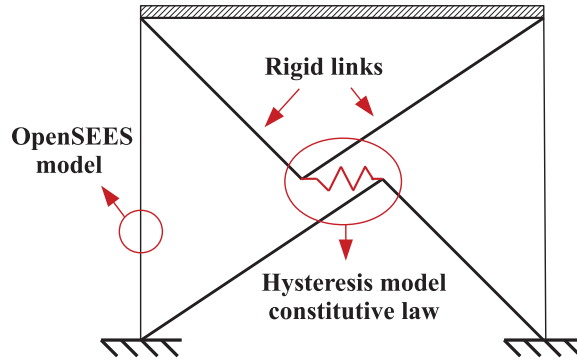


Figure 1: Scheme of the macro-model for the infilled frame: the bare frame is modeled with the software OpenSees, while the infill panel is simulated as a spring whose constitutive law is given by the Bouc-Wen model.

1 INTRODUCTION

The cyclic behavior of infilled frames has been widely investigated from both theoretical and experimental points of view [1–5]. However, the mechanical modeling of such structural systems is still an open challenge [6]. This mainly because the response of an infilled frame depends on many geometric and mechanical factors, which influence the damage mechanisms.

Many authors developed equivalent strut macro-models to predict the in-plane behavior of infilled frames [7–10]. However, predictions from different models lead to results that are often conflicting. Moreover, these modeling strategies are often complex and involve a large number of parameters that are very difficult to predict a priori. For the above reasons, recently many mechanics-based data-driven models have been developed [9, 11, 12]. In this case, the parameters governing the mechanical behavior of the structural system are estimated using regression formulas based on large data-sets. This modeling strategy has been proven to be reliable for many typologies of infilled frames.

This contribution proposes a new hysteresis data-driven macro-model based on the Bouc-Wen equation [13–15]. Deterioration of both stiffness and strength are taken into account through degrading functions for cyclic damage [16]. Pinching is introduced by adding in series a slip-lock element [17] to simulate the effect of masonry cracking. The model is governed by 11 parameters that have clear physical meanings. The regression formulas of these parameters were obtained with a data-driven approach, based on calibrations performed with experimental data of real infilled frames. The validation of the model was performed through additional blind prediction tests. The model proposed in this work is an effective tool for nonlinear dynamic analyses and probabilistic assessments [18, 19].

2 MACRO-MODELING OF THE INFILLED FRAME

The infill panel was replaced by a spring connected to the bare frame through rigid links, as shown in Figure 1. The constitutive behavior of the spring was defined by the hysteresis model proposed in this work. The bare frame was modeled on the OpenSees 2.5 software platform by using fiber-section beam-column elements with distributed plasticity.

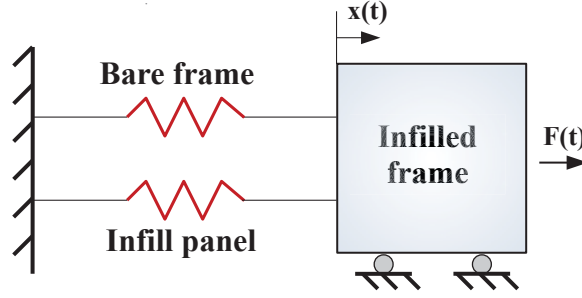


Figure 2: The infilled frame is regarded as a simple oscillator composed of two springs in parallel, one representing the bare frame and one representing the infill panel.

As depicted in Figure 1, the rigid links constrain the frame and the spring to undergo the same horizontal displacement. Thus the infilled frame was regarded as a simple oscillator whose stiffness is given by two contributions. One is provided by the bare frame, the other by the spring representing the panel. The sum of these two contributions gives the response of the entire infilled frame (Figure 2).

In the following section, the analytical formulation of the hysteresis model for the spring (infill panel) is presented.

3 ANALYTICAL DESCRIPTION OF THE PROPOSED HYSTERESIS MODEL

The proposed hysteresis model consists in a degrading Bouc-Wen element [13, 14] integrated with a slip-lock element [17, 20]. The latter was specifically introduced to simulate the typical pinching of masonry due to cracks opening and closure.

The equation of motion for a single-degree-of-freedom system is

$$m\ddot{x} + c\dot{x} + F_s[x(t), z(t), t] = F(t) \quad (1)$$

where x is the relative displacement of the mass of the system m with respect to the ground, c is the linear viscous damping coefficient, $F_s[x(t), z(t), t]$ is the non-damping restoring force, $z(t)$ is the hysteresis displacement and $F(t)$ is the external excitation. The overdots indicate the derivative with respect to the time, thus \dot{x} and \ddot{x} represent velocity and acceleration, respectively.

The Bouc-Wen model gives the following expression for the restoring force:

$$F_s[x(t), z(t), t] = \alpha kx(t) + (1 - \alpha)kz(t) \quad (2)$$

where k is the elastic stiffness of the system and α is the ratio between the final tangent stiffness k_f and the elastic stiffness ($0 \leq \alpha \leq 1$). Eq. (2) is composed of two contributions: the linear elastic component $\alpha kx(t)$ and the hysteresis component $(1 - \alpha)kz(t)$, which depends on the past history of stresses and strains. The hysteresis displacement $z(t)$ is given by the differential equation

$$\dot{z}(t) = A\dot{x}(t) - [\beta|\dot{x}(t)||z(t)|^{n-1}z(t) + \gamma\dot{x}(t)|z(t)|^n] \quad (3)$$

with the initial condition $z(0) = 0$. The parameters β , γ , and n control the shape of the hysteresis cycles. The parameter A determines the tangent stiffness but it is set to unity to avoid redundancy [21, 22]. The hysteresis energy dissipated by the system is defined as the area under the hysteresis restoring force $F^h[z(t), t] = (1 - \alpha)kz(t)$ along the total

displacement $x(t)$. The hysteresis energy can be normalized with respect to the mass, as follows [23, 24]:

$$\varepsilon(t) = \int_{x(0)}^{x(t)} \frac{F^h[z(t), t]}{m} dx = (1 - \alpha)\omega_0^2 \int_0^t z(\tau)\dot{x}(\tau)d\tau. \quad (4)$$

According to some results in literature [25–27] a linear proportionality between β and γ was introduced

$$\gamma = \eta_0\beta. \quad (5)$$

A dimensionless damage index, involving both energy dissipated and maximum displacement reached, was considered

$$d_i(t) = \frac{\varepsilon(t)}{kx_y^2/m} + \frac{|x_{max}(t)|}{x_y} \quad (6)$$

where x_y is the yielding displacement and $x_{max}(t)$ is the maximum displacement of the system until time t . Since the degrading Bouc-Wen model is a smooth hysteresis model, there is not a standard way to compute the yielding displacement x_y . Hence, this is an unknown parameter that must be identified along with the other model parameters.

The damage index $d_i(t)$ allows to define the following stiffness and strength degrading functions [16]:

$$\begin{aligned} A(d_i) &= e^{-\delta_k d_i(t)p_k(d_i)} \\ \beta(d_i) &= \beta_0 e^{[\delta_k p_k(d_i) - n\delta_f]d_i(t)} \end{aligned} \quad (7)$$

where δ_k and δ_f are two parameters that control respectively the amount of stiffness and strength degradation. The function $p_k(t)$ controls the stiffness degradation rate

$$p_k(t) = e^{-\psi d_i(t)} \quad (8)$$

where ψ is a parameter that controls the rising of stiffness degradation.

With the introduction of the degrading functions, the differential equation defining $z(t)$ assumes the following expression:

$$\begin{aligned} \dot{z}(t) &= A(d_i)\dot{x}(t) - \beta(d_i)(|\dot{x}(t)||z(t)|^{n-1}z(t) + \eta_0\dot{x}(t)|z(t)|^n) = \\ &= e^{-\delta_k d_i(t)p_k(d_i)}\dot{x}(t) - \beta_0 e^{[\delta_k p_k(d_i) - n\delta_f]d_i(t)}(|\dot{x}(t)||z(t)|^{n-1}z(t) + \eta_0\dot{x}(t)|z(t)|^n). \end{aligned} \quad (9)$$

A slip-lock element [17] was introduced in order to properly simulate the opening and closure of cracks in the infill panel. The slip-lock element was linked in series with the Bouc-Wen element (see Figure 3). Therefore, the relationship between the two elements is the following:

$$x(t) = x_1(t) + x_2(t) \quad (10)$$

where x is the total displacement of the system. The term x_1 is the contribution of the degrading Bouc-Wen differential equation, expressed by Eq. (9). The term x_2 is the contribution of the slip-lock element, which is given by the following relation:

$$\dot{x}_2(t) = a(t)f(z)\dot{z}(t). \quad (11)$$

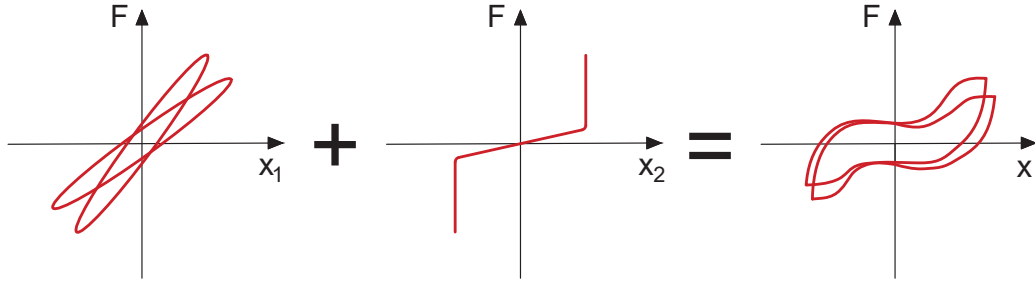


Figure 3: Schematization of the proposed hysteresis model, which is obtained as composition of degrading and slip-lock elements.

The function $f(z)$ defines the shape of the slip-lock phenomenon and is defined as

$$f(z) = e^{-z^2/Z_s^2} \quad (12)$$

where Z_s is the range of z in which the slip lock occurs. Parameter a is a function of the attained ductility

$$a(t) = A_s \frac{|x_{max}(t)|}{x_y} \quad (13)$$

where A_s is a parameter that controls the slip length, x_{max} is the maximum displacement reached at the prior cycle and x_y is the yielding displacement.

Combining Eqs. (9), (10) and (11), the following differential equation for the hysteresis displacement is obtained:

$$\dot{z}(t) = \dot{x}(t) \frac{A(d_i) - \beta(d_i)[\text{sgn}(\dot{x})|z(t)|^{n-1}z(t) + \eta_0|z(t)|^n]}{\{1 + a(t)f(z)[A(d_i) - \beta(d_i)(\text{sgn}(\dot{x})|z(t)|^{n-1}z(t) + \eta_0|z(t)|^n)]\}}. \quad (14)$$

4 CALIBRATION OF THE PROPOSED MODEL USING EXPERIMENTAL DATA

The model parameters were calibrated according to experimental data. The data involve 10 real specimens and were collected from the experimental campaigns by Cavaleri and Di Trapani [6], Mehrabi et al. [2], Colangelo [28], Kakaletsis and Karayannis [29] and Mansouri et al. [30]. For each specimen, the force-displacement hysteresis curve was acquired. All the campaigns considered performed displacement control tests in quasi-static loading.

For each specimen, the contribution of the infill panel was represented by the difference between the entire response of the infilled frame and the response of the bare frame modelled on OpenSees. The parameters of the proposed model for the infill panel were calibrated on the basis of this contribution. For each specimen, the calibration of the parameters was performed through a genetic algorithm implemented in a MATLAB® code. The genetic algorithm finds the set of parameters that minimizes the following objective function

$$OF(\boldsymbol{\theta}) = \frac{\int_{x_0}^{x_f} |[F_e(x) - F_s(\boldsymbol{\theta}, x)]| dx}{\int_{x_0}^{x_f} |F_e(x)| dx} \quad (15)$$

where $\boldsymbol{\theta}$ is the parameter vector, x_i and x_f are the initial and final displacement records, $F_e(x)$ is the force derived from the experimental data and $F_s(\boldsymbol{\theta}, x)$ is the one simulated by

Table 1: Calibrated values of the hysteresis model parameters for the reference specimens.

Reference	Spec.	α	β_0	η_0	n	k	x_y	δ_k	δ_f	ψ	Z_s	A_s
Cavaleri and Di Trapani [6]	S1A	0.003	0.35	0.2	1.5	76	1.45	0.017	0.003	0.001	0.16	5.91
Mehrabi et al. [2]	4	0.003	0.41	0.2	1.5	84	1.23	0.035	0.013	0.001	0.16	4.55

The values of k and x_y are given respectively in kN/mm and mm.

Table 2: Predicted values of the hysteresis model parameters for the reference specimen for validation.

Reference	Spec.	α	β_0	η_0	n	k	x_y	δ_k	δ_f	ψ	Z_s	A_s
Cavaleri and Di Trapani [6]	S1C	0.003	0.32	0.2	1.5	79	1.70	0.022	0.006	0.001	0.18	4.86

The values of k and x_y are given respectively in kN/mm and mm.

the proposed model. Parameters α , η_0 , n and ψ assumed very similar values for different specimens. Thus their values were fixed for simplicity, since small variations did not affect the results significantly.

The calibration was performed for 10 specimens, but in this work only 2 of them are shown for the sake of brevity. The two specimens considered are S1A from [6] and specimen 4 from [2]. The parameters identified in the calibration are listed in Table 1 and the results are shown in Figure 4.

5 EMPIRICAL CORRELATION LAWS AND BLIND VALIDATION TEST

The dependence of the parameters k , x_y , β_0 , δ_k , δ_f , Z_s , A_s on the geometrical and mechanical properties of infilled frames was investigated. The most representative quantities like the elastic modulus of masonry, the compression strength of masonry or the compression strength of concrete were gathered in a functional. Considering all the specimens, the values assumed by each model parameters were put in relation with the values assumed by the functional. In this way, the correlation laws defined are able to predict the values of the model parameters in function of the geometrical and mechanical properties of a generic infilled frame.

The proposed correlation laws were validated on the basis of experimental data related to a specimen different from the one used for calibration. The specimen used for validation is specimen S1C from the experimental campaign in [6]. The geometrical and mechanical properties of specimen S1C are different from those of the specimens used for in the calibration. Therefore, this specimen was considered eligible for the blind validation test. Table 2 collects the parameters predicted by the correlation laws and Figure 5 shows the result of the validation test.

6 CONCLUSIONS

The data-driven macro-model proposed in the present contribution is able to predict the hysteresis response of infilled frames. The combination of degrading Bouc-Wen element and slip-lock element provides a model that captures hysteresis and damage of such structural systems.

The model involves 11 parameters with clear physical meanings. Four of them have less impact on the final behavior, so they were fixed. The 7 remaining parameters were

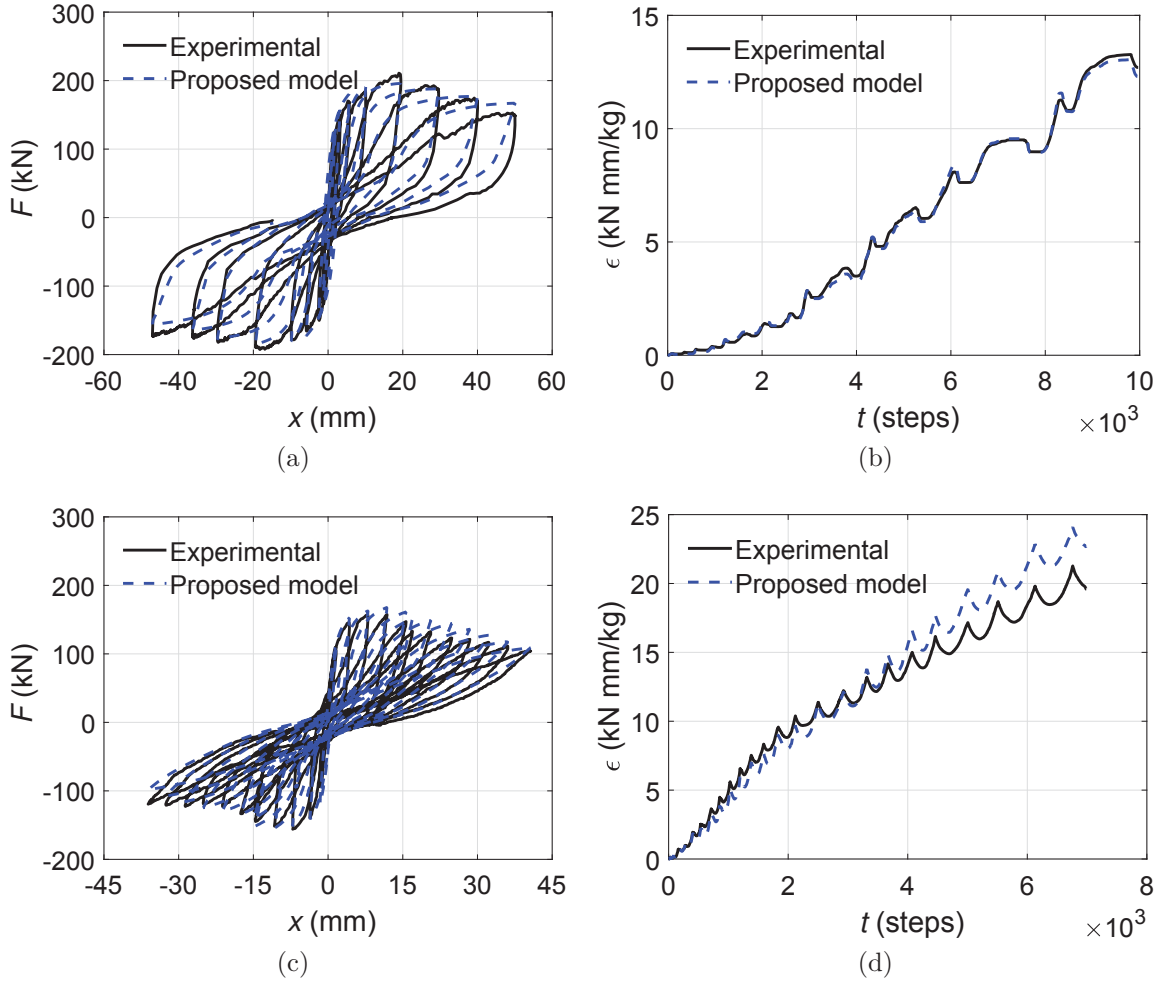


Figure 4: Results of the calibration for specimen S1A from Cavaleri and Di Trapani [6] and specimen 4 from Mehrabi et al. [2]: (a) S1A hysteresis cycles; (b) S1A dissipated energy; (c) Mehrabi 4 hysteresis cycles; (d) Mehrabi 4 dissipated energy.

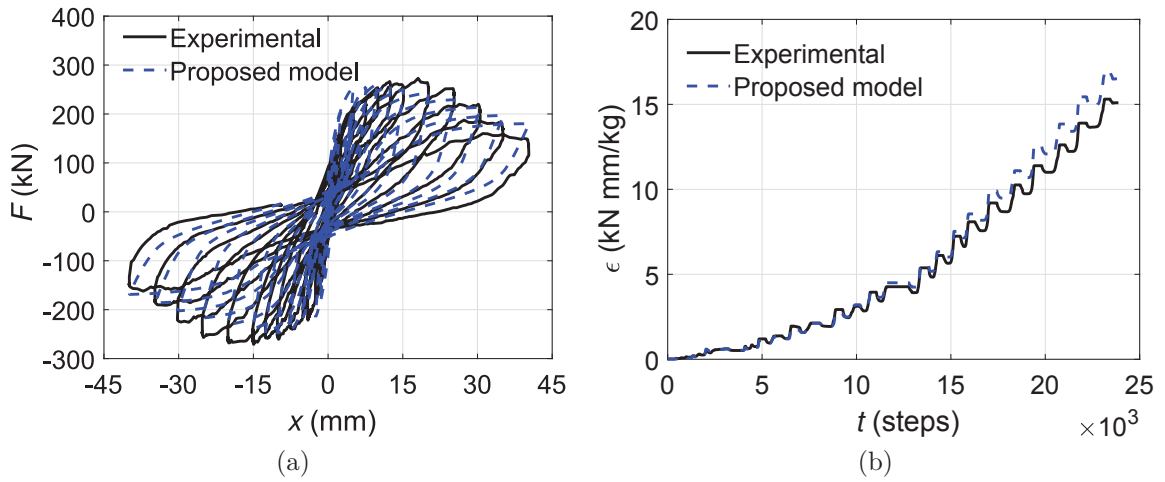


Figure 5: Blind validation test on specimen S1C from Cavaleri and Di Trapani [6]: (a) hysteresis cycles; (b) dissipated energy.

calibrated on the basis of experimental data of real infilled frames. Regression laws were estimated and final blind validation tests were performed. The results demonstrated that the model is accurate.

The simplicity and the smooth nature of the governing equations make the proposed model suitable for numerical simulations, such as nonlinear time-history and stochastic analyses.

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