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AN APPLICATION OF THE PROPER ORTHOGONAL DECOMPOSITION METHOD FOR NONLINEAR DYNAMIC ANALYSIS OF REINFORCED CONCRETE STRUCTURES SUBJECTED TO EARTHQUAKES

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Abstract

This paper presents an extension of the Proper Orthogonal Decomposition method (POD) to nonlinear dynamic analysis of reinforced concrete multistory frame structure where the material nonlinearity is modeled by the multi-fiber section. To test the effectiveness of this approach, we first perform a nonlinear dynamic analysis under a seismic excitation using a direct implicit time integration scheme. Then, based on structural response observations, POD modes were extracted and used to reduce the structural system subjected to different earthquakes. A comparison was made between full model and reduced model analysis in order to assess the effectiveness of this technique.

Keywords: Reinforced concrete beam, Material nonlinearity, Multi-fiber section, Dynamic analysis, Reduced model, Proper Orthogonal Decomposition.

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1 INTRODUCTION

This paper presents an extension of the Proper Orthogonal Decomposition method (POD) to nonlinear dynamic analysis of reinforced concrete multistory frame structure where the material nonlinearity is modeled by the multi-fiber section.

The multi-fiber section model consists in dividing the structural element cross section into a set of longitudinal fibers. Each fiber is made up of a single material and has the potential to undergo nonlinear inelastic longitudinal deformation according to the uniaxial stress-strain behavior of its corresponding material [1-4]. The layered shell element consists in dividing the 2D structural element into layers along its thickness. Each layer is made up of a single material and can behave nonlinearly in 2D [5-7]. When dealing with nonlinearities, nonlinear solving techniques should be adopted. The classical and most used nonlinear solvers are the Newton-Raphson method and its derivatives, displacement control approach and the arc length technique.

Dynamic excitations in structures are usually studied using direct integration time history analysis. In this approach, temporal discretization is considered and the direct time integration is conducted using implicit methods like Newmark-β [8], Wilson θ [9], HHT-α [10] or explicit methods like central difference and Runge-Kutta. The main concern in using the direct time integration analysis for linear and nonlinear models is its high computational cost especially when applied in structural seismic analysis. In fact for seismic analysis, the structure is subjected to dynamic excitations at its base. These excitations are generally based on the accelerograms of previously recorded quakes in the region. In order to cover all probable scenarios, the structure should be subjected to multiple accelerograms vibrating in all different directions which greatly increases the time cost of this analysis technique.

Due to this setback, several model reduction techniques have been proposed to decrease the time cost of the dynamic time history analysis. For linear systems, modal truncation can be used to define the most influential mode shapes of the structure and then this truncated modal base is used to reduce the dynamic equation of the structural system [11-12]. For nonlinear structures, research based on the work of [13] has been conducted to determine an analogy between nonlinear normal modes and linear ones. However, this nonlinear modal analysis is not widely used due to the limitation when non-smooth nonlinearities are present in the structure [14].

The Proper Orthogonal Decomposition (POD) is a data driven method based on the statistical Principal Component Analysis (PCA) of observations dataset. In other words, data obtained from observations at different time intervals (snapshots) are analyzed to determine the optimal subspace that can be used to recreate the entire dataset with minimum errors. This subspace is later used to reduce the model under consideration in calculation. The POD method dates back to the 1930's and today is applied in fluid mechanics for model reduction of turbulent flow, model reduction of structural dynamics, damage detection, reduction of dynamic models for microelectromechanical systems and in lots of other domains.

Seismic analysis considering nonlinear material behavior of reinforced concrete structures is classically conducted by two approaches. The first one is the pushover analysis which is a static nonlinear approach that tries mimicking the dynamic behavior of the structure by considering it to respond dynamically according to its fundamental mode shape only. Horizontal loads are distributed on the structure proportionally to this fundamental mode shape vector and are increased progressively while nonlinear material behavior is taken into account. This approach is limited to structures where the fundamental mode shape is the dominant mode of vibration and thus limiting it to regular low-rise buildings where no response in function of time is required (only maximum values are provided). For other cases, the previously mentioned direct integration nonlinear time history analysis is used.

According to the authors' knowledge, the POD method was never used for reducing the direct integration nonlinear time history analysis of a Reinforced Concrete (RC) structure where material nonlinearity is modelled by the multi-fiber section approach. This paper presents a nonlinear multi-fiber RC multistory frame structure subjected to seismic excitations at its base and the POD is used to reduce the direct integration time history analysis cost. Section 2 presents the modeling of material nonlinear behavior for a RC beam element using the multi-fiber section approach. Section 3 is dedicated to the dynamic analysis of a RC element with material nonlinearities using classical and reduced POD procedures. Section 4 presents an application of the POD reduction method to the multistory frame structure under consideration while comparing the results with full model analysis. Section 5 summarizes the conclusions and future perspectives.

2 MULTI-FIBER BEAM MODEL

The concentrated (lumped) plasticity is the simplest and most popular approach to model material nonlinearity in structural elements. However, this technique is based on the assumption that nonlinear material behavior occurs only at specified concentrated points of the structural member (which is a major simplification). In addition, interaction between bending moments and varying axial forces at the plastic hinge is not taken into account. Moreover, the plastic hinges behavior is defined by characteristic curves (Moment versus Rotation or Force versus Displacement) provided by the seismic codes. These curves are based on rough estimations and assumptions which reduce their accuracy. On the other hand and as we will detail in this section, the multi-fiber beam model assures the distribution of nonlinear material behavior all along the structural element length and all over its cross section (distributed plasticity approach). In addition, this technique takes into account the interaction between bending moments and axial loads and can be applied to elements having non-typical cross sections. The fiber model approach is more computationally demanding than the concentrated plastic hinge technique however it remains efficient and very beneficial especially for wall elements modeled by the equivalent beam approach.

Since 2D finite elements are not addressed in this paper, the multi-fiber beam approach is adopted in this work to model the nonlinear material in 1D finite elements (beams, columns, equivalent beam model for walls) while considering the Euler-Bernoulli hypothesis (planar sections before deformation remain planar and perpendicular to the element's center line after deformation).

As already mentioned, the multi-fiber beam approach consists in dividing the structural element cross section into a set of longitudinal fibers. As a consequence, using this modeling technique requires a 3-level analysis.

Fibers are the fundamental level of analysis. Each fiber is made up of only one single material: for reinforced concrete members, the fibers can be made of steel reinforcements, confined or unconfined concrete. The fiber axial stress σ_{fiber} and the longitudinal tangent Young modulus $E_{T\,fiber}$ are determined as a function of the longitudinal fiber axial strain ε_{fiber}.

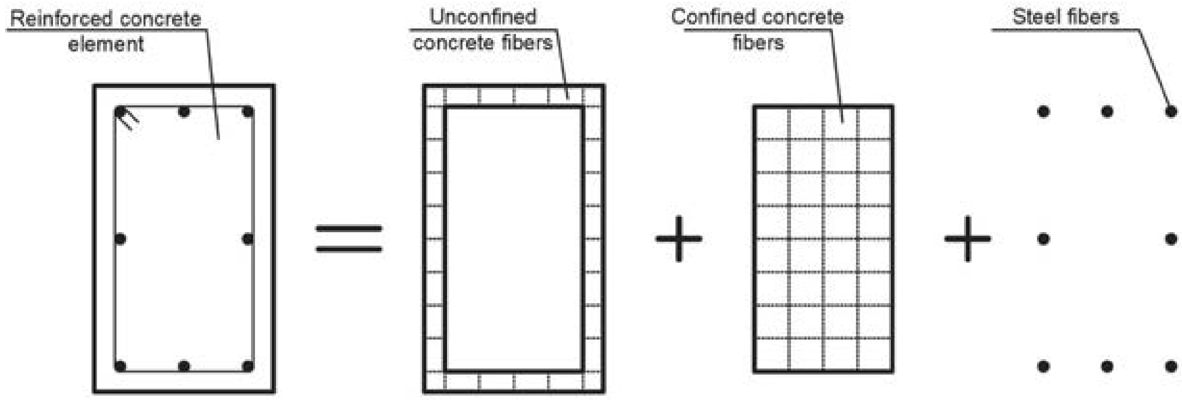

Figure 1. Multi-fiber reinforced concrete section

The element's cross section is the second level of analysis. Applying the Euler-Bernoulli hypothesis will result in perfect bond conditions between fibers (no sliding of a fiber with respect to another is allowed). In 2D structural analysis, for a fiber having its centroid located at the ordinate y in the section reference, the axial longitudinal strain in the fiber $\varepsilon(y)$ can be determined as a function of the section's uniform axial strain along x axis ε_x and the section's curvature along z axis ϕ_z

$$\varepsilon(y) = \varepsilon_x - y\phi_z = \{1 \quad -y\}\begin{Bmatrix}\varepsilon_x \\ \phi_z\end{Bmatrix} \tag{1}$$

since in nonlinear analysis the calculation is done by increments, we get:

$$\Delta\varepsilon(y) = \Delta\varepsilon_x - y\Delta\phi_z = \{1 \quad -y\}\begin{Bmatrix}\Delta\varepsilon_x \\ \Delta\phi_z\end{Bmatrix} \tag{2}$$

this axial strain increment of the fiber $\Delta\varepsilon(y)$ causes an increment in the section's internal axial force ΔN and bending moment along z axis ΔM_z.

$$\Delta N = E_{T\,fiber} A_{fiber} \Delta\varepsilon(y) \tag{3.a}$$

$$\Delta M_z = -y\Delta N = -yE_{T\,fiber} A_{fiber} \Delta\varepsilon(y) \tag{3.b}$$

for a single fiber, the resulting increment of internal forces in the section is

$$\{\Delta F_{Section}\} = \begin{Bmatrix}\Delta N \\ \Delta M_z\end{Bmatrix} = E_{T\,fiber} A_{fiber} \begin{bmatrix}1 & -y \\ -y & y^2\end{bmatrix}\begin{Bmatrix}\Delta\varepsilon_x \\ \Delta\phi_z\end{Bmatrix} \tag{4}$$

for all the fibers, the entire resulting increment of internal forces in the section is

$$\{\Delta F_{Section}\} = \begin{Bmatrix}\Delta N \\ \Delta M_z\end{Bmatrix} = \underbrace{\sum_{i=1}^{n_{fiber}} E_{T\,fiber\,i}\, A_{fiber\,i} \begin{bmatrix}1 & -y_i \\ -y_i & {y_i}^2\end{bmatrix}}_{[K_T]}\begin{Bmatrix}\Delta\varepsilon_x \\ \Delta\phi_z\end{Bmatrix} \tag{5}$$

where $n_{fiber}$is the total number of fibers in the section and $[K_T]$ is the section's tangent stiffness matrix.

The entire element is the third level of analysis. Linear shape functions are considered for longitudinal translation and Hermite cubic shape functions are used for bending. Applying the principle of virtual work we get

$$\{F_{int}\} = \int [B(x)]^T \begin{Bmatrix}1 \\ -y\end{Bmatrix} \sigma_{fiber}(x,y)dV \tag{5}$$

$$[K_T] = \int [B(x)]^T \begin{Bmatrix} 1 \\ -y \end{Bmatrix} E_{T\,fiber}(x,y)\{1 \quad -y\}[B(x)]dV \tag{6}$$

where $\{F_{int}\}$ is the internal nodal force vector of the element, $[K_T]$ is the element's tangent stiffness matrix and $[B(x)]$ is the gradient operator containing the derivatives of shape functions.

The volume integral required for the calculation of $\{F_{int}\}$ and $[K_T]$ is split into a surface integral on the cross section and a 1D integral along the longitudinal axis of the element. Since the element's cross section is already divided into fibers, we substitute the surface integration by the summation of fiber areas. Next, the longitudinal 1D integration is done by Gauss points.

3 FULL AND REDUCED DYNAMIC MODELS

As already mentioned, the classical time costly approach for capturing the nonlinear seismic response of a structure in function of time is the full model implicit direct integration nonlinear time history analysis. The Newmark-β method [8] is one of the famous implicit direct integration techniques used for linear and nonlinear time history analysis. For this method, knowing the structural system state at instant t_i (displacement, velocity and acceleration vectors) and assuming a variation pattern for acceleration between instants t_i and t_{i+1} (i.e. constant average acceleration) makes it possible to express the dynamic equation of the structural system at instant t_{i+1} with only one unknown (the displacement vector at instant t_{i+1}) and thus solving easily the system.

The proper orthogonal decomposition POD also known as the Principal Component Analysis PCA and the Karhunen-Loève Decomposition KLD is a statistical analysis of observation data. Let's consider a data matrix $[X]$ containing n observation vectors $[X] = [\{X_1\} \quad \cdots \quad \{X_n\}]$ and each observation vector is made of m dimension

$$[X] = [\{X_1\} \quad \cdots \quad \{X_n\}] = \begin{bmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{bmatrix} \tag{7}$$

$\{S_i\} = \{x_{i1} \quad \cdots \quad x_{in}\}$ is row i in matrix $[X]$ and represents all the data collected on dimension i. If data set $\{S_i\}\ \forall\ i$ has a zero mean, the variance of $\{S_i\}$ becomes

$$\sigma^2(\{S_i\}) = \frac{1}{n-1} \times \sum_{k=1}^{n} \left(x_{ik} - mean(\{S_i\})\right)^2 = \frac{1}{n-1} \times \sum_{k=1}^{n} (x_{ik})^2 = \frac{1}{n-1} \{S_i\}\{S_i\}^T \tag{8}$$

and the covariance of $\{S_i\}$ and $\{S_j\}$ becomes

$$COV(\{S_i\},\{S_j\}) = \frac{1}{n-1} \times \sum_{k=1}^{n} \left(x_{ik} - mean(\{S_i\})\right)\left(x_{jk} - mean(\{S_j\})\right) \tag{9.a}$$

$$COV(\{S_i\},\{S_j\}) = \frac{1}{n-1} \times \sum_{k=1}^{n} (x_{ik})(x_{jk}) = \frac{1}{n-1} \{S_i\}\{S_j\}^T \tag{9.b}$$

High value of $\sigma^2(\{S_i\})$ indicates high action on dimension i and vice versa. High value of $COV(\{S_i\},\{S_j\})$ indicates high similarity between the actions on dimension i and dimension j. On the other hand, $COV(\{S_i\},\{S_j\}) = 0$ indicates zero resemblance (total independence) between the actions on dimension i and dimension j.

If data set $\{S_i\}\ \forall\ i$ has a zero mean, the covariance of matrix $[X]$ becomes

$$COV([X]) = \frac{1}{n-1} [X][X]^T \tag{10.a}$$

$$COV([X]) = \begin{bmatrix} \sigma^2(\{S_1\}) & COV(\{S_1\},\{S_2\}) & \cdots & COV(\{S_1\},\{S_n\}) \\ COV(\{S_2\},\{S_1\}) & \sigma^2(\{S_2\}) & \cdots & COV(\{S_2\},\{S_n\}) \\ \vdots & \vdots & \ddots & \vdots \\ COV(\{S_n\},\{S_1\}) & COV(\{S_n\},\{S_2\}) & \cdots & \sigma^2(\{S_n\}) \end{bmatrix} \quad (10.b)$$

Determining the principal components of data matrix $[X]$ starts by finding a new orthonormal reference $[N]$. The initial data matrix $[X]$ is expressed in this new reference as $[X'] = [N]^T[X]$. For $[N]$ to be containing the principal components of the data observation, $COV([X'])$ should be a diagonal matrix. In other words, we have zero similarity between actions on different new dimensions in reference $[N]$ ($COV\left(\{S'_i\},\{S'_j\}\right) = 0 \; for \; i \neq j$).

Since $COV([X])$ is made up of $[X][X]^T$ it is a symmetrical matrix and thus has real eigenvalues.

$$[X][X]^T[\emptyset] = [\emptyset][\lambda] \quad (11)$$

where $[\emptyset]$ is the eigenvectors matrix and $[\lambda]$ is the diagonal matrix containing the eigenvalues. Eigenvectors are orthonormal vectors and we can demonstrate that the new reference $[N]$ we were talking about in the previous paragraph is in fact the eigenvectors matrix ($[N] = [\emptyset]$) of $[X][X]^T$. In fact for $[X'] = [\emptyset]^T[X]$ we get

$$COV([X']) = \frac{1}{n-1}[X'][X']^T = \frac{1}{n-1}[\emptyset]^T \underbrace{[X][X]^T[\emptyset]}_{[\emptyset][\lambda]} = [\lambda] \quad (12)$$

$COV([X'])$ is a diagonal matrix and $\sigma^2(\{S'_i\}) = \lambda_i$. We notice that the higher λ_i is the more we have actions on dimension i in the eigenvectors reference. As a conclusion, principal components of the data set $[X]$ are the eigenvectors of $[X][X]^T$ and modes with high eigenvalues are the most influential in representing $[X]$.

The orthogonal eigenvectors obtained are called POD modes and the corresponding eigenvalues are called proper orthogonal values. The POD modes can be used to reconstruct the initial data matrix $[X]$. The higher the eigenvalue of a POD mode is, the more essential this mode is in recreating $[X]$.

By considering the most important s POD modes ($s < m$) and placing them in $[T] \in \mathbb{R}^{m\times s}$, the $\{X_t\}$ snapshot vector previously expressed in m dimensions can now be approximated in the lower s dimensions

$$\underbrace{\{X_t\}}_{\in\mathbb{R}^{m\times 1}} \cong \underbrace{[T]}_{\in\mathbb{R}^{m\times s}} \underbrace{\{Q_t\}}_{\in\mathbb{R}^{s\times 1}} \quad (13)$$

where $\{Q_t\}$ contains the coordinates of the snapshot vector in the new reference $[T]$. The choice of the number s of POD modes to consider in the reduced new reference should satisfy 2 conditions:

1- The representation in the new reference should be accurate so the error should be minimal. The higher s is, the more accurate the approximation is.

$$error = \sum_{i=1}^{n} \left\| \{X_{t_i}\} - [T]\{Q_{t_i}\} \right\| \quad (14)$$

2- For the dimensions reduction to be efficient, the number of chosen POD modes s should be relatively small.

In order to balance between accuracy and efficiency, an energy criterion is considered to determine the optimal value of s. The Proper Orthogonal Value of a mode gives an indication on the energy carried by this mode. Generally, the first s POD modes carrying at least 99% of the total system energy are considered for the new reduced reference.

$$\frac{\sum_{i=1}^{s}\lambda_i}{\sum_{j=1}^{m}\lambda_j} \geq 99\% \tag{15}$$

In structural dynamics, the POD reduction can be applied to the direct integration time history analysis for linear or nonlinear structures. In order to get the observation data required for the POD, we initially do a classical implicit direct integration time history analysis of the full structural finite element model subjected to a specific base excitation. Let's consider a nonlinear structural system with m degrees of freedom and n snapshots were taken. We calculate the POD modes and proper orthogonal values of the data matrix $[X]$ and then choose the subspace $[T] \in \mathbb{R}^{m\times s}$ containing the first s POD modes satisfying the 99% energy criterion. The dynamic equation of the system is

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + R(\{X(t)\}) = \{F(t)\} \tag{16}$$

By replacing $\{X(t)\}$ and its derivatives by $[T]\{Q(t)\}$ and multiplying both sides of the dynamic equation by $[T]^T$ we get

$$\underbrace{[T]^T[M][T]}_{[M_r]\in\mathbb{R}^{s\times s}}\{\ddot{Q}(t)\} + \underbrace{[T]^T[C][T]}_{[C_r]\in\mathbb{R}^{s\times s}}\{\dot{Q}(t)\} + \underbrace{[T]^T R([T]\{Q(t)\})}_{R_r([T]\{Q(t)\})\in\mathbb{R}^{s\times 1}} = \underbrace{[T]^T\{F(t)\}}_{\{F_r(t)\}\in\mathbb{R}^{s\times 1}} \tag{17}$$

The previously m degrees of freedom dynamic system is reduced to s degrees of freedom. However, the nonlinear restoring force $R([T]\{Q(t)\})$ cannot be reduced and always needs to be calculated in the full coordinate model which makes this step the most time consuming part of the entire process. In this case, the most effective direct time integration technique to adopt will be the one with the least recurrence for the expensive nonlinear restoring force calculation.

Implicit direct time integration techniques are usually used in conjunction with the Newton-Raphson approach for solving nonlinear systems. In order to reach convergence with this approach, multiple iterations are required at each time step and for every iteration we need to calculate the tangent stiffness matrix, its inverse and the nonlinear restoring force which are all time costly. Using the constant stiffness Newton-Raphson approach will save us the need for the tangent stiffness calculation and its inverse but will increase the number of iterations required for convergence.

On the other hand and for explicit direct time integration techniques, the popular central difference method requires only one iteration per time step and no expensive calculation of the tangent stiffness matrix and its inverse are needed (only the nonlinear restoring force is required). However, the central difference approach is conditionally stable and needs to satisfy the following stability condition

$$\Delta t < \frac{2}{\omega_{max}} \tag{18}$$

where Δt is the time step and ω_{max} is the largest natural pulsation of the system. Generally, the full model of the structure has a relatively large number of degrees of freedom and will result in high natural pulsations (for high modes) hence requiring small time steps to maintain calculation stability and consequently increasing the computational cost. Nevertheless, when working with a reduced structural model, significantly fewer number of degrees of freedom are considered and thus the reduced system will have smaller natural pulsations which makes it possible to use larger time steps while maintaining numerical stability. For this reason, in this work the central difference method is considered to be the most effective direct time integration technique for reduced models.

Various applications of this POD nonlinear dynamic model reduction are possible. As already mentioned for the dynamic seismic analysis, the structure is studied for a range of possible earthquakes and is analyzed and checked for each excitation (earthquake record) separately. Since we need to conduct an analysis for each excitation, we start with the classic full model implicit direct integration nonlinear time history analysis for the first excitation. By collecting snapshots from this initial analysis, we can determine the essential POD modes and use them to reduce the dynamic model in the analysis of the remaining excitations. [15] proposed this approach and applied it on a small scale steel frame in addition to using it for studying seismic base isolators. In the current article, we will use this approach and extend it on a reinforced concrete multistory frame structure while the material nonlinearity is modeled by the multi-fiber section technique.

4 APPLICATION

At first we need to consider the base vibrations to use. The following 4 earthquake recordings obtained from the Center of Engineering for Strong Motion Data CESMD (www.strongmotioncenter.org) were considered (refer to Table 1).

Earthquake	Location	Date	Magnitude	Measurement station	Vibration direction	Total duration	Time step
Northridge	Los Angeles, USA	01/17/1994	6.4 ML	Newhall LA county fire station	0°	60s	20ms
Elcentro	California, USA	05/18/1940	6.9 Mw	Elcentro	0°	53.74s	20ms
L’Aquila	L’Aquila, Italy	04/06/2009	6.3 Mw	L'Aquila V.Aterno Centro Valle	90°	60s	20ms
Chile	Off the coast of central Chile	02/27/2010	8.8 Mw	Constitucion city	90°	120s	20ms

Table 1: Considered earthquakes.

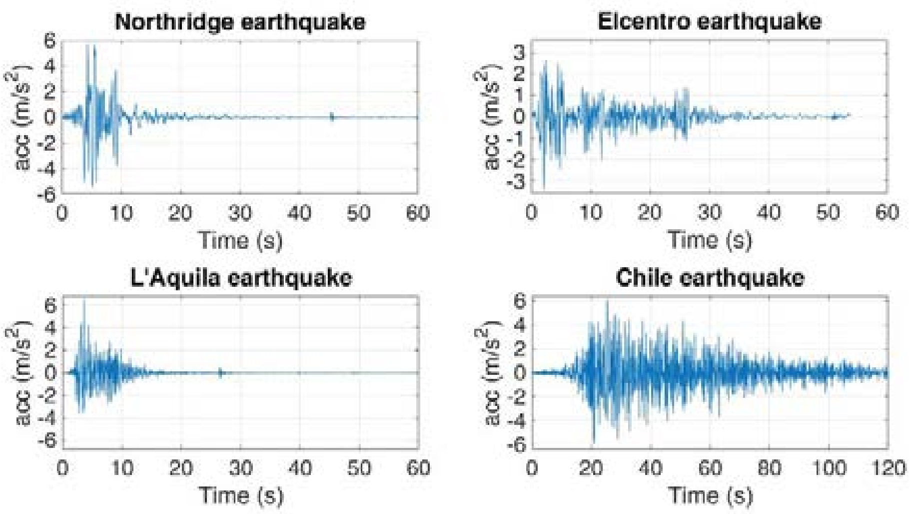

Figure 2. Earthquakes accelerograms

The structure is a 2D Reinforced Concrete (RC) multistory frame made up of 10 stories and 5 spans with a 3 m story height, a 5 m span length, a 4 t/m linear load is considered on beams and the structural self-weight is neglected (refer to Figure 3). All concrete columns and beams are divided into 1 m length finite elements and considered to have the same square cross section 40x40 cm with four 20 mm High Bond HB reinforcing bars at both top and bottom sides (refer to Figure 5). The cross section is divided into 4 concrete fibers and 2 steel fibers. Rayleigh damping was used to get a 5% damping ratio for the first two eigenmodes (more than 90% of the total mass is participating in the first two eigenmodes). The energy criteria for POD modes selection is set to 99.9% (higher than 99% due to the complexity of reinforced concrete elements).

Material nonlinearity is considered to occur in the elements near the beam column connections at the first 5 stories (refer to Figure 4). The steel rebar is considered to have a bilinear backbone curve (initially linear elastic then plastic with strain hardening) with a yielding stress of 400 MPa, a yielding strain of 2‰, an elastic Young modulus of 200 GPa, an ultimate stress of 420 MPa and an ultimate strain of 2.5%. Under cyclic loading, if nonlinearity is reached, the steel material will undergo a kinematic hysteresis behavior (refer to Figure 6). Concrete is considered to be unconfined and modeled according to a simplified version of Mander model [16] that takes into account the damaging phenomena. The maximum concrete compressive strength is 25 MPa at a corresponding strain of 2‰, the ultimate compressive strain is 4‰, the maximum tensile strength is considered 2.5 MPa (10% of the compressive strength) at a corresponding strain of 0.1‰ and the elastic Young modulus is 25 GPa (refer to Figure 7).

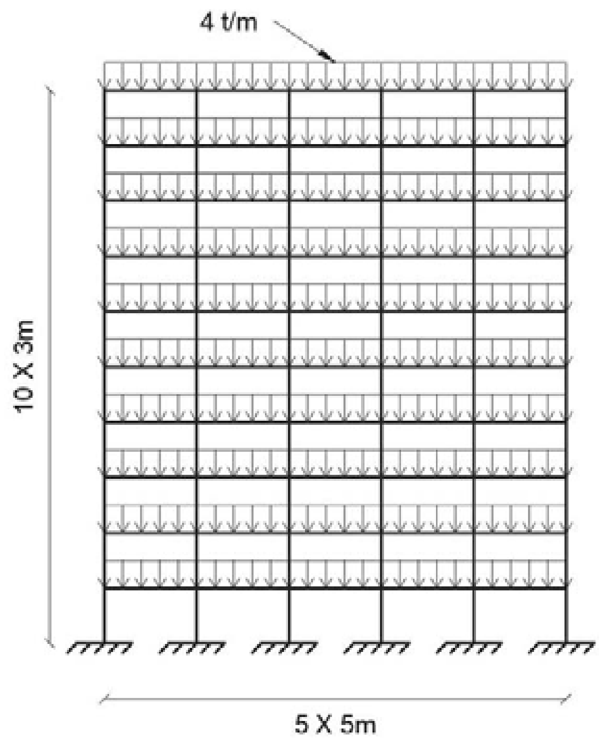

Figure 3. RC frame geometry and loading

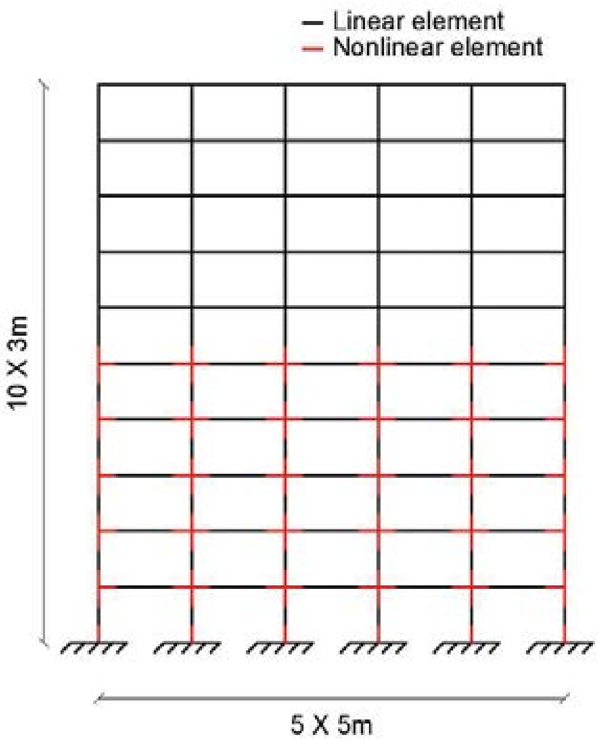

Figure 4. Position of nonlinear elements in RC frame

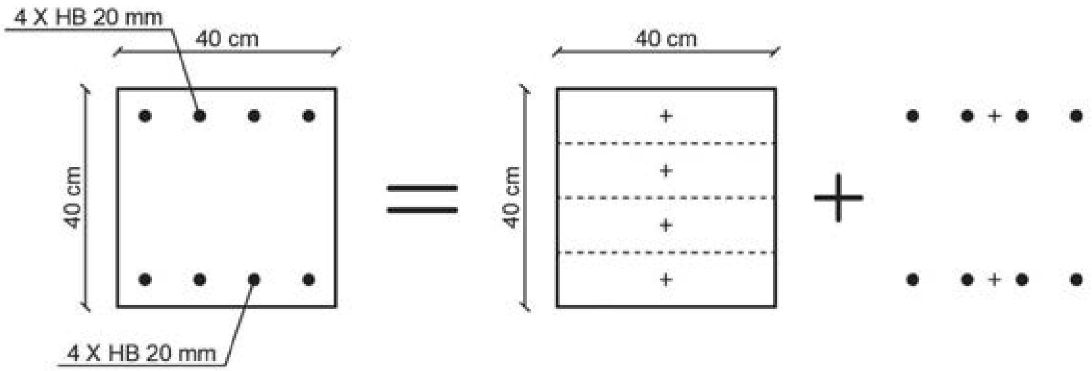

Figure 5. RC element multi-fiber section

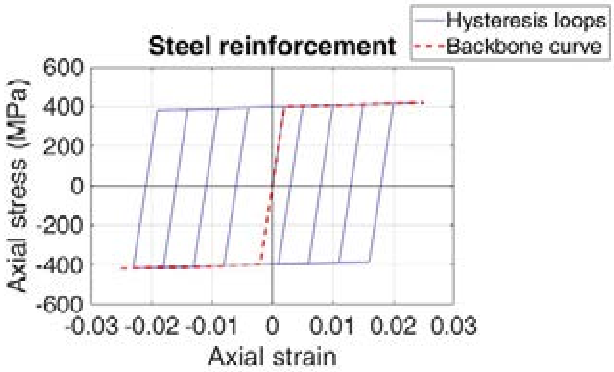

Figure 6. Steel reinforcement axial stress-strain curve

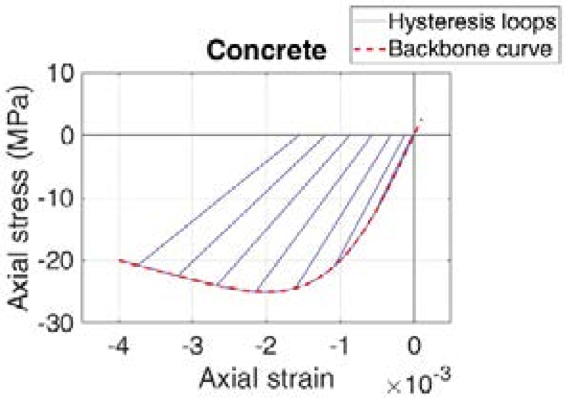

Figure 7. concrete axial stress-strain curve

Northridge earthquake is considered as the initial vibration. Full Model (FM) implicit nonlinear time history analysis with a 20 ms time step was carried out for this earthquake. 50 snapshots were taken for the resulting displacement vector during the first 15 seconds of the vibration (where most of the powerful excitation occurs) and at equally spaced time intervals. POD modes were extracted from the snapshot matrix and the dynamic system was reduced based on Northridge earthquake results. Then, Reduced Model (RM) explicit nonlinear time history analysis with a 20 ms time step was carried out for the remaining earthquakes (Elcentro, L'Aquila and Chile).

It should be noted that for comparison purpose, FM implicit nonlinear time history analysis was conducted separately for Elcentro, L'Aquila and Chile earthquakes in order to have a base reference. Also RM analysis was performed for the Northridge earthquake for comparison with the initially calculated full dynamic model.

For this structure, the reduced base is made of the first 4 POD modes since they represent more than 99.9% of the system's energy. As a result, this structure with initially 1140 degrees of freedom is reduced to only 4.

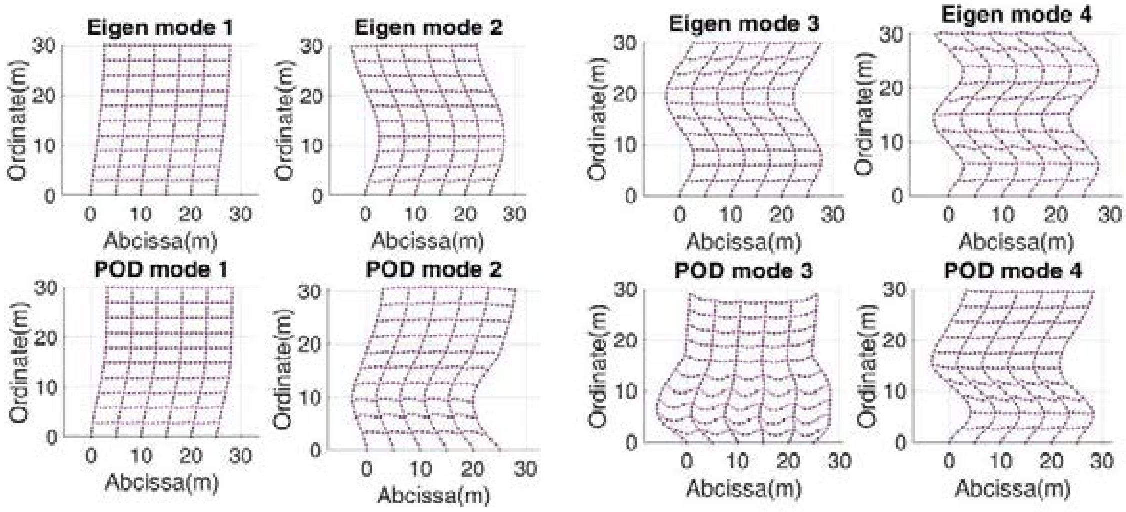

Figure 8. Classic structural eigenmodes Vs POD modes

By comparing the POD modes with the classical eigenmodes of the structure, we can clearly see the nonlinearity effect in the POD modes at the first 5 stories of the structure especially for modes 2 and 3.

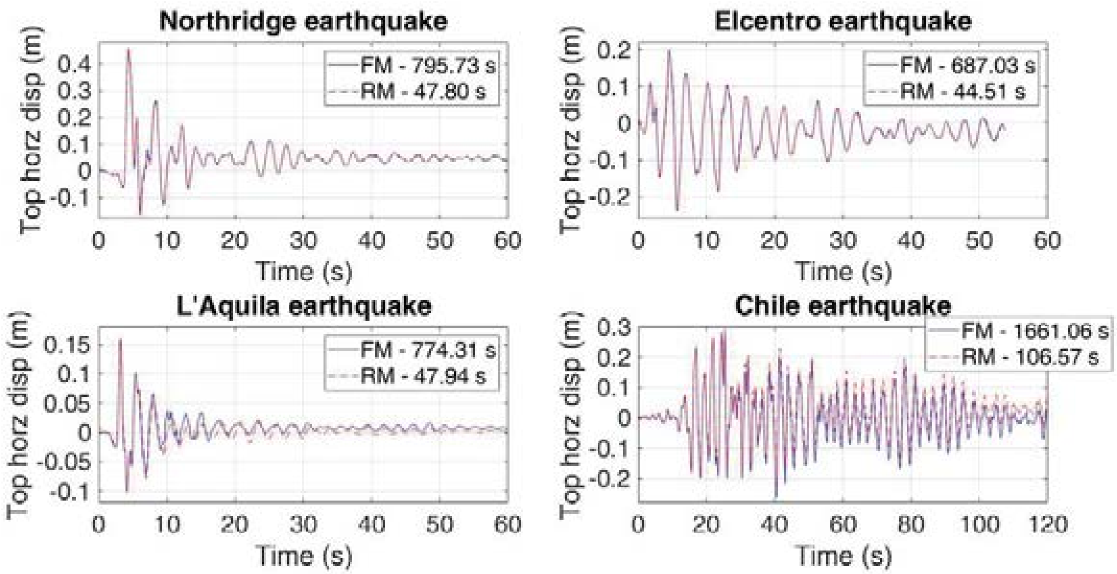

Figure 9. Structural top left corner horizontal displacement in function of time for Full Models (FM) and Reduced Models (RM)

As we can see in Figure 9 the reduced models results are very close to the full models and at a fraction of the time cost, for further details refer to the following table 2.

Earthquake	FM time	RM time	Time saving	Speedup	Average error	Max horiz displacement
Northridge	795.73 s	47.80 s	93.99%	16.6	0.28 cm	45.06 cm
Elcentro	687.03 s	44.51 s	93.52%	15.4	0.36 cm	23.96 cm
L'Aquila	774.31 s	47.94 s	93.81%	16.2	0.79 cm	15.85 cm
Chile	1661.06 s	106.57 s	93.58%	15.6	2.71 cm	27.24 cm

Table 2: Accuracy and time saving of the Reduced Model (RM) with respect to the Full Model (FM).

We can clearly see the time saving benefits of the POD modes in reducing the nonlinear structural system. In addition, the POD modes extracted from the Full Model (FM) analysis of Northridge earthquake are working well in the reduction of the structural model subjected to other excitations (Elcentro, L'Aquila and Chile Earthquake).

5 CONCLUSIONS AND PERSPECTIVES

In this paper we extended the application of the Proper Orthogonal Decomposition to reduce nonlinear dynamic analysis of reinforced concrete multistory frame structure where the material nonlinearity was modeled by the multifiber section approach. We succeeded in reducing a 1140 degrees of freedom system to only 4 degrees while achieving a speedup of around 16 (the reduced model calculation is 16 times faster than the full model) and maintaining an acceptable accuracy level. It was also shown that POD modes obtained from the analysis of a full structural model subjected to a certain base vibration were also convenient for reducing the same model when subjected to different base excitations. The key point here is having a well representative snapshot matrix of the dynamic system.

As perspectives for future work, we are looking forward for increasing the time saving, results accuracy and applying this approach on 2D reinforced concrete structural elements (plates, shells and membranes) by using the layered 2D element model.

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