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INFLUENCE OF THE CONTINUITY OF THE BALLASTED TRACK ON THE DYNAMIC RESPONSE OF A SIMPLY SUPPORTED HIGH-SPEED RAILWAY BRIDGES

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Abstract

In a system of soil-structure interaction, in railway lines particularly, the difference in rigidity at the area of interaction ground-pillar or ground-bridge generates a bending moment at the ends of the structure during the passage of high-speed trains. To analyze the phenomenon and its influence on the response of the work, a more realistic schematization of the studied system is necessary. Although the introduction of rotary springs at the ends of the deck enhances the choice of the appropriate analysis model, most of the contributions reported do not consider this effect. In the present work, an analytical approach to analyze the dynamic response of railway bridges especially those with the ballasted track is investigated. The idea of the proposed model is based on analyzing the continuity effect of the ballasted track (rails and ballast) on the dynamic response of railway bridges, with taking into account an axial force that models the effect of prestressing, ballast interface, axial displacement, force braking. The analytic solution is based on Hamilton's principle, two dynamic case studies of a simply supported and simply supported partially clamped Euler-Bernoulli beam are presented. The results revealed that the compression force presents an additional stiffness which affects the critical velocity and the continuity of the track modeled by rotational springs at the beam's end, so as it increases the dynamic response decreases.

Keywords: Resonance; Force axial; Ballasted track; Partially clamped beam.

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1 INTRODUCTION

Nowadays, high-speed train plays an important role in the public transportation strategy of the states. Thus, vertical acceleration in the platform of short or long railway bridges in high-speed lines is still a matter of concern. As dynamic analyzes and experimental tests have shown that vertical acceleration is responsible for this result, these analyzes were carried out by the European Railway Research Institute (EERRI) [1]. So from this, we conclude that the deck vertical acceleration is among the most important specifications required for the design and evaluation of short railway bridges, which have been identified as $3.5m / s^2$ for ballast tracks [2]. Therefore, the development of precise models working to respect this acceleration factor has become important in applications. So the dynamic performance of railway bridges is playing an increasingly important role in public transportation systems due to the extensive construction of new high-speed lines and the use of old high-speed lines. The regular and repetitive nature of the wheel loads that make up the railway bridges can stimulate resonance situations in these structures. For this latter, it is necessary to continuously modernize the railway infrastructure and especially the bridge structures. Even the axle loads of modern trains are not greater than the forces transmitted in older vehicles, higher design speeds can lead to the occurrence of resonance phenomena. Resonance in a railway bridge occurs when the excitation frequency of the periodic loading of a train approaches a natural frequency of the bridge. In this case, the loads enter the bridge in phase with the natural vibrations of the structure, resulting in a progressive increase in the vibration response of the deck.

In the reported works, Mesuros et al. [3] based on an analytical approach, analyzed and studied the free vibration response of simply and elastically supported beams under moving constant and moving loads, in which the conditions of resonance and cancelation are proved. Resonance vibrations have been observed on railway bridges subjected to high-speed trains. An elementary theoretical model of a bridge has been studied using the integral transformation method, which provides an estimate of the amplitudes of the free vibration. Besides, the analysis gives the critical speeds at which resonant vibration can occur. They are caused by two main reasons: the repeated action of axle loads and the high speed itself [4]. Moreover, it is reported in a work that generally consists of studying the effect of the continuity of the ballasted track on the dynamic response of simply supported railway bridges to assess the influence of the track components and its continuity on the response to the acceleration of the bridge under the passage of railway convoys [5-6].

Additionally, some authors have investigated the effect of axial load on natural frequencies and modal shapes with various types of uniform beam boundary conditions with a concentrated mass at the tip [7]. Some authors investigate the vibration and stability of an infinite Bernoulli-Euler beam on an elastic foundation of the Winkler type when the system is subjected to a static axial force and a moving load with constant or harmonic amplitude variation [8]. The present study takes into account the fact of an axial force that models the effect of prestressing, ballast interface, axial displacement, force braking, and so on. This same idea was discussed by Zhong et al. [9] in which they studied and analyzed the effect of prestressing on the dynamic responses of bridges.

A study that presents a methodology for the comprehensive analysis of railway transition zones (for example, near bridges), which includes: an advanced measurement technique that uses a DIC (Digital Image Correlation) device to measure the dynamic displacements of the rails at multiple locations along the track in transition zones [10]. Also, the authors of this work carried out an experimental study on the problem of the degradation of the railways in the transition zones towards the railway bridges, by numerous measurements on the ground [11].

For this reason, in this study, the interest is to analyze the continuity of the ballasted track in the maximum dynamic response of the bridge (acceleration, displacement) due to rail traffic as a criterion of ballast stability in the case of a ballasted track with taking into account an axial force for the two models, simply supported (SS) beam and simply supported partially clamped (SSPC) beam.

2 BASIC ASSUMPTIONS AND PROBLEM FORMULATION

In this section, as cited previously, to understand the track's continuity effects in the dynamic response of a simply supported bridge, the studied bridge is modeled by an E-B simply supported partially clamped beam, as shown in Fig. 1, which connected in its end's by two rotational springs with a constant stiffness K_r, and an axial Force K_s is presented which simulate the effect of prestressing, ballast interface, axial displacement, force braking.

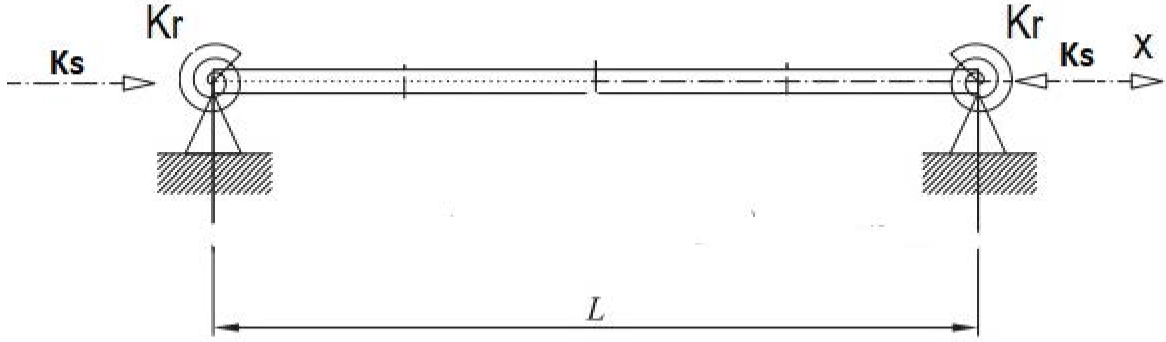

Figure 1: Simply supported partially clamped beam.

By neglecting the damping ratio, the problem of dynamic vibrations of the system can be expressed by Hamilton's principle which is

$$\delta\int_{t_1}^{t_2}\left(U-T-W\right)dt=0 \tag{1}$$

Which U , T and W are the potential energy, the kinetic energy, and the work of the external forces and defined as

$$U=\frac{1}{2}EI\int_L\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx+\frac{1}{2}\int_L k_s\left(\frac{\partial v}{\partial x}\right)^2 dx \tag{2a}$$

$$T=\frac{1}{2}\rho A\int_L\left(\frac{\partial v}{\partial t}\right)^2 dx \tag{2b}$$

$$W=\int_L p(x,t)v dx+W_{r,0}\left(\frac{\partial v}{\partial x}\right)+W_{r,L}\left(\frac{\partial v}{\partial x}\right) \tag{2c}$$

With $v(x,t)$ is the vertical displacement, k_s is the axial force, EI , ρA and $p(x,t)$ are the bending stiffness, the mass per unit length, and the force acting in the structure are assumed to be constant whereas the vehicle-bridge interactions are neglected, then the moving load model HSMA-A is chosen here, which is expressed as follows:

$$p(x,t)=\sum_{k=1}^{N}P_k\left(H\left(t-\frac{d_k}{c}\right)-H\left(t-\frac{d_k+L}{c}\right)\right)\delta(x-ct+d_k) \tag{2d}$$

The loads presented through Dirac delta function acting along $(x_v=ct-d_k)$, where c is the constant train speed, d_k is the distance when $t=0$ from the kth to the entrance of the beam, N is the total number of axle loads, P_k the value of the kth load, H is the Heaviside function and L stands for the beam length.

$W_{r,i}$ $i = 0, L$ are the general work done by the moments at the boundary sections $x = 0, L$, and the moment at these sections are obtained by

$$M_{r,0} = -\frac{\partial W_{r,0}\left(\frac{\partial v}{\partial x}\right)}{\partial\left(\frac{\partial v}{\partial x}\right)}, \quad M_{r,L} = \frac{\partial W_{r,L}\left(\frac{\partial v}{\partial x}\right)}{\partial\left(\frac{\partial v}{\partial x}\right)} \tag{3a}$$

Where $$W_{r,0}\left(\frac{\partial v}{\partial x}\right) = -\frac{1}{2}K_r\left(\frac{\partial v}{\partial x}\right)^2_{x=0}, \quad W_{r,L}\left(\frac{\partial v}{\partial x}\right) = -\frac{1}{2}K_r\left(\frac{\partial v}{\partial x}\right)^2_{x=L} \tag{3b}$$

With K_r is the stiffness of the rotational springs which are identical in this present model, in the same context, the strain energy due to internal axial forces N and the strain energy produced by the sheer force V is neglected according to the Euler-Bernoulli beam theory. Introducing Eqs. (2) and (3) into Eq. (1) and integrating by parts lead to Eq. (4):

$$\int_{t_1}^{t_2}\left\{\int_L\left(EI\frac{\partial^4 v}{\partial x^4} - k_s\frac{\partial^2 v}{\partial x^2} + m\frac{\partial^2 v}{\partial t^2} - p\right)\delta v\, dx - \left[V - k_s h^2\frac{\partial v}{\partial x}\right]_0^L \delta v - \left[M - M_r\right]_0^L \delta\theta\right\}dt = 0 \tag{4}$$

With M, V are the total bending moment and the total shear force defined by

$$M = EI\frac{\partial^2 v}{\partial x^2}\,;\; V = EI\frac{\partial^3 v}{\partial x^3} \tag{5}$$

With $m = \rho A$.

As the variations δv, $\delta\theta$ must vanish at the t_1 and t_2 according to Hamilton's principle, the partial differential governing equation of the system is given by

$$EI\frac{\partial^4 v}{\partial x^4} - k_s\frac{\partial^2 v}{\partial x^2} + m\frac{\partial^2 v}{\partial t^2} = p(x,t) \tag{6a}$$

With the pertaining boundary conditions

$$\left(EI\frac{\partial^3 v(x,t)}{\partial x^3} - k_s\frac{\partial v(x,t)}{\partial x}\right)\delta v_{x=0}^{L} + \left(EI\frac{\partial^2 v(x,t)}{\partial x^2} - M_r\right)\delta\frac{\partial v}{\partial x}_{x=0}^{L} = 0 \tag{6b}$$

As mentioned in the Eqs. (6b), it is evident that the beam affected by the presence of the moments $(M_{r,0}, M_{r,L})$ due to the rotational springs and the axial force, were (M, V) are the total moment and the total shear force in the considered beam.

As mentioned in the introduction, the interest is to analyze the continuity of the ballasted track in the maximum peak bridge dynamic response (acceleration, displacement) due to the rail traffic as a criterion for the ballast stability in the case of ballasted track. However, as the frequency band should range from 0 to $30Hz$ in the dynamical analysis, this is as a reason which allows this limitation of the frequency range, why we use the mode superposition method for solving the equation of motion Eq. (6a), which the general solutions expressed as follows

$$v(x,t)=\sum_{i=1}^{N_m}\Phi_i(x)q_i(t) \tag{7}$$

Where Φ_i is ith the mode shape which depends on the boundary conditions, and q_i is the time functions response of the beam and N_m stands for the number of the considered modes.

2.1 Simply supported partially clamped beam

As known for a simply supported beam, the mode shape is a sinusoidal function and following the same procedure, attaching the simply supported beam with two rotational springs introduce end moments at the supports. Fig.1 shows the model of a simply supported beam leaning on identical rotational spring with identical rotational stiffness which is constant and equal to K_r.

From the eq. (7) which is the vertical deflection of the beam, by using the equations 6(a) and 6(b) the mode shapes and the frequency equation of the uniform simply supported partially clamped beam, the frequency equation is given by

$$\begin{aligned}&\frac{4k^2(r_1L)(r_2L)}{\sqrt{\Delta}L^2}\left(\cosh(r_1L)\cos(r_2L)-1\right)+k\left(r_2L\right)\cos(r_2L)\sinh(r_1L)-k\left(r_1L\right)\cosh(r_1L)\sin(r_2L)\\&-\left(\frac{16k^2k_b+\left(\sqrt{\Delta}L^2\right)^2}{8\sqrt{\Delta}L^2}\right)\sin(r_2L)\sinh(r_1L)=0\end{aligned} \tag{8}$$

With

$$\Delta=\left(\frac{k_s}{EI}\right)^2+\frac{4m\omega_i^2}{EI}\ ;r_1=\sqrt{\frac{\sqrt{\Delta}+\frac{k_s}{EI}}{2}}\ ;r_2=\sqrt{\frac{\sqrt{\Delta}-\frac{k_s}{EI}}{2}}\ ;\ k_b=\frac{k_sL^2}{EI}\ ;k=\frac{K_rL}{4EI} \tag{9}$$

The parameters k and k_b are the ratio of the rotational stiffness of the rotational springs to the flexural rigidity of the beam, and the ratio of the axial force to the flexural rigidity of the beam, respectively.

The stiffness and flexural strength of the abutments should be analyzed in detail for each particular structure. Nevertheless, from the retrofit studies performed by the authors on real cases, it can be concluded that stiffness and flexural strength of abutments will be usually railway structures [1].

As shown in equation (8), if we neglected the influence of the axial force ($k_b = 0$), the equation becomes

$$2k^2 \cosh(rL)\cos(rL) - 2k^2 + krL\sinh(rL)\cos(rL) - krL\cosh(rL)\sin(rL) - \frac{(rL)^2}{4}\sinh(rL)\sin(rL) = 0 \quad (10).$$

Which is the same frequency equation founded in [14] with the root $rL = rL(k)$ and $rL = \pi\sqrt{\frac{\omega_i}{\omega_{SS}}}$.

The analytical model shape expressed as follows

$$\psi_i(x) = B_i\left(b_1\left(\cosh\left(r_1L\frac{x}{L}\right) - \cos\left(r_2L\frac{x}{L}\right)\right) + \sin\left(r_2L\frac{x}{L}\right)\right) + \left(b_1\left(\cosh\left(r_1L\frac{x}{L}\right) - \cos\left(r_2L\frac{x}{L}\right)\right) + \sin\left(r_2L\frac{x}{L}\right)\right) \quad (11a)$$

Where $$B_i = -\frac{b_1\left(\cosh\left(r_1L\right) - \cos\left(r_2L\right)\right) + \sin\left(r_2L\right)}{a_1\left(\cosh\left(r_1L\right) - \cos\left(r_2L\right)\right) + \sinh\left(r_1L\right)} \quad (11b)$$

$$\Phi_i(x) = \frac{\psi_i(x)}{\max(\psi_i(x))} \quad (11c)$$

And $$\eta_1 = \frac{4k}{\left(r_1L\right)_i}\ ;\ \eta_2 = \frac{4k(r_2L)_i}{\left(r_1L\right)_i^2}\ ;\ x_1 = \left(\frac{\left(r_2L\right)_i}{\left(r_1L\right)_i}\right)^2 ;\ a_1 = \frac{\eta_1}{1+x_1}\ ;\ b_1 = \frac{\eta_2}{1+x_1} \quad (11d)$$

Finally, introducing the Eqs. (7) and (12c) into the Eq. (6a) and multiplying by the jth mode $\Phi_j(x)$ with integration over the length of the beam L, the ith modal equation of motion is obtained by using the orthogonality conditions, and by introducing a modal damping ratio ξ_i, the modal damped equation of motion is expressed as:

$$\ddot{q}_i(t) + 2\xi_i\omega_i\dot{q}_i(t) + \omega_i^2 q_i(t) = \frac{1}{M_i}\sum_{k=1}^{N}\left(H\left(t - \frac{d_k}{c}\right) - H\left(t - \frac{d_k + L}{c}\right)\right)P_k\Phi_i(ct - d_k) \quad (12)$$

Where M_i is modal mass associated to the $i-th$ mode

$$M_i = m\int_0^L \Phi_i^2(x)dx = \rho A\int_0^L \Phi_i^2(x)dx \quad (13)$$

ω_i Represents the ith circular frequency in rad/s.

2.2 Free vibration of simply supported partially clamped beam

As can be seen, the mode shape (Eq.11) composed by symmetric and axisymmetric modes, we used this expression, if we neglected the damping effect, the solution of the equation of motion of simply supported partially clamped beam for each value of $r_j L = r_j L(k, k_b) \quad j = 1,2$ subjected to moving load can be derived as follows

$$\frac{q_i(t)}{q_{st}} = B_i\left[\frac{\eta_1}{1+x_1}\frac{\left(\cosh\left(O_{i1}t\right)-\cos\left(\omega_i t\right)\right)}{1+K_{i1}^2}+\frac{\left(\sinh\left(O_{i1}t\right)-K_{i1}\sin\left(\omega_i t\right)\right)}{1+K_{i1}^2}-\frac{\eta_1}{1+x_1}\frac{\left(\cos\left(O_{i2}t\right)-\cos\left(\omega_i t\right)\right)}{1-K_{i2}^2}\right]$$
$$+\left[\frac{\eta_2}{1+x_1}\frac{\left(\cosh\left(O_{i1}t\right)-\cos\left(\omega_i t\right)\right)}{1+K_{i1}^2}-\frac{\eta_2}{1+x_1}\frac{\left(\cos\left(O_{i2}t\right)-\cos\left(\omega_i t\right)\right)}{1-K_{i2}^2}+\frac{\left(\sin\left(O_{i2}t\right)-K_{i2}\sin\left(\omega_i t\right)\right)}{1-K_{i2}^2}\right] \quad 0 \le t \le L/c \tag{14}$$

Where q_{st} is the static solution which is

$$q_{st} = \frac{P}{\omega_i^2 m \int_0^L \Phi_i^2(x)dx} \tag{15}$$

And $O_{i1} = \frac{(r_1 L)_i c}{L}$; $O_{i2} = \frac{(r_2 L)_i c}{L}$; $K_{i1} = \frac{O_{i1}}{\omega_i}$; $K_{i2} = \frac{O_{i2}}{\omega_i}$ (16)

The speed parameters K_{i1} and K_{i2} are defined by the ratio of the load frequencies, O_{i1} and O_{i2} the simply supported partially clamped beam circular frequency ω_i, c is the speed of the moving load, B_i, η_1, η_2 and x_1 are defined in the Eq. (11d), $(r_1 L)_i$ and $(r_2 L)_i$ are the roots of the frequency equation correspond to the ith mode, which is equal in the case of axial force is neglected ($k_b = 0$).

To understand the effect of the axial force in the case of free vibration of a simply supported partially clamped composite beam, as derived in [3] by using the analytical mode shapes of the considered beam, the amplitude of the ith modal response of the beam after the passage of a unit constant load P at a constant speed c in the so-called free vibration phase normalized by the static solution is computed by [3]

$$R_i = \frac{1}{q_{st}}\sqrt{\frac{\dot{q}_i\left(t = L/c\right)^2}{\omega_i^2} + q_i\left(t = L/c\right)^2} \tag{17}$$

Where $q_i(t = L/c)$ and $\dot{q}_i(t = L/c)$ are the initial conditions for the free vibration phase correspond respectively to the ith modal deflection of the beam and its derivative $t = L/c$ during the forced vibration phase, $0 \le t \le L/c$. Analytical expressions of these quantities are derived in Eq. (14). Then as can be observed, this equation at $t = L/c$ is only dependents on the parameters k which is the ratio of the stiffness of the rotational springs to the flexural stiffness of the beam, the beam length L and its flexural stiffness EI, the two no dimensional speed K_{i1} and K_{i2} which are defined in the Eq.(16) and the k_b which is the ratio between of the axial force and the beam bending stiffness.

3 NUMERICAL RESULTS

In this section, two cases study is presented, dynamic response of simply supported beam and dynamic response of partially clamped beam, to investigate the effect of the continuity of the ballast track and the axial force in the dynamic response of the aforementioned structure.

3.1 Simply supported beam

the dynamic response of a simply supported and simply supported partially clamped beam is analyzed, the Vinival Bridge [13] is a simply supported, single-track bridge, the main mechanical properties are cited in the Table. 1, in which these properties schematize the total properties of the bridge (include the mass of the track superstructure), the height of the ballast in this bridge is computed by $h_b = 0.52m$, then the dynamic response of the bridge under the circulation of High-Speed Load Model-A (HSLM-A) is presented and the maximum dynamic response takes place at mid-span section [14], as noted in the Eurocode and accounting for modes with natural frequencies below $30Hz$, Fig .2 shows the maximum vertical acceleration in terms of the circulating velocity in the range [140,310]km/h every 1km/h. The maximum vertical acceleration reaches $6.16\, m/s^2$ and occurs when the train HSLM-A2 crosses the bridge at $220\, km/h$ which corresponding to the fourth resonance of the first bending mode. For these reasons, in the following section, the dynamic response of the bridge will be accounted for by the passage of the train $A2$.

From Table.1, the main properties are calculated considering the mass and the inertia of the track superstructure which is composed of rails, sleepers, and ballast.

Length $L(m)$	9.70
Mass per unit length, kg/m	9754
Natural frequency, $f_{0b}(Hz)$	12.8
Modal damping ratio, $\xi(\%)$	2

Table1: mechanical properties of the considered bridge [13]

Besides, the axial force has the following value: $k_s = 5.37MN$ (18)

The dynamic response of the bridge under the load model HSLM-A2 is evaluated in the time domain with taking into account the first mode of vibration (allow the Eurocode criterion), the figs. 2 (a) and 2(b) show the vertical acceleration and displacement of the bridge at mid-span section, The blue line in the two figures shows the variation of the acceleration and displacement without taking into account the effect of axial force, and the red line shows the variation of the acceleration and displacement with taking into account the axial force with the mentioned value (Eq. 18). From this figure, it can be concluded that critical speed increases as the axial force are introduced, which explain that the axial force such as bracing force, axial displacement introduces a remarkable stiffness (additional stiffness) to the structure which increases the critical velocity, and the displacement amplitude of vibration decreases, especially in the vicinity of resonance at the critical speed $220\,km/h$ which corresponding to a fourth resonance of the first bending mode.

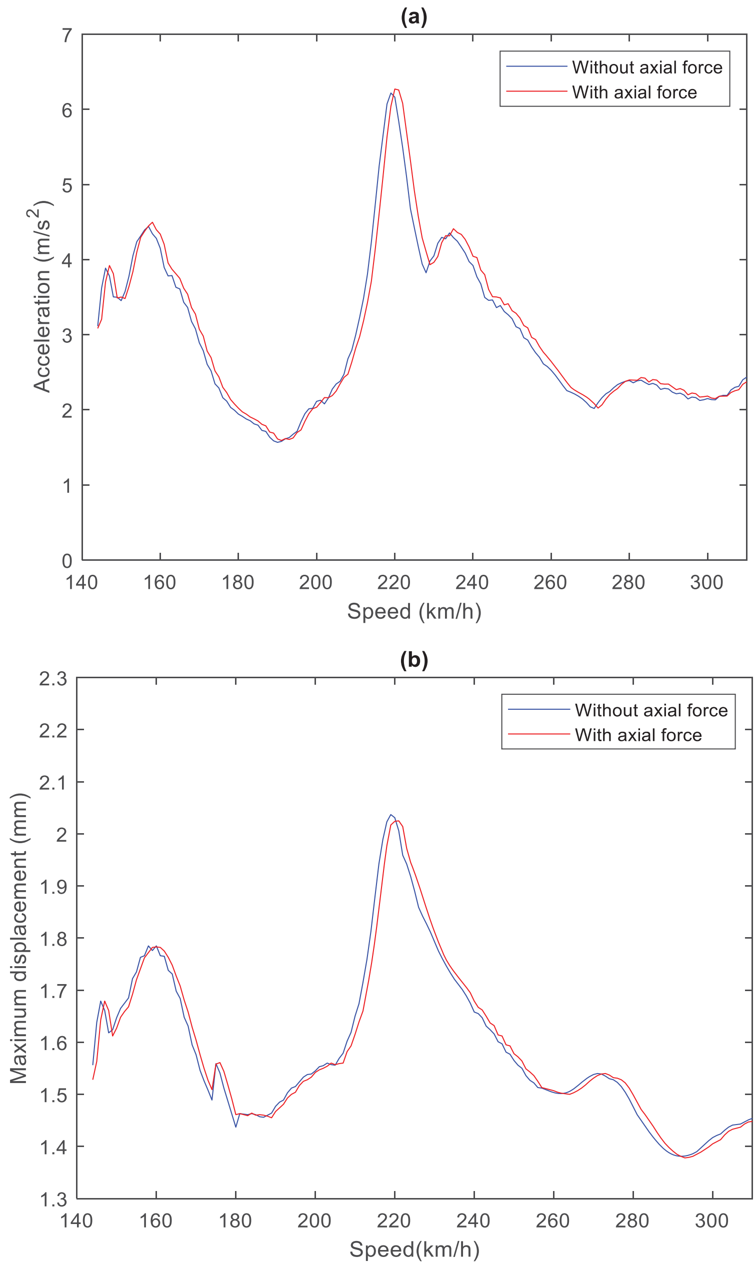

Figure 2: Dynamic response of the beam at mid-span section as a function of the speed of HSLM-A2
(a) Acceleration, (b) displacement.

3.2 Simply supported partially clamped beam

In the following part, as cited in the literature remarkable scientific contributions focused to study the effect of ballast in the response of simply supported bridge, but as the continuity of track introduces a moment of resistance at the beam ends an influence can occur, it is important to study this effect by modeling the bridge as a partially clamped beam schematized by two identical linearly rotational springs attached to the beam ends, to ensure this continuity of the track.

As demonstrated in Eq. (17) the normalized amplitude of the free vibrations of the simply supported partially clamped beam depends on two parameters (k, k_b), which represent the effect of the rotational springs and the effect of the axial force. Figure 3 represents the first modal response amplitude of the simply supported partially clamped beam as a function of the speed parameters K_{11} (Eq. 16) $k = 1.1$, which represents the ratio between the stiffness of the rotational springs and the beam bending stiffness and which means that the fundamental frequency of the partially clamped beam increases by 50%, with the value of k_b in the range $[0, 5k_b]$. The value k_b is calculated by using the value of axial force (Eq.18) which corresponds to the value $k_b = 0.1255$.

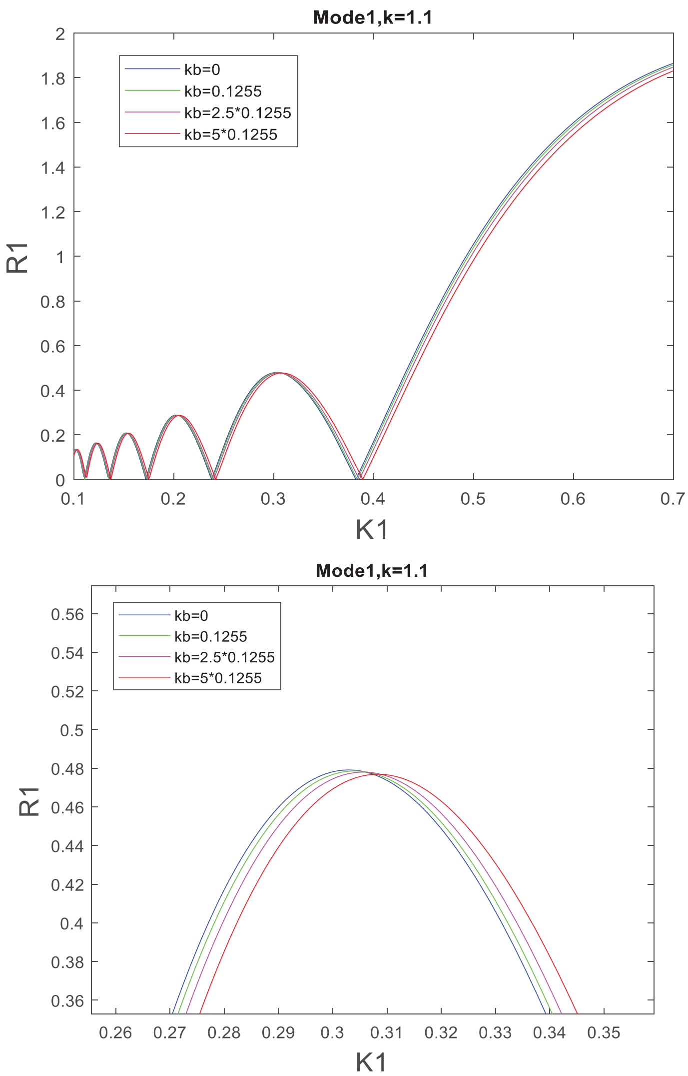

Figure 3: First modal response amplitude vs the speed parameters.

Then as shown in Fig.3, in the case of free vibration of simply supported partially clamped beam, as the ratio k_b increases, the speed parameters K_{11} increases which correspond to an increase in the critical speed.

In the case of forced vibration, due to higher modes have frequencies above $30Hz$, the response of the beam is computed from the contribution of its first bending mode, the Figs. 4(a) and 4(b) represent the displacement and acceleration of the simply supported partially clamped beam at the mid-span section of the bridge under the circulation of HSLM-A2 for circulating velocities in the range $[150,350]km/h$ in steps $1km/h$, this wide range of velocities permits the occurrence of a second resonance of the fundamental bridge frequency. Then, from this figure, in comparison with the section (3.1) for a simply supported beam, the critical speed corresponds to $v_c = 220\,km/h$ which theoretical is $v_{i=1,n=4}^{\mathrm{Res}} = \frac{f_{SS}D}{4} = \frac{12.8.19}{4}.3.6$ [15-16], where f_{SS} is the fundamental frequency of the SS beam, i the model number, n the resonance number and $D = 19m$ is the characteristic distance of the train HSLM-A2, corresponds to the fourth resonance of the fundamental mode. In the same context, by fixing the value $k = 1.1$, the fundamental frequency of the beam is changed, due to the rotational spring's stiffness and corresponds to $\omega_1 = 1.5\omega_{SS}$ which leading to $f_1 = 19.2Hz$ the value $k_b = 0$. Also, as it can be seen, for $k_b = 0$ the maximum acceleration reaches the value $5.21\,m/s^2$ which corresponding to a critical velocity $v_c = 1.5v_{i=1,n=4}^{\mathrm{Res}} = 330\,km/h$, for $k_b = 5\times 0.1255$ the acceleration is attained $5.24\,m/s^2$ which corresponding to the critical velocity $v = 333\,km/h$, the thing explains that for a fixed value $k = 1.1$, the peak of resonance increases as the ratio k_b increases. Also, for the displacement, we can conclude that as the ratio k_b increases the displacement amplitude decreases, and critical velocity increases. Then, this is the same conclusion found in section (3.1) which indicates that the axial force introduces an additional stiffness and independent of the boundary conditions of the beam.

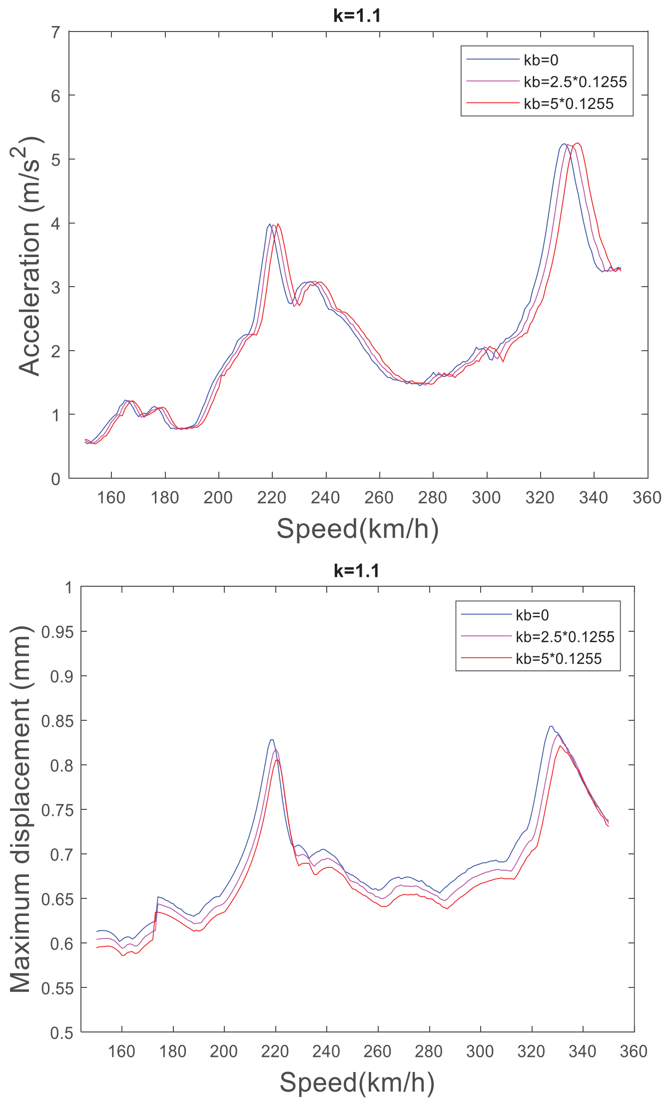

Figure 4: Dynamic response of the beam at mid-span section as a function of the HSLM-A2
(a) Acceleration, (b) displacement; effect of k_b .

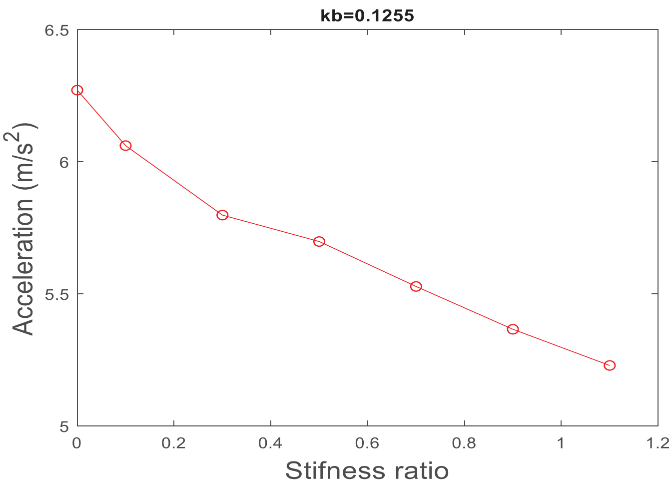

Figure 5: Acceleration as a function of the stiffness ratio k .

Another parameter that should be also analyzed is the effect of the rotational stiffness due to the rotational springs located at the beam ends, Fig. 5 represents the variation of the acceleration at the mid-point section of the simply supported partially clamped beam under the passage of HSLM-A2 at the critical velocity, as a function of the ratio k for a fixed value of the ratio $k_b = 0.1255$, from this figure we conclude that as the stiffness ratio k increases, the vertical acceleration decreases, which means that the continuity of track as its rigidity increases the vertical acceleration decreases independent of the influence of the axial forces.

4 CONCLUSION

To summarize, the dynamic response of a simply supported uniform beam and a simply supported partially clamped beam have been studied, with take into account the axial forces and the continuity of the track effects, the results indicate that the axial forces introduce an additional stiffness which modifies the critical velocity and the continuity of the ballast affects largely only the vertical acceleration of the bridge.

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