Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2021) 1476-1491

ECCOMAS
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COMPDYN 2021
8th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Streamed from Athens, Greece, 28 - 30 June 2021

A TECHNIQUE FOR TIME INTEGRATION WITH STEPS LARGER THAN THE EXCITATION STEPS: REVIEW OF THE PAST ADDRESSING THE EXISTING CHALLENGES AND A PERSPECTIVE OF THE FUTURE

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Abstract

Time history analysis using a time integration method is a powerful broadly accepted tool for structural dynamic analysis. In practical areas, such as earthquake engineering, computations based on time history analysis might be computationally very expensive. In 2008, a technique was proposed to considerably reduce the computational effort by reducing the data in the earthquake record. This technique, which is recently named as a SEB THAAT, is reviewed in this paper. After a brief review on the formulation and implementation, the positive points, the limitations, and the challenges facing versatile implementation of the SEB THAAT are addressed, and a future perspective is presented.

Keywords: Time History Analysis, Earthquake Engineering, Computational Effort, SEB THAAT, Existing Challenges, Future Perspective.

ISSN:2623-3347
Peer-review under responsibility of the organizing committee of COMPDYN 2021.
doi:10.7712/120121.8574.19609

1 INTRODUCTION

The true behavior of structural systems is nonlinear and dynamic. Time history analysis using a time integration method is a versatile tool to study structures' nonlinear dynamic behavior. The analysis process is briefly reviewed in Fig. 1. Evidently, the analysis leads to approximate responses and is computationally expensive. Specifically, the run-time may be considerable in many real analyses. The significance of these features highlights in earthquake engineering, where seismic codes require structures' time history analysis considering several ground acceleration records. Also there exist advanced seismic computations such as IDA

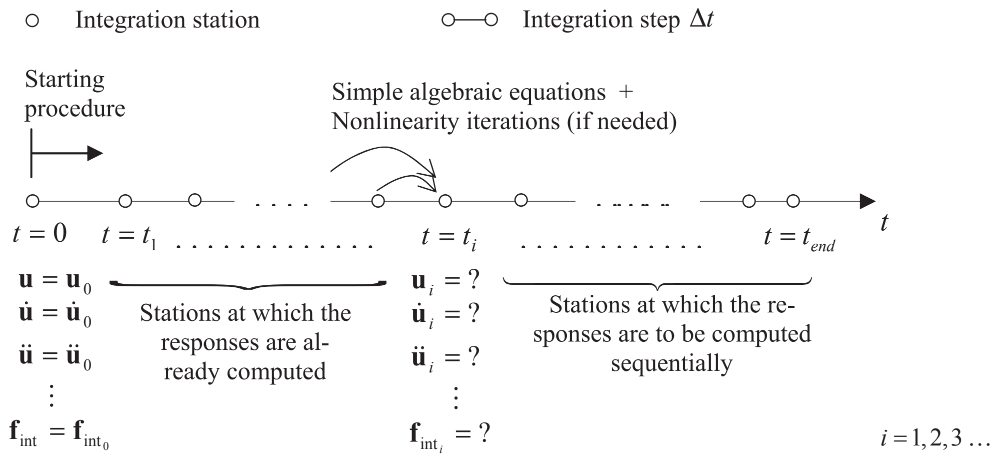


Figure 1: Brief description of the process of time history analysis using a time integration method [1].

(Incremental Dynamic Analysis) [2], for which many time history analyses are essential and the computational effort can be very high. Some main approaches to lessen the computational effort are: (1) reducing the structural systems by replacing the structures finite element models with models with less degrees of freedom [3, 4], (2) reducing the number of essential earthquake records, e.g. see [5-7], (3) reducing the number of oscillatory modes [8, 9], and (4) using higher order time integration methods [10-12]. Meanwhile, in the last two decades, approaches are developed to reduce the computational effort by reducing the earthquake records' data [13-15].

The semi-discretized equation of motion can be expressed as [1, 11, 16-19]:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t)+\mathbf{f}_{\text{int}}(t)=\mathbf{f}(t) \qquad 0\le t\le t_{end}$$

$$\text{Initial Conditions :}\quad \left|\begin{array}{l}\mathbf{u}(t=0)=\mathbf{u}_0\\ \dot{\mathbf{u}}(t=0)=\dot{\mathbf{u}}_0\\ \mathbf{f}_{\text{int}}(t=0)=\mathbf{f}_{\text{int}_0}\end{array}\right. \tag{1}$$

$$\text{Additional Constraints :}\quad \mathbf{Q}$$

where, t and t_{end} imply the time and the duration of the dynamic behavior; $\mathbf{M}$ is the mass matrix; $\mathbf{f}_{\text{int}}$ and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation; $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$, denote the vectors of displacement, velocity, and acceleration; $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}_0}$, define the initial status of the model ($\mathbf{f}_{\text{int}_0}$ is essential in presence of material nonlinearity; see [20]); and finally, $\mathbf{Q}$ represents restricting conditions, e.g., additional constraints in problems involved in impact or elastic-plastic behavior [21, 22].

For time history analysis of Eq. (1), using a time integration method, the broadly recommended integration-step [1, 23-26] is as follows:

$$\Delta t = \text{Min}\left(\frac{T}{\chi}, \Delta t_{cr}, {}_f\Delta t\right) \tag{2}$$

where, T is the smallest oscillatory period with worthwhile contribution in the response [26], Δt_{cr} stands for the largest integration step leading to numerically stable responses [10, 11, 23], ${}_f\Delta t$ is the step by which the excitation is digitized [13, 25], and as addressed in [1, 23-26],

$$\chi = \begin{cases} 10 & \text{when the behavior is linear} \\ 100 & \text{when the behavior is nonlinear and there is no impact} \\ 1000 & \text{when the behavior is nonlinear and there is impact} \end{cases} \tag{3}$$

In view of Eqs. (1)-(3), there are techniques [14] that replace the earthquake digitized record and specifically replace t_{end} with $(t_{end})_{new}$, such that: (1) after an ordinary time integration analysis, the main features of the response remain almost unchanged, and (2)

$$(t_{end})_{new} < t_{end} \tag{4}$$

There are also techniques [13, 15] that replace the excitation record, such that: (1) the main features of the response, e.g. peak response and frequency content, are almost preserved, and (2) the integration-step Δt, can be replaced with $(\Delta t)_{new}$, satisfying

$$(\Delta t)_{new} > \Delta t \tag{5}$$

The purpose of both of these groups of techniques is to reduce the number of integration steps, and accordingly, lessen the computational run-time and effort; also see [1].

With attention to the following features of a technique in the second group, i.e. the SEB THAAT (Step-Enlargement-Based Time History Analysis Acceleration Technique) [13, 27]:

1. Significant reduction in computational effort; see Table 1,
2. Simple implementation; see [1, 13, 28]
3. Good accuracy for the time history of the response [28]
4. Versatility (see Table 1) [1, 28, 31-34],
5. Having a mathematical basis [13].

the objective of this paper is to review the SEB THAAT, address the existing challenges, and present a perspective of the future.

2 THE SEB THAAT AND ITS PAST

The SEB THAAT is developed, based on two mathematical statements, a broadly accepted convention, and a realistic assumption, such that to preserve convergence and its order, for responses obtained from time integration. The two statements are:

1. Consider analysis of Eq. (1) by a time integration method of order q. Consider an approximation of the $\mathbf{f}(t)$ in Eq. (1), i.e. $\mathbf{f}_{new}(t)$, which converges to $\mathbf{f}(t)$ with respect to the integration step, with an order q' satisfying $q' \geq q$. Analysis of Eq. (1) by that integration method, after replacing $\mathbf{f}(t)$ with $\mathbf{f}_{new}(t)$, would lead to responses converging with order q (see [13, 35]).

System	Effort reduced in the price of negligible change in accuracy (%)	Details
Shear frames	60-80	A few linear shear frames subjected to different excitations
Residential buildings	50-90	More than 200 buildings structures with linear and nonlinear behavior and regularity and irregularity in plan or height subjected to different excitations
A thirty-storey building	50	A thirty-storey steel three-dimensional frame subjected to two different excitations
Bridges	30-80	About 20 real bridges with linear and nonlinear behaviors, some with pre-stressed elements, subjected to different excitations
Power station, Cooling tower, Space structure, Silo	>50	One or two of each special structure, considering linear and nonlinear behavior and different near-field and far-field excitations and different integration schemes
Earth dams	<80	Several earth dams each subjected to many earthquake records
Milad telecommunication tower	50-70	Considering linear and nonlinear behavior, near-field and far-field excitations, and different integration schemes
Structural systems damped non-classically	30-90	Considering linear and nonlinear behavior, and different integration schemes

Table 1: A summary of the tests carried out on the SEB THAAT [1, 27-30].

2. In view of the Taylor series expansion [36], for an arbitrary continuous function of x, i.e. $H(x)$,

$$H(x+\Delta x)+H(x-\Delta x)=2H(x)+\mathrm{O}(\Delta x^2) \tag{6}$$

The convention is the second order of accuracy of majority of time integration methods [1, 10, 11]. An extension of the SEB THAAT disregarding this convention is addressed in [13], the numerical tests are however few; see [37]. And finally, the realistic assumption is:

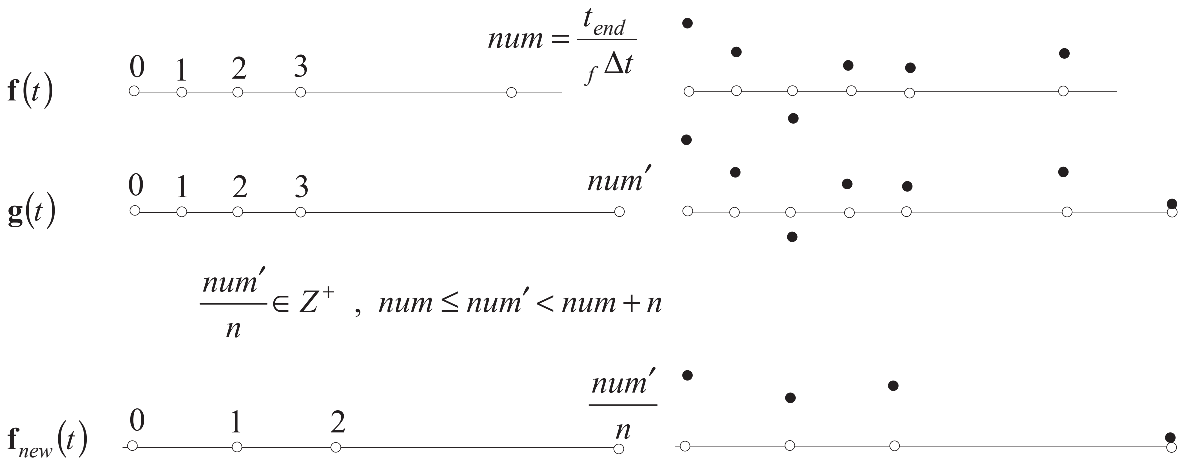
The $\mathbf{f}(t)$ in the right hand side of Eq. (1), though is available in digitized format, can be considered originally continuous with respect to time.

Provided these considerations, implementation of the SEB THAAT means ordinary time integration analysis after replacing the $\mathbf{f}(t)$ in Eq. (1), digitized in step ${}_f\Delta t$, with a new excitation $\mathbf{f}_{new}(t)$, digitized in step $({}_f\Delta t)_{new}$,

$$({}_f\Delta t)_{new} = n\,{}_f\Delta t \tag{7.1}$$

$$n = 2,3,4,\ldots \tag{7.2}$$

where, n stands for the enlargement scale of the digitization-step (see also Fig. 2), and



(a) (b)

Figure 2: Typical changes in the $\mathbf{f}(t)$ in Eq. (1) because of implementation of the SEB THAAT: (a) Digitization stations, (b) Digitized data.

$$\mathbf{f}_{new}(t) = 0, \quad \text{unless when} \quad t = t_0, t_1, t_2, \ldots (\text{the digitization stations}) \tag{8.1}$$

$$\mathbf{f}_{new}(t = t_i) = \begin{cases} \mathbf{g}(t_i) & \text{when} \quad t_i = t_0 = 0 \\ a\mathbf{g}(t_i) + (1-a)\sum_{k=1}^{n'} b_k \left[\mathbf{g}(t_i + k\,{}_f\Delta t) + \mathbf{g}(t_i - k\,{}_f\Delta t)\right] & \text{when} \quad 0 < t_i < t'_{end} \\ \mathbf{g}(t_i) & \text{when} \quad t_i = t'_{end} \end{cases} \tag{8.2}$$

$$i = 0, 1, 2, \ldots$$

$$t_i = i(n\,{}_f\Delta t) = 0, n\,{}_f\Delta t, 2n\,{}_f\Delta t, 3n\,{}_f\Delta t, \ldots t'_{end} \tag{9}$$

$$\mathbf{g}(t_i) = \begin{cases} \mathbf{f}(t_i) & \text{when} \quad 0 \le t_i \le t_{end} \\ \mathbf{O} & \text{when} \quad t_{end} < t_i \le t'_{end} \end{cases} \tag{10}$$

$$a = \frac{1}{2} \tag{11.1}$$

$$b_k = \frac{1}{2n'} \tag{11.2}$$

$$n' = \begin{cases} n-1 & \text{when} \quad t_i = t_1 = n_f \Delta t \ \text{ or } \ t_i = t'_{end} - n_f \Delta t \\ \begin{cases} \dfrac{n} {2} & (\text{even values of } n) \\ \dfrac{n-1}{2} & (\text{odd values of } n) \end{cases} & \text{when} \quad n_f \Delta t < t_i < t'_{end} - n_f \Delta t \end{cases} \tag{12}$$

$$t'_{end} = \begin{cases} t_{end} & \text{when} \quad \dfrac{t_{end}}{n_f \Delta t} \in Z^+ \ \ (Z^+ \text{ implies the set of positive integers}) \\ l\left(n_f \Delta t\right) & \text{when} \quad \dfrac{t_{end}}{n_f \Delta t} \notin Z^+, \ \exists l \in Z^+, \ t_{end} < l\left(n_f \Delta t\right) < t_{end} + n_f \Delta t \end{cases} \tag{13}$$

and extension of Eqs. (8-13) to fractional/real values of n (see Eq. (7.2)), is discussed in [1, 38, 39]. Evidently, in view of Eq. (2), the SEB THAAT is effective and may reduce the run-time, only when

$$_f\Delta t < \text{Min}\left(\frac{T}{\chi}, \Delta t_{cr}\right) \tag{14}$$

and more, in view of Eqs. (8-13), the effort needed for computing $\mathbf{f}_{new}(t)$ is negligible compared to that of the time history analysis. Accordingly, the enlargement scale n implies reduction in run-time for linear analyses, where no additional computation is needed to model the nonlinearities. Furthermore, in view of Fig. 1 and notion of computational effort [40, 41], the reductions in run-time and computational effort because of the SEB THAAT are identical, for linear analyses.

There is no guarantee about sufficiency of the accuracy of the results of the SEB THAAT when using arbitrary value of n. The reason is that convergence (and its rate) is the only accuracy-related basis of the SEB THAAT. Furthermore, when the SEB THAAT is successful for some value of n, upper-bounds will exist on n. Considering this and some different theoretical questions, research on the SEB THAAT has been followed in two directions. In one direction, to study the performance of the SEB THAAT, many cases are studied, considering changes of structural system, time integration method, digitized excitation (e.g. far- and near-field earthquakes), etc.. For each case, responses are investigated for the following three questions:

(1) Does $_f\Delta t$ governs Eq. (2), for the case under study (see Eq. (14))?

(2) For cases with $_f\Delta t$ as the governing term of Eq. (2), can, for some value of n, implementation of the SEB THAAT cause no notable loss of accuracy, i.e. is the following statement correct?

$$\exists\, n > 1: \quad \mathbf{R}_{new} \cong \mathbf{R} \tag{15}$$

where, $\mathbf{R}_{new}$ and $\mathbf{R}$ are the responses obtained from time integration with and without implementation of the SEB THAAT, respectively.

(3) For cases, where $_f\Delta t$ governs Eq. (2), is the largest value of n satisfying Eq. (15) ($n_{\max}$) consistent with Eq. (2), i.e. is the following statement correct?

$$\forall\, n \quad \mathbf{R}_{new} \cong \mathbf{R}: \quad \exists\, n_{\max} \cong \frac{1}{_f\Delta t}\text{Min}\left(\frac{T}{\chi}, \Delta t_{cr}\right): \quad 1 < n \le n_{\max} \tag{16}$$

The consequence was positive responses in almost all cases [1, 27-30, 37-39, 42-56]. Specifically, a very interesting numerical observation, for two cases reported in [42, 43], was the more accuracy after implementation of the SEB THAAT (compared to that of the ordinary analysis). This leads to the fact that the inaccuracy because of the SEB THAAT can be added or subtracted from the inaccuracy originated in the approximate time integration, and accordingly,

$$\exists \text{ Cases}: \quad \left\| \mathbf{R}_{new} - \mathbf{R}_{exact} \right\| < \left\| \mathbf{R} - \mathbf{R}_{exact} \right\| \tag{17}$$

where, $\mathbf{R}_{exact}$ stands for the exact response and $\|.\|$ represents arbitrary norm [57]. Finally, it is worth noting that the reduction in run-time because of the SEB THAAT is generally considerable, as addressed in Table 1 and Fig. 3. And, as already stated, for linear analyses, the reduction in run-time is equal to the reduction in computational effort.

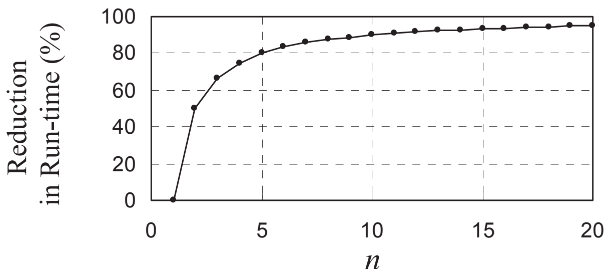


Figure 3: Reduction in the run-time with respect to the enlargement scale *n*, for linear analyses accelerated by the SEB THAAT.

In the other direction, some theoretical studies were carried out. First, enlarging the digitization step, without changing the record at the non-eliminated time stations (see Fig. 4), which is implemented as a simple approach for faster analysis since decades e.g. [58], was studied. It was demonstrated that though in some cases the simple approach leads to good accuracy and reduced computational effort compared to the ordinary analysis, the efficiency is in all cases better, when using the SEB THAAT [59]. Specially, cases were observed, where the result of the simple approach was totally erroneous, while the corresponding result of the SEB THAAT was satisfactory [59].

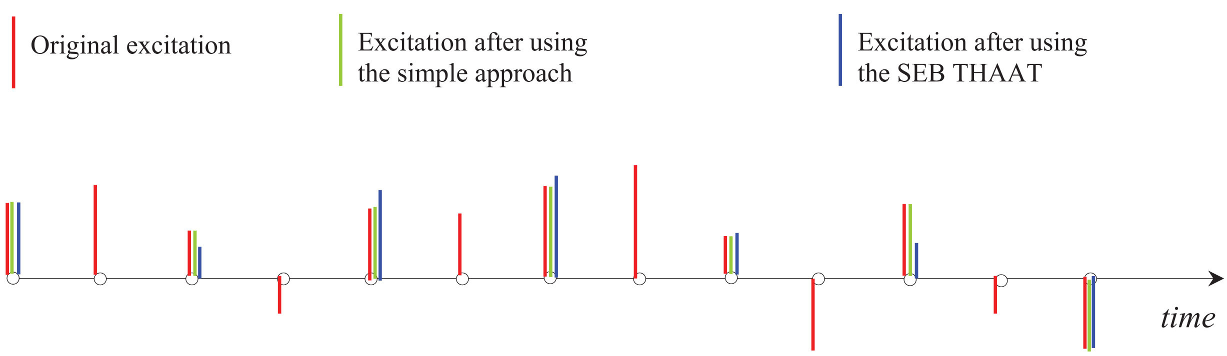


Figure 4: Typical comparison between excitation records in implementation of the SEB THAAT and the traditional simple step enlargement approach considering $n = 2$.

It is then studied whether the inaccuracy because of the SEB THAAT can lead to instability, or in other words, whether the SEB THAAT may negatively affect the responses stability. The response was negative [1, 60], i.e. when Eq. (2) is satisfied the responses are numerically stable regardless of implementation of the SEB THAAT. The main reasons are the convergence basis of the SEB THAAT and the relation between convergence and numerical stability in time integration analyses [61-63]. In the next step, selecting values for *a* and *b* in Eq. (8.2), different from those stated in Eqs. (11), and subjected to (see [13]):

$$\sum_{k=1}^{n'} b_k = \frac{1}{2} \tag{18}$$

was studied [64]. As the result, though in some examples Eqs. (11) did not present the best alternative, the overall best selection can e considered as stated in Eqs. (11). Another question was on how to consider non-integer values as the enlargement scale *n*. This is essential especially with attention to Fig. 3 and when the $n_{\max}$ available from Eq. (16) satisfies:

$$1 < n_{\max} < 2 \tag{19}$$

Two simple approaches to extend the SEB THAAT to:

$$n, n_{\max} = \frac{p}{q} \quad , \quad p, q \in Z^+ \tag{20}$$

and

$$n, n_{\max} \in R^+ \ (R^+ \text{ stands for the set of positive real numbers}) \tag{21}$$

can be found in [38, 39].

The effect of nonlinear behavior on the performance of the SEB THAAT was another area for theoretical study. The carried out brief study led to the good performance of the SEB THAAT, when adequate values are assigned to the nonlinearity parameters [49, 65]. Further study seems essential.

Meanwhile, in response to a question, after a conference presentation, it was demonstrated that the *T* in Eq. (2) should be related to the response, the accuracy of which, is under study [66]. Some initial study was also carried out on the frequency content of the inaccuracies because of the SEB THAAT [67].

And finally, the most recent study was on the performance of the SEB THAAT when the structural viscous damping is non-classical. The study had a numerical/theoretical nature. As the consequence, for majority of integration methods, the performances of the SEB THAAT in application to classically and non-classically damped analyses are conceptually similar [27]. For other integration methods, the case may be the same as well, though can not be guaranteed before the analysis [27].

3 THE EXISTING CHALLENGES

As implied in the previous sections and specifically in Table 1 and Fig. 3, the reduction in computational effort because of implementation of the SEB THAAT can be considerable. Considering this, the simplicity and versatility of the SEB THAAT, the comparisons made between the SEB THAAT and other analysis acceleration methods [1, 68], the importance of computational effort in practical structural analyses, and the fact that structural systems become larger and more complicated every new day, the SEB THAAT can be considered a successful analysis acceleration technique in need of further research for broad practical implementation. Accordingly, special attention should be paid to the challenges facing the SEB THAAT. Some of the most important challenges are as follows:

The first challenge relates to the T in Eqs. (2), (14) and (16). The T is not only vague, because of the expression “worthwhile contribution” in its definition (see the definition just after Eq. (2)), but also is not computable prior to the analysis. The consequence is a serious problem in determination of $n_{\max}$ and n; see Eq. (16). Practical implementation of the SEB THAAT will hence be significantly affected and indeed becomes complicated, the least; i.e. what is the value to be assigned to step enlargement scale n resulting in the determination of the $\left({}_f\Delta t\right)_{new}$ and $\mathbf{f}_{new}(t)$? Recently, efforts for overcoming this problem are reported (see [31]). These efforts make use of a convergence-based conventional error-control method [23, 69, 70] and the practical suggestions stated in seismic standards [25, 71]. According to these efforts, the analysis is carried out via sequential time history analyses, each throughout $\left[0 \;\; t'_{end}\right]$ using a special enlargement scale. The repetitive analyses are continued, till two sequential responses are sufficiently close. The resulting analysis seems powerful and versatile, in the limits of structural dynamics problems, but not restricted to satisfaction of Eq. (14). More study on this effort and analyzing problems in real need of run-time reduction are essential.

The second challenge relates to the true nature of many oscillatory behaviors, which is a combination of inertial (structural dynamics) behavior and wave propagation. As implied in Eqs. (1) and (2) and Table 1, the SEB THAAT is mostly applied to analysis of structural dynamic problems. Study on application of the SEB THAAT in analysis of wave propagation problems is complicated, especially because of the CFL condition [72] (this condition upper-bounds the time integration step by a coefficient of the element sizes in spatial discretization by finite elements). Despite, successful numerical efforts presented in this regard [29, 30, 48], further detailed study is essential, both in theory, as well as in application. The relaxation method recently proposed for the CFL condition [73, 74] seems a good starting point.

The third challenge returns to the purpose of the SEB THAAT, i.e. reduction of the computational effort in time integration analysis of structural systems against digitized excitations. As implied in Eqs. (2) and (14), the SEB THAAT cannot reduce the analysis computational effort, when the excitation digitization step ${}_f\Delta t$ does not dominate Eq. (2), i.e.

$$\mathrm{Min}\left(\frac{T}{\chi}, \Delta t_{cr}\right) \leq {}_f\Delta t \tag{22}$$

Overcoming this restriction is a main challenge facing the SEB THAAT for highly oscillatory and highly nonlinear problems, for which T can be sufficiently small to satisfy Eq. (22). Though the ongoing efforts on the first challenge seem effective in easing this challenge, direct efforts to cancel this restriction would be instructive and reasonable. No such effort is reported yet. Enhancement in the accuracy or order of accuracy of the integration scheme and/or the SEB THAAT seems an appropriate solution.

A fourth challenge is implementation of the SEB THAAT in analysis of structural systems subjected simultaneously to several excitations digitized in completely different step-sizes. Considering that many important structural systems, e.g. offshore or super-tall structures, are in this group, overcoming this challenge is of high practical importance. No special effort is reported yet.

And finally, for practical implementation of the SEB THAAT, besides clear determination of the enlargement scale (n) discussed as the first challenge, it is important to have an idea about the resulting reduction in computational effort. Figure 3 clarifies this ambiguity for linear analyses. The case is however different for nonlinear analyses, and even there is yet no theoretical guarantee for the reduction of the run-time and computational effort. Especially the method of nonlinearity iteration and the related parameters can significantly affect the reductions. Though in many nonlinear analyses, the reduction in run-time is less than that reported in Fig. 3, the opposite can be correct, as well; see [42, 43, 75]. Considering that nowadays structural dynamic analyses are generally nonlinear, without overcoming this challenge, implementation of the SEB THAAT would hardly be accepted in practice. Yet, no effort is reported.

4 A LOOK AT THE FUTURE

In view of the discussions presented in Sections 1-3, the SEB THAAT can be considered as a successful analysis acceleration technique in need of further research. Nevertheless, especially to manage and access the research resources, it is important to have a perspective of the future of the SEB THAAT and the probable needs to this technique.

With attention to Section 3, the SEB THAAT seems in progress in both numerical experiments' direction, as well as the theoretical aspects' direction. The growing sizes of structural models, and complicatedness of analysis requirements from different points of view, e.g. material and excitation, highlight the significance of analysis acceleration. In addition, the seismic recording instrumentation is in progress towards smaller digitization step ${}_f\Delta t$ [76]. Besides, because of the everyday advancement in structural design optimization and the growing popularity of the optimization, the T in Eq. (2) will likely become larger in future. These lead to more chance for ${}_f\Delta t$ to be the governing term in Eq. (2). The consequence is more need to the SEB THAAT, in near- and mid-future. Accordingly, it is reasonable to anticipate efforts to overcome the first challenge addressed in Section 3, i.e. clear determination of the enlargement scale n, in near future. The study reported in [31] implies an initial step that should be established, after which extending the SEB THAAT to wave propagation problems can be anticipated for future studies. The next step can be plugging the SEB THAAT in a commercial soft ware, e.g. seismo-struct [77]. With an analysis soft ware equipped with the SEB THAAT, many different numerical tests, considering different structural materials, such as wood, aluminum, epoxy glass composites, different nonlinear behaviors, interaction with completely different behaviors, subjected to different earthquake records, temperature changes, soil-structure-interaction, etc. can be simply carried out. Specifically, efforts on the fourth challenge addressed in Section 3 can be carried out much simpler. In mid- and far-future, it will be reasonable to try for improving the SEB THAAT, especially to overcome the third challenge addressed in Section 3. This seems possible, either directly with attention to the very details of the technique [1], the integration methods, or even by combining the SEB THAAT with other techniques, such as those proposed in [5-7, 14].

Furthermore, and in view of the mathematical basis of the SEB THAAT [1, 13], this technique can be tested in non-seismic problems, where the excitation is not originated in earthquakes. First steps in this area are already taken; see [32, 78]. Even more, the digitization might be limited to some part or some components of the $\mathbf{f}(t)$ in Eq. (1), or more, to terms of Eq. (1) other than the excitation. Some related studies are already reported [33, 34]. In far future, the SEB THAAT seems having the potential to be accepted as a general tool for data simplification for arbitrary numerical computation. The most important pre-requisite is however to overcome the first challenge addressed in Section 3.

Meanwhile, no anticipation is possible for the future of the fifth challenge, addressed in Section 3. The reason is the same ambiguity for ordinary time history analysis, persisting since decades. A unique solution can however be anticipated for both ambiguities.

5 CONCLUDING REMARKS

As a technique for accelerating time history analysis, by enlarging the excitation records' digitization steps (${}_f\Delta t$),

1. The SEB THAAT can be considered a successful analysis acceleration technique, in need of further research.
2. For practical implementation in real problems, there are still challenges to be overcome; the most important challenges can be summarized as:
   (a) clear determination of the enlargement scale (*n*),
   (b) providing the capability to successfully implement the SEB THAAT in analysis of wave propagation problems,
   (c) providing the capability to successfully implement the SEB THAAT in analysis of highly oscillatory/nonlinear problems,
   (d) providing the capability to successfully implement the SEB THAAT in analysis of problems with several excitations completely different in digitization step and time length,
   (e) clarifying the amount of run-time and computational effort reduction in nonlinear analyses prior to the analyses.
3. The future of the SEB THAAT is promising, especially considering the improvements in the recording instrumentation.
4. Besides, the challenges addressed in the Point 2 above, some areas for further research are:
   (a) Combination of the SEB THAAT, with other analysis acceleration techniques.
   (b) Plugging the SEB THAAT in commercial structural analysis soft ware, and application of the SEB THAAT to different time history analyses.
   (c) Applying the SEB THAAT to computations other than those in structural and earthquake engineering.
   (d) Study on the details of the SEB THAAT, to improve the results accuracies.

Finally, the author anticipates that the SEB THAAT will eventually be extended to a data simplification technique.

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