Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2021) 2121-2131

ECCOMAS Proceedia

COMPDYN 2021
8th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Streamed from Athens, Greece, 28 - 30 June 2021

SIMPLIFIED METHODS FOR FREQUENCY ESTIMATION OF SMALL WIND TURBINE TOWER

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Abstract

The design process of a slender structure, such as a wind turbine tower, goes through the analysis of the natural frequency, since it is important that this frequency is away from the frequencies generated by the excitation sources. The simplified analysis models and formulations are an important tool in the frequency evaluation on design phase, even though most formulations consider the structure with constant mass and inertia, which is not the case of wind turbine tower, that have a tapered shape. The main objective of this study is to evaluate the accuracy of the simplified models when applied to structures with variable inertia. The results of the simplified models applied to a small wind tower were compared with the finite element method and experimental results. The large scatter of the simplified method's results, frequencies in the range of 0.8 – 2.12 Hz, emphasizes the importance of a reliable model to evaluate the frequency. The Rayleigh and SDOF reduction methods were found to be accurate in evaluating the first frequency of the investigated tower under consideration.

Keywords: Wind turbine tower, Simplified frequency estimation, Eigenfrequency.

ISSN:2623-3347 © 2021 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of COMPDYN 2021.
doi:10.7712/120121.8625.20280

1 INTRODUCTION

Wind energy is one of the sources of renewable energy that has been used by mankind since the dawn of civilization. Recent wind turbine towers (WTT) studies intend to optimize the costs of installation and production, not only for large ones but also for small ones; the later can be used for domestic or specific energy production. The wind turbine tower is a part of the system that affects energy production efficiency, and because of being usually slender and flexible structures, is considered the critical part of the wind turbine since it should support the dead loads of equipment and additional loads caused by wind. There are essentially two types of towers used for this purpose: lattice towers and tapered hollow towers. Tapered hollow towers are usually preferable because they require less implantation area and are aesthetically more pleasant. For modeling these structures, it is necessary the estimate the natural frequency in the lowest sway mode of the structure.

The importance of the natural frequency determination for wind turbine towers design is because it must be away from the frequencies generate by the excitation sources as wind, the rotor (1P) or blade passing (3P-for 3 blades turbines)[1]. For complex structures finite-element computer programs will be used to access this information, however, for cost reduction and in the early design the simplified methods are quite useful.

In literature can be found several methods for frequency estimation, like empirical formulas [2] simplified formulas [1, 3] as well as Rayleigh's method [4, 5]. The tower can be considered as a generalized single-degree-of-freedom structure with a concentrated tip mass on top, that is analyzed with SDOF reduction [5] or through the solution of the transcendental equation. The main problem is that the mass of the rotor and nacelle located on top of the tower is usually neglected, and generally tapered towers have a behavior different from the tubular towers; these are sufficient sources capable of causing certain errors.

The objective of this research work is to analyze the accuracy of simplified models for the determination of the first eigenfrequency of a cantilever metallic hollow tower (used as a small WTT), by evaluating the differences between them and the results obtained by finite-element model analysis in established computational software; still comparing these results, with those of evaluated frequencies from ambient or structural health monitoring vibrations measurements on the real structure. The analysis shows that the main difficulty of simplified formulas is the determination of the mass and moment of inertia that should be used, which can significantly affect the results.

2 SIMPLIFIED MODELS REVIEW

The studies of standard vibratory movements of structures are based on the hypothesis that for small amplitudes of vibration the structures exhibit a linear behavior; that is, the forces of inertia, stiffness and damping are respectively proportional to acceleration, displacement and velocity, defining a linear systems [6]. The equation of motion (on the displacement variable $w(t)$) of an elastic linear undamped system (mass M and stiffness K) in the absence of external forces (Eq.1), leads to the natural frequency f (in Hz) expressed by Eq.2.

$$M\ddot{w}(t) + Kw(t) = 0 \tag{1}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} \tag{2}$$

The simplest way to determine the frequency of a cantilever tower is to consider the structure as a beam with a concentrated mass at the top and with constant stiffness and mass, that is, as a structure with one degree of freedom (SDOF). In such case for towers of height *H* with constant inertia and constant distributed flexural stiffness *EI*, the lateral stiffness is calculated by

$$K_L = \frac{3EI}{H^3} \tag{3}$$

According to Ko [1] if the tower is modelled as a SDOF system with a tip mass, using a constant cross-section than the natural frequency is,

$$f = \frac{1}{2\pi}\sqrt{\frac{K_L}{M_{tip} + \alpha M}} \tag{4}$$

where α is the equivalent mass ratio of the tower concentrated at the top. The wind turbine design standard DNV-OS-J101 [7] establishes $\alpha = 0.25$, while the IEC 61400-1 standard [8] indicates that $\alpha = 0.5$; another value of $\alpha = 33/140$ is proposed by Blevins [9].

A different procedure is to model the wind turbine tower as Euler-Bernoulli beam system [10], with distributed mass *m* and distributed flexural stiffness *EI*. The governing equation of motion for undamped free vibrations of a uniform Euler-Bernoulli beam [11, 12, 13] is,

$$EI\frac{\partial^4 w(z,t)}{\partial z^4} + m\frac{\partial^2 w(z,t)}{\partial t^2} = 0 \tag{5}$$

and, the clamped-free boundary conditions with a tip mass are,

$$w(O,t) = 0, \qquad \left[\frac{\partial w(z,t)}{\partial z}\right]_{z=0} = 0 \tag{6}$$

$$\left[EI\frac{\partial^2 w(z,t)}{\partial z^2} + I_{tip}\frac{\partial^3 w(z,t)}{\partial t^2 \partial z}\right]_{z=L} = 0, \quad \left[EI\frac{\partial^3 w(z,t)}{\partial z^3} - M_{tip}\frac{\partial^2 w(z,t)}{\partial t^2}\right]_{z=L} = 0 \tag{7}$$

Separating the spatial and temporal functions using the method of separation of variables [11], applying Eq. 8 leads to Eq. 9

$$w(z,t) = \Phi(z)\eta(t) \tag{8}$$

$$\frac{EI}{m}\frac{1}{\Phi(z)}\frac{\partial^4 \Phi(z)}{\partial z^4} = -\frac{1}{\eta(t)}\frac{\partial^2 \eta(t)}{\partial t^2} = \gamma = \omega^2 \tag{9}$$

From which, the solution for Eq. 8 can be expressed as follows,

$$\Phi(z) = A\cos\left(\frac{\lambda}{H}z\right) + B\cosh\left(\frac{\lambda}{H}z\right) + C\sin\left(\frac{\lambda}{H}z\right) + D\sinh\left(\frac{\lambda}{H}z\right) \tag{10}$$

$$\eta(t) = E\cos(\omega t) + F\sin(\omega t) \tag{11}$$

where A, B, C, D, E and F are unknown constants and,

$$\lambda^4 = \frac{\omega^2 m H^4}{EI} \rightarrow \omega = \frac{\lambda^2}{H^2}\sqrt{\frac{EI}{m}} \tag{12}$$

The non-trivial solution for $\Phi(z)$ is obtained by the differential eigenvalue problem, using the spatial form of the boundary conditions to find the values of λ [12]. Then the characteristic equation of the system is,

$$\begin{aligned} 1 + \cos\lambda\cosh\lambda + \frac{\lambda M_{tip}}{mH}[\cos\lambda\sinh\lambda - \sin\lambda\cosh\lambda] \\ - \frac{\lambda^3 I_{tip}}{mH^3}[\cosh\lambda\sin\lambda + \sinh\lambda\cos\lambda] + \frac{\lambda^4 M_{tip} I_{tip}}{mH^4}[1 - \cos\lambda\cos\lambda] = 0 \end{aligned} \tag{13}$$

If the tip mass is modeled as a point mass, the rotational inertia of the tip mass is negligible, $I_{tip} = 0$. The first root of the characteristic equation is the first natural frequency, calculated by solving Eq. 13.

The main difficulty is to determine the values of *m* and *EI* to be used, since the tower has distributed mass and elasticity, i.e., the structure is non-uniform. Nevertheless, this structure can be modeled as a generalized SDOF system and provide an approximate solution. This procedure is called SDOF reduction and consists in creating a SDOF-equivalent system with distributed mass and stiffness, where according to the principle of virtual work, the generalized mass $M^*$and stiffness K^* can be obtained [6, 13, 14],

$$M^* = M_{tip} + \int_0^H m(z)\,\Phi^{*2}(z)dz \tag{14}$$

$$K^* = \int_0^H EI(z)\left[\frac{\partial^2\Phi^*(z)}{\partial z^2}\right]^2 dz - \int_0^H N(z)\left[\frac{\partial\Phi^*(z)}{\partial z}\right]^2 dz \tag{15}$$

Paz [5] recommends the Eq. 16 as a simplified mode shape of the tower,

$$\Phi^*(z) = 1 - \cos\left(\frac{\pi z}{2H}\right) \tag{16}$$

On the other hand, an alternative method to establish the stiffness of the structure is through the linear relationship between the force and displacement Eq.17. But the difficulty with this procedure is that one must know the force and the displacement caused by that force.

$$F = Kx \rightarrow K = \frac{F}{\delta} \tag{17}$$

The displacement can be determined by theoretical calculations or by structural analysis software. Balagopal *et al.* [14] present a simplified model for calculating the displacement at the top considering the application of a unit force on the structure. The procedure defined the cross sectional area of the tower and the moment of inertia are calculated based on ASCE 48-11 [15].

$$K = \frac{1}{\delta_{unit}} \tag{18}$$

$$\delta_{unit} = \delta_{lat} + \delta_{vert} \tag{19}$$

$$I = Cd_m^3 e \qquad C = 0{,}403 \qquad d_m = (d_t + d_b)/2 \tag{20}$$

$$A = 3{,}19 d_m e \tag{21}$$

The displacement δ_{unit} is calculated using the following set of equations,

$$\delta_{lat} = \int_0^H \frac{z^2}{ECe(d_t + s_p z)^3} dz \tag{22}$$

$$\delta_{vert} = \int_0^H \frac{Mz}{ECe(d_t + s_p z)^3} dz \tag{23}$$

$$M = \delta_{lat} V = \delta_{lat} M_{tip} * g \tag{24}$$

$$s_p = \left(\frac{d_b - d_t}{H}\right) \tag{25}$$

When the law of conservation of energy is used to obtain the differential equation of motion for an undamped system in free vibration, the so-called Rayleigh's method is derived to find the natural frequency, by equalizing the maximum kinetic energy to the maximum potential energy (at different specific instants); but for this, it is necessary to assume a deformed shape. Usually, the shape is determined as that produced by the gravitational loads acting in the direction of the expected displacements [5, 6]. The natural frequency is expressed as,

$$f = \frac{1}{2\pi} \sqrt{\frac{g \sum_i W_i \delta_i}{\sum_i W_i {\delta_i}^2}} \tag{26}$$

where δ_i is deflection at the coordinate i and W_i is the concentrated weight at this coordinate.

Holmes [2] uses a semi empirical formula for cantilevered tapered circular tower, that was also recommended by the European Convention for Structural Steelwork in 1978, given by

$$f = \frac{\lambda}{2\pi H^2} \sqrt{\frac{EI_b}{m_b}} \tag{27}$$

$$\lambda = 1{,}9 * \exp\left(\frac{-4d_t}{d_b}\right) + \frac{6{,}65}{0{,}9 + (e_t/e_b)^{0{,}666}} \tag{28}$$

Another simplified expression is presented in the Annex F of EN 1991-1-4 [3],

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{w,max}}} \tag{29}$$

where $\delta_{w,max}$ is the maximum displacement (expressed in meters) due to self-weight applied in the vibration direction.

3 TOWER AND INVESTIGATION DESCRIPTION

To determine the natural frequency of a structure, there are three main approaches: theoretical calculation, application of the finite element method (FEM) using robust computational software, and experimental determination on the real structure. In general, the FEM method and the experimental determination show more accurate results, but they are also more expensive to achieve and perform.

To analyze the accuracy of the simplified models in determining the first natural frequency of conical hollow towers, the results of the formulations presented in the literature review are compared to the numerical FEM model and the experimental results.

3.1 Description of the tower

A metal tower 17.8 m height located at the School of Technology and Management of the Polytechnic Institute of Bragança (Portugal), consists of a steel S275 structure with a hexadecagonal section with an outside diameter of 0.5890 m at the base and 0.1954 m at the top, with a constant wall thickness of 4 mm. The tower is fixed to the gravity-base foundation by 16 anchor bolts connected to a flange at the base; the flange has an external diameter of 0.7960 m and a thickness of 20 mm. The main properties of the tower are presented in Table 1, and the geometric plan view configuration is shown in Figure 1.

Density	7850.000	[kg/m³]
Young's modulus	210.000	[GPa]
Poisson's ratio	0.300	--
Yield strength	275.000	[MPa]
H	17.800	[m]
$\boldsymbol{d_t}$	0.195	[m]
$\boldsymbol{d_b}$	0.589	[m]
$\boldsymbol{d_m}$	0.392	[m]
$\boldsymbol{s_p}$	22.135x10^{-3}	
$\boldsymbol{d(z)}$	$0.589 - 22.135\text{x}10^{-3}z$	[m/m]
$\boldsymbol{e_b = e_t = e}$	0,004	[m]
$\boldsymbol{A(z)}$	$0{,}765(9{,}36 - 0{,}35376z)10^{-3}$	[m^2/m]
$\boldsymbol{m(z)}$	$6{,}008(9{,}36 - 0{,}35376z)$	[kg/m]
$\boldsymbol{I(z)}$	$(2{,}987 - 0{,}339z + 0{,}013z^2 - 1{,}612\text{x}10^{-4}z^3)10^{-4}$	[m^4/m]
$\boldsymbol{M_{tip}}$	75,000	[kg]
$\boldsymbol{M_{tower}}$	664,277	[kg]

Table 1: Tower properties

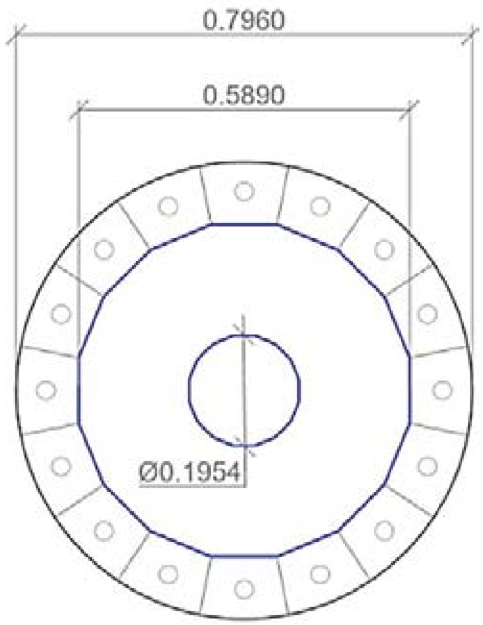


Figure 1 : Tower geometries.

3.2 Numerical model

The analysis of the structure using the finite element method is performed using the SAP2000 Software. The tower is modeled with the geometry presented above, considering the bottom boundary condition of the type BC1f (CEN, 2010) [3] for the location of the 16 anchor bolts; the top boundary is considered free with the action of a tip mass of 75 kg that corresponds to the weight of the wind turbine. The simulation is performed considering the linear behavior of the material.

3.3 Experimental analysis

To determine the dynamic characteristics of the structure, the experimental modal analysis was carried out using: a data acquisition system composed of NI USB-4431 data acquisition board with 4 input channels, 24-bit ACD resolution, sampling rate from 1 kS/s to 102.4 kS/s, resolution 2.10 mS/s; piezoresistive accelerometer ICP® with a frequency range from 0.5 to 2000 Hz; and a laptop with SignalExpress © software.

Two accelerometers are placed at 1.41 meters high in two distinct perpendicular directions (x and y). The input force is caused by the impact hammer PCB 086b20. The first (and others, if required) natural frequency is defined by the peak-picking technique.

3.4 Summary of analysis

In Table 2 are synthesized the methodologies used in this work for the determination of the WTT fundamental frequency, ranging from several simplified formulations presented in the literature to the numerical model and experimental analysis; some formulas require the use of a software to find the displacement, in which case the FTOOL software was use.

Equation	Stiffness parameters	General parameters	Name/ reference
$\boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{K_L}{M_{tip} + \alpha M_{tower}}}}$	$K_L = \frac{3EI_m}{H^3}$	$\alpha = 0{,}25$	DNV-OS-J101
		$\alpha = 0{,}5$	IEC 61400-1
		$\alpha = 33/140$	Blevins, 1979
$\boldsymbol{f = \frac{1}{2\pi}\frac{\lambda^2}{H^2}\sqrt{\frac{EI}{m}}}$	$I = I_m$	$1 + \cos\lambda \cosh\lambda$ $m = m_m$	Characteristic equation
		$1 + \cos\lambda\cosh\lambda + \frac{\lambda M_{tip}}{mH}[\cos\lambda\sinh\lambda - \sin\lambda\cosh\lambda]$ $m = m_m$	Characteristic equation + $\boldsymbol{M_{tip}}$
$\boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{K^*}{M^*}}}$		$N(z) = g\ M_{tip}$	SDOF reduction
$\boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}}$	$K = \frac{1}{\delta_{unit}}$	$\delta_{unit} = \delta_{lat} + \delta_{vert}$ $M = 3{,}19 d_m e H \rho$	Balagopal, 2018
		δ_{unit} – Ftool 1 segments d_m $M = M_{tip} + M_{tower}$	
$\boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{g\sum_i W_i\delta_i}{\sum_i W_i{\delta_i}^2}}}$		δ_i – Ftool 10 segments $d_{m,i}$	Rayleigh's method
$\boldsymbol{f = \frac{\lambda}{2\pi H^2}\sqrt{\frac{EI_b}{m_b}}}$		$\lambda = 1{,}9 * \exp\left(\frac{-4d_t}{d_b}\right) + \frac{6{,}65}{0{,}9 + (e_t/e_b)^{0{,}666}}$	Holmes, 2011
$\boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{w,max}}}}$		$\delta_{w,max}$ – Ftool 1 segments d_m	EN 1991-1-4
$\boldsymbol{f}$		FEM	SAP2000
$\boldsymbol{f}$		Experimental	EMA – Hammer impact

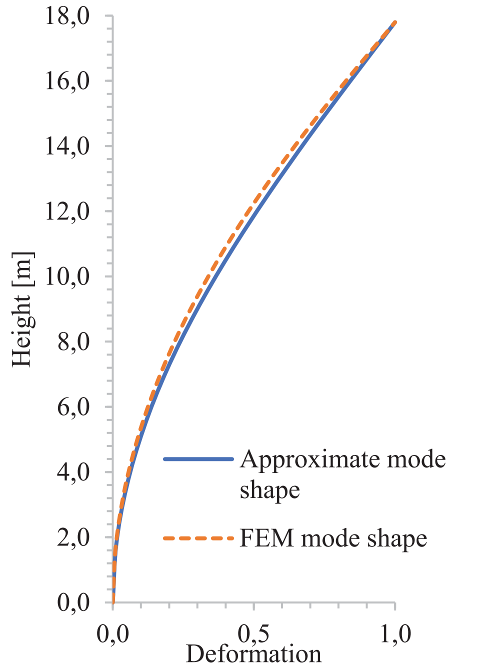
Table 2: Analysis summary

4 RESULTS AND DISCUSSION

The advantage of using a finite element model is the determination of the mode shapes of the structure, as the fundamental or 1st mode shape shown in Figure 2(a). As mentioned in Paz and Kim [5], the accurate determination of the mode shapes is one of the crucial points for the correct calculation of the frequency by the application of SDOF reduction [5]. Figure 2 (b) compares the FEM mode shape and the approximate mode shape given by Eq. 16.

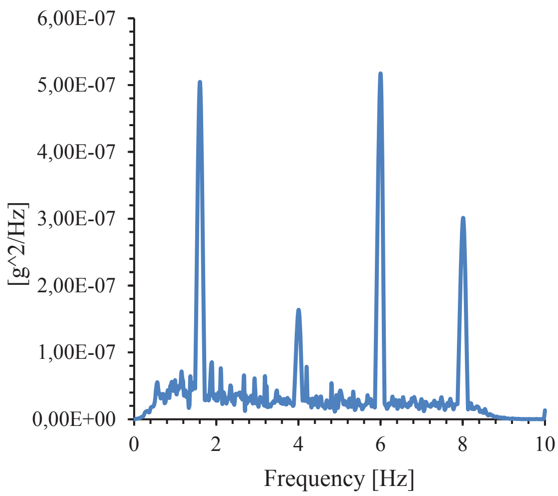
In relation to the results of the experimental analysis, the power spectra density is presented in Figure 3 (a) for the x-direction and in Figure 3 (b) for the y-direction. As shown, the first natural frequency is coherently evaluated as 1.610 Hz.

(a) FEM mode shape

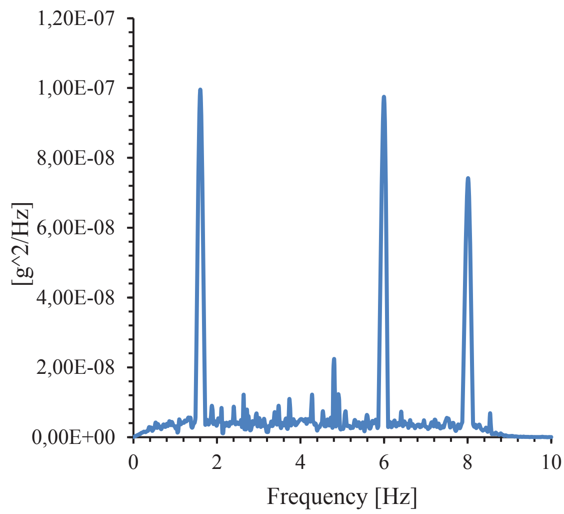


(b) Approximate mode shape in comparison to FEM mode shape

Figure 2: Tower mode shapes



(a) x-direction



(b) y-direction

Figure 3: Power spectral density

The values obtained by the different methods for the first natural frequency are shown in Figure 4. It is possible to observe a large dispersion of results with relative differences in the order of 50%, in comparison with the experimental value; this is due to the fact that most of the formulations are established for a tower of constant mass and constant inertia per unit length. Although they are simplifications, the problem lies mainly in the definition of the inertia to be used, since it varies cubically.

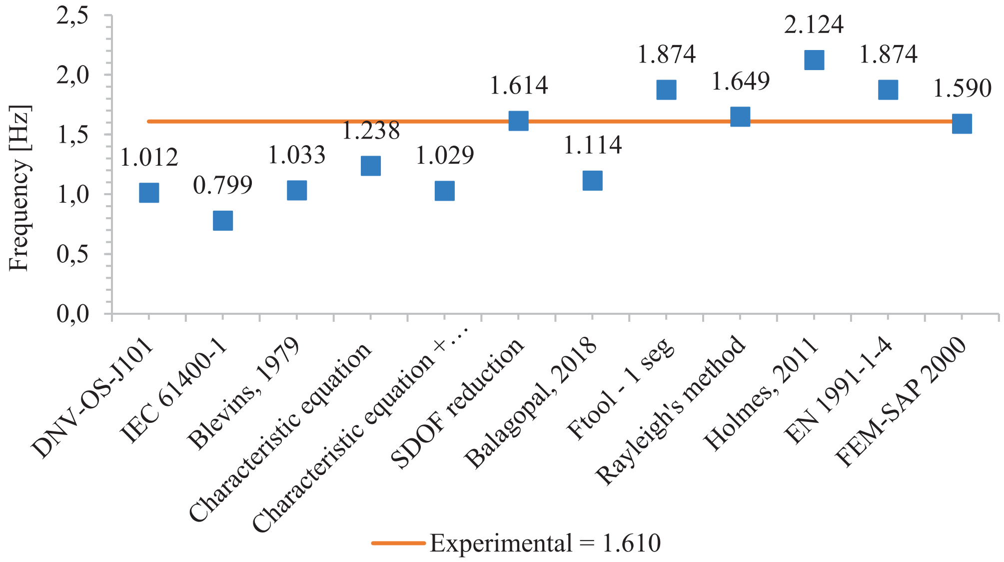


Figure 4: First natural frequency, by different methodologies

It is observed that the simplified models that present the results closest to the experimental value are the FEM model, the Rayleigh method and the SDOF reduction method; however the first two methods require the use of software, and are more time consuming. The difficulty in applying the SDOF reduction method consists in solving the integrals, but they are easily solved with a graphic scientific calculator; in fact this method presents the best results when analyzing the frequency value and the time taken to calculate it.

Notice that the vast majority of results by different methods underestimate the value of the fundamental frequency, that is, consider the structure more flexible than it really is. This is mainly due to consider the inertia and the mass defined in the first half of the structure (up to H/2), and because the tower having a conical or tapered shape its mass is more concentrated near the base. Also the results of the expressions using the displacement value at the top distributed by a simplified model of average diameter in the FTOOL, overestimate the frequency value by about 16%.

5 CONCLUSIONS

The metallic hollow cantilever tapered tower analyzed has the first natural frequency of 1.61 Hz. This frequency can be accurately evaluated by the application of the finite-element model, even though this method is time consuming. The Rayleigh's method also shows accurate result for the WTT analyzed; but the single degree of freedom reduction method is the simplified method that has the best performance, since it is simple and the result is accurate.

The analysis shows that the main difficulty of simplified formulas is the determination of the moment of inertia and mass, which significantly affect the results. The results show large difference between the frequencies analytically evaluated by different methods (with values in a range of 0.8 to 2.12 Hz), some of them with a significant difference from the real natural frequency, and the majority of them underestimate the value of the frequency.

The dispersion of the results reflects the importance of accurate assessment of the structural inertia and mass on the use of simplified methods, for tapered tower with tip mass; nevertheless further investigation, on the accurate estimation of mass and inertia for the tapered towers, should be carried out.

ACKNOWLEDGEMENTS

This paper is within the scope of the first author's Ph.D. degree in progress, financially supported by the Portuguese Foundation for Science and Technology (FCT) through the PhD grant reference SFRH/BD/147789/2019 under the programme POCH (N2020 – P2020) and subsidized by the European Social Fund (FSE) and national funds from MCTES. This work was financially supported by: Base Funding - UIDB/04708/2020 of the CONSTRUCT - Instituto de I&D em Estruturas e Construções - funded by national funds through the FCT/MCTES (PIDDAC).

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