

## **APPLICATION OF A GENERAL PURPOSE DISTINCT ELEMENT MODEL (DEM) FOR THE SEISMIC VULNERABILITY PREDICTION OF MASONRY AGGREGATES**

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### **Abstract**

*Masonry aggregates are an important class of existing masonry structures. They can be seen as assemblages of several structural units that are independent of the viewpoint of the intended use but that are structurally connected one each other. Clusters are large and complex structures involving several adjacent structural units with different heights, number of stories, and inter-storey heights, erected in continuity one to each other, making it rather difficult to distinguish the independent units and to identify a global behavior of the whole construction. The authors have provided a novel limit analysis method to discretize a masonry pagoda through infinite resistant hexahedrons in some recent papers. For the vulnerability analysis of aggregates, the paper extends this assumption to a no-tension material. Under such assumptions, considering a generic portion of an aggregate, it is here discretized by means of hexahedron (Hexa8) elements. The material inside each element is supposed infinitely resistant and all the internal plastic dissipation of the mechanical system is assumed to occur exclusively at the interfaces between adjoining elements. Two aggregates located in Arsita, Italy are chosen to benchmark the code. They are “La Vecchia Forestale” and “Church of Santa Vittoria”. This research assumes that the tensile strength of the masonry material decreases gradually from a very high value to 0 (no-tension material), observes the change of the collapse mechanisms of the masonry aggregates, and deduces the associated failure mechanism. The approach shows that when the tensile strength is high, the collapse acceleration is unrealistically large. When the tensile strength of the material is reduced to a small value (close to the no-tension material), the overturning mechanism is most easily seen.*

**Keywords:** limit analysis, masonry aggregates, seismic vulnerability, collapse mechanisms.

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## 1 INTRODUCTION

The poor behavior of masonry structures undergoing earthquakes is mainly due to the main characteristics of masonry aggregates. The masonry aggregates have very low tensile strength compared with high compressive strength and moderately high friction angle. Therefore, when a high-rise architecture with masonry material encounters an earthquake, its resistance will be insufficient, and the poor characteristic will cause structural damage and collapse. Considering these causes, trying to protect and repair historical masonry aggregates has become an important mission for contemporary people. Seismic vulnerability assessment of masonry aggregates in the historical centers represents a specific problem to be solved. The research should predict the behavior of aggregates under earthquakes and implement seismic protection measures in the event of defects [1-5].

Nowadays, through different kinds of numerical simulation methods, doing finite element analysis of these historical masonry aggregates may bring a deep understanding of architectural dynamic characteristics. These simulations have peculiar application values for seismic, reinforcement, and maintenance of structures. However, the numerical simulation method still has the disadvantages of inaccurate solutions and long calculation time.

To overcome this shortcoming, this research introduces the method of limit analysis. The plastic limit analysis is carried out under the assumption that the material has ideal rigid-plastic properties, thus avoiding the complex calculation of elastic-plastic analysis. The ultimate load obtained from the solution of the limit analysis is exactly equal to the ultimate load obtained from the elastoplastic analysis. The classical limit analysis is generally based on the following assumptions: (1) the ideal plasticity assumption of the material, (2) the small deformation assumption, (3) the structure will not lose its stability before reaching the ultimate load, (4) the external loads acting on the structure are increased in the same proportion, that is, the proportional loading conditions are satisfied [6][7]. The upper bound limit analysis method must satisfy the maneuvering condition and the yield condition, while the lower bound limit analysis method must satisfy the static equilibrium condition and cannot violate the yield condition. In theory, the result obtained by the upper bound limit analysis is greater than the exact solution, the result obtained by the lower bound limit analysis is smaller than the exact solution, and the real solution is between the two [8-10].

Although the limit analysis method can effectively make up for the shortcomings of numerical simulation and laboratory experiments, it still cannot bridge the gap between theory and engineering applications. Therefore, a novel limit analysis method is proposed in this paper, which discretizes the masonry structure into hexahedral elements, which can quickly and conveniently analyze the vulnerability of complex masonry aggregates. This paper verifies the feasibility of this approach by taking two masonry aggregates in historical centers as examples.

## 2 DESCRIPTION OF THE CASE STUDY

There are two masonry aggregates in the Arsita region of Italy were selected for research here, they are “La Vecchia Forestale” and “Church of Santa Vittoria” respectively. On April 6, 2009, a magnitude 6.3 earthquake occurred in Italy, with the epicenter in the town of L'Aquila, Italy. The earthquake caused a large number of people to be displaced, and masonry structures collapsed and cracked. The masonry buildings in the historical center of the Arsita are old and mostly do not meet today's safety standards. The area is around 35.78 kilometers away from the epicenter (Figure 1), and the masonry structures affected by the earthquake suffered serious damage. To comprehensively evaluate the post-earthquake state of the historical masonry structures in the area, and to better conceive the reconstruction plan, a team

composed of the Italian government, universities, and various research institutions has conducted an exhaustive survey of the buildings in the area [11].

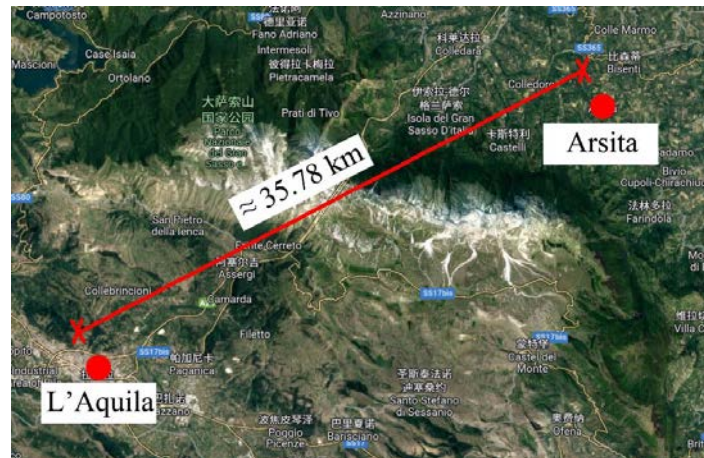


Figure 1. Distance from the epicenter to Arsita.

## 2.1 “La Vecchia Forestale”

The first aggregate case is called “La Vecchia Forestale”. It has an irregular plane with the characteristics of being narrow in the northwest and wide in the southeast. The building can reach up to 25 meters in the north-south direction. According to Ref. [11], the building is divided into five different units A – E (Figure 2-a), and each unit is divided according to structural features and functions with different floors and heights, and the highest point of the building is up to 11.8 meters. The performance of the masonry materials used here is not good. Mortar is used to bond the masonry bricks, which causes the wall to be greatly affected by the seismic horizontal load, and multiple damages can be seen with the naked eye. The interlocking relationship between vertical walls is weak, and the tie relationship between the walls is almost non-existent. Here, the research selected the wall combination of two different parts of the aggregate for analysis. The name of the group and the name of the wall are shown in Figure 2. Among them, Group 1 is a corner unit, and Group 2 is a continuous facade unit.

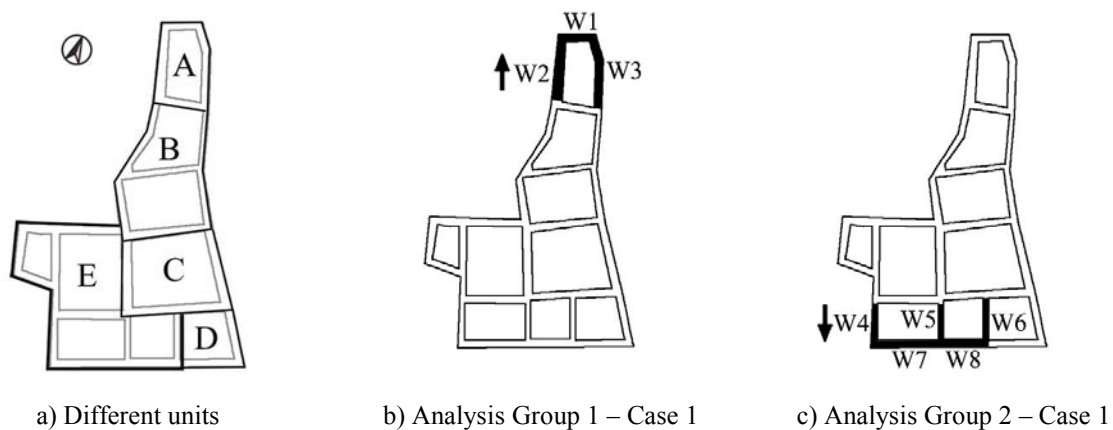


Figure 2. Units and analysis groups of “La Vecchia Forestale”.

## 2.2 “Church of Santa Vittoria”

The second case is called "Church of Santa Vittoria". Its plane shape is almost a regular rectangle, with the characteristics of being narrow in the northeast part and wide in the southwest part, and the longest north-south direction of the building can reach 25 meters. In Ref. [11], the building is divided into three different units A – C (Figure 3-a). Among them, units A and B are used as residences, and unit C is used as a church. The heights of each unit are different, but the highest point of the floor is 15. The material properties of this masonry aggregate are not good, and the internal arch structure plays a certain connection role. The chain relationship between vertical walls is good, so simple facade overturning is not easy to happen. The wall combination of two different parts in the aggregate is selected for analysis. The name of the groups and the name of the wall are shown in Figure 3. Among them, Group 1 is a corner unit, and Group 2 is a continuous facade unit.

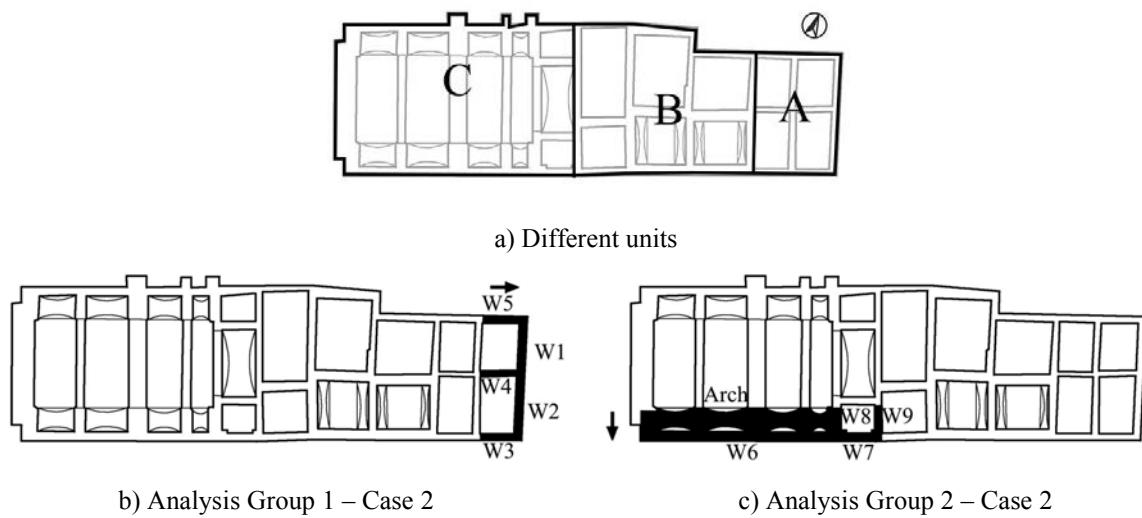


Figure 3. Units and analysis groups of “Church of Santa Vittoria”.

## 3 LIMIT ANALYSIS RESULTS

### 3.1 Description of the limit analysis model

The novel limit analysis method proposed in this paper is developed from the technique published by the author in the past [15][16]. The theoretical basis of this method is the theorem of kinematics. A kinematically admissible load multiplier, or one for which the power associated with external forces is equal to the power associated with plastic dissipation, in a failure mechanism obeying compatibility, is an upper bound of the true collapse load multiplier. This theorem can apply to no-tension or quasi-no-tension material. When a mechanism is engaged, the modeling of the structure in stiff macro-blocks makes it simple to convert the structure into a kinematic chain or a problem with a few variables [16]. When studying the out-of-plane collapse mechanism in some walls of the masonry aggregate, it is necessary to isolate the research part and define certain boundary conditions. The study here assumes a general-purpose part of the polymer, which is discretized into hexahedral elements, and the material inside each unit has infinite resistance. It is also assumed that all plastic dissipation within the system occurs between adjacent element interfaces (Figure 4).

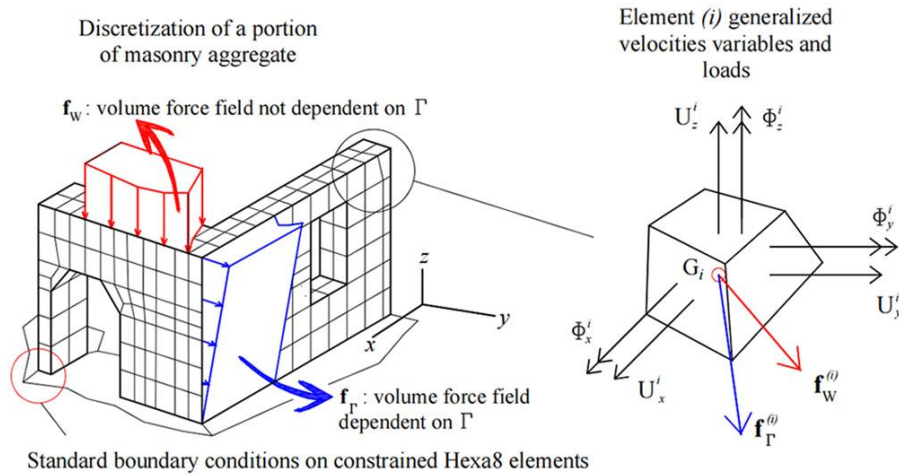


Figure 4. Discretization of an infinite resistance hexahedron for aggregates.

### 3.2 Limit analysis method results of “La Vecchia Forestale”

The previous model of analysis groups walls was made in a common commercial software Straus7, and the limit analysis was done in Matlab software. Here, it was assumed that the tensile strength ( $f_t$ ) of the masonry material decreases gradually from a very high value to 0 (no-tension material).

Figure 5 shows three situations when the failure mechanism of the masonry wall is changed. When the tensile strength of the material is larger ( $f_t = 0.15$  MPa), the collapse acceleration is large ( $a_g/g = 4.5094$ ). The aggregate was damaged at the opening part of W1. On the W3, there was overturning along the bottom edge and longitudinal edge of the opening. At the same time, the masonry block above the opening wall slipped upwards. When the tensile strength of the material is reduced to 0.05 MPa, the collapse acceleration is also reduced to 1.927. The collapse mechanism of the aggregate is generally similar to the first hypothesis, but there are some changes in W3. The masonry block cracks longitudinally along the center of the upper edge of the opening, creating overturning. Finally, when the tensile strength of the masonry material is very small ( $f_t = 0.008$ ),  $a_g/g = 0.55168$ . The overturning also occurs at the aggregate. However, on W3, the masonry block above the opening slides downward.

Figure 6 shows the simulation and derivation results for aggregate at different values of masonry tensile strength. In Figure 5-10 (a), the author assumed a high tensile strength  $f_t = 0.3$  MPa, and gradually decrease until the material becomes no-tension. When  $f_t$  goes from 0.3 MPa to 0, the internal power dissipated decreases as the  $f_t$  decreases. But the power expended by gravity loads does not follow this rule, it only depends on the shape of the failure mechanism and remains almost constant (0.74697) during the initial  $f_t$  decrease, it drops sharply until  $f_t$  approaches 0, and the failure mechanism will change accordingly. The change of this mechanism is activated by sliding. In Figure 5-10 (b), the collapse acceleration for different tensile strengths was plotted. The red dashed curve appears linear when the  $f_t$  is large. However, when  $f_t$  is very low, the collapse acceleration drops almost to 0, where the graph is nonlinear. A straight tangent from the high tensile strength was extrapolated to find the intercept with the vertical axis, it is the collapse acceleration  $a_g/g = 0.74697$  for the no-tension material model with this failure mechanism.

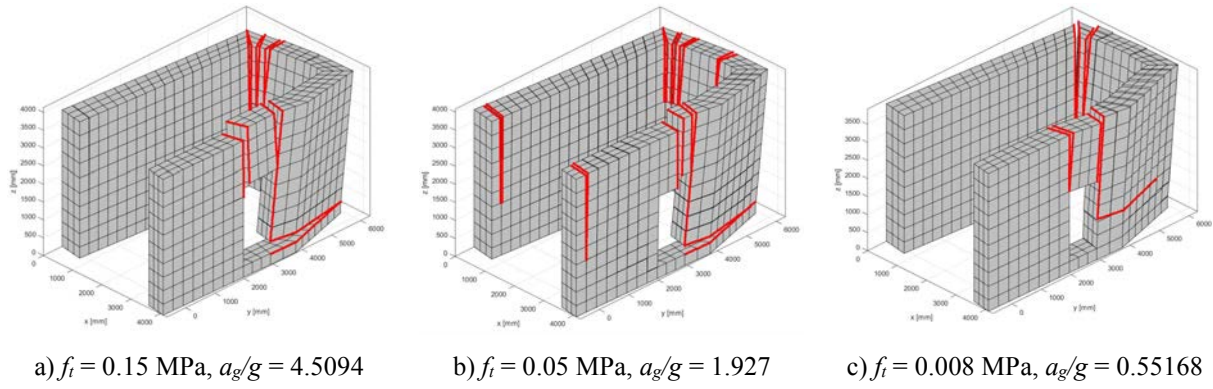


Figure 5. Group 1 – Case 1, collapse mechanisms and collapse accelerations with three different tensile strengths.

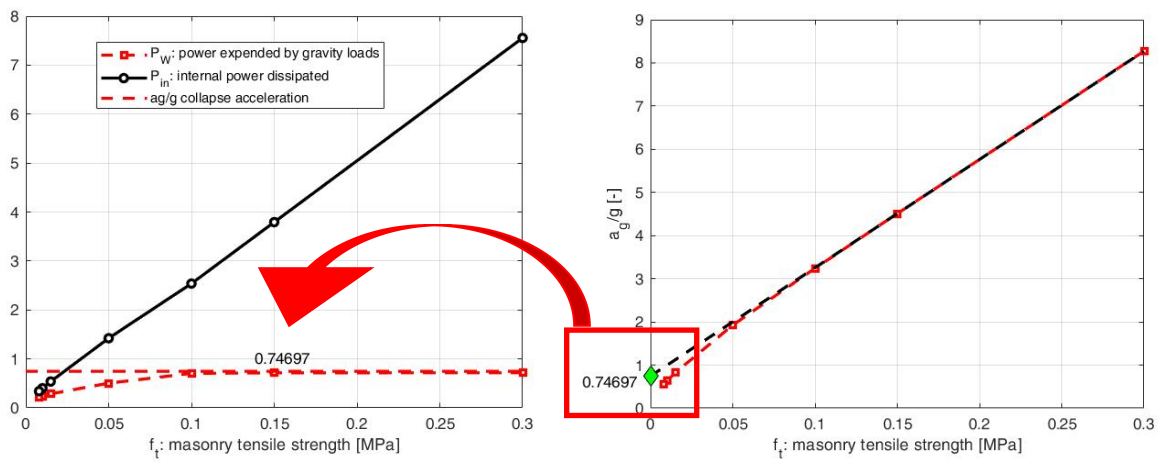


Figure 6. Group 1 – Case 1, the power dissipated on interfaces and that expended by gravity loads at decreasing values of masonry tensile strength, and the decrease of the normalized collapse accelerations.

Figure 7 shows three situations when the failure mechanism of the masonry wall is changed. When the tensile strength is 0.15 MPa, the collapse acceleration is large ( $a_g/g = 1.6144$ ). Walls W7 and W8 showed damage along the edges of the opening, in particular the activation of the horizontal bending mechanism at the bottom edge of the opening and the activation of the vertical bending mechanism at the longitudinal sides of the opening. Cracks along the edge of the opening also appeared in W5. When the tensile strength of the material is gradually reduced to 0.05 MPa, the bending mechanism on W7 and W8 is no longer obvious, and instead, the overturning mechanism along the edge of the opening is activated. At the same time, the blocks near the openings of W5 and W6 slipped to varying degrees due to reduced material cohesion. Finally, when the assumed material strength is reduced to 0.008 MPa, the vertical bending and horizontal bending mechanisms are activated again on W7 and W8, while the block above the opening in W5 appears to slide down.

Figure 8 shows the simulation and derivation results for aggregate at different values of masonry tensile strength. The meaning of the sub-figures is the same as that described in the previous part. When  $f_t$  is from 0.3 MPa to 0.15 MPa, the power expended by gravity loads did not change much, it is 0.18551, and when the tensile strength changed from 0.15 MPa to 0, the value gradually decreased, and the amplitude became larger and larger. The collapse



acceleration of a model with this failure mechanism is 0.18551 when the material is no-tension material.

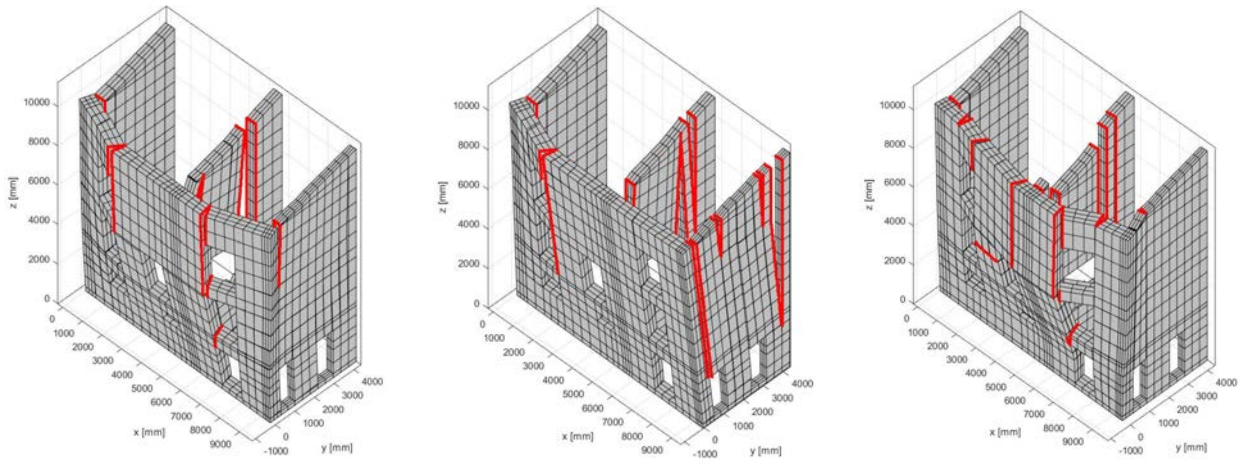


Figure 7. Group 2 – Case 1, collapse mechanisms and collapse accelerations with three different tensile strengths.

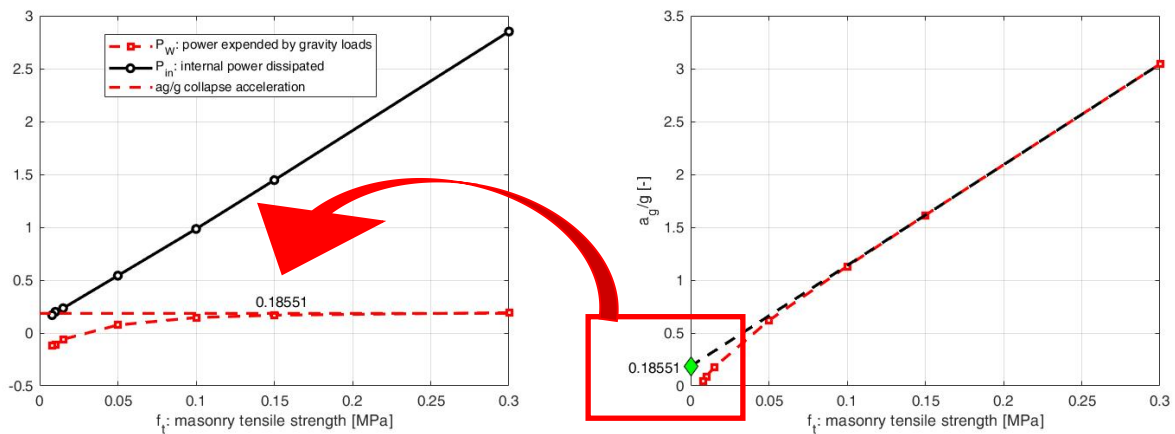


Figure 8. Group 2 – Case 1, the power dissipated on interfaces and that expended by gravity loads at decreasing values of masonry tensile strength, and the decrease of the normalized collapse accelerations.

### 3.3 Limit analysis method results of “Church of Santa Vittoria”

Figure 9 shows three situations when the failure mechanism of the masonry wall is changed. When the tensile strength of the material is 0.15 MPa, the collapse acceleration is very high. Damage along the openings appeared on the walls of W3, W4, and W5, and the blocks above the openings exhibited sliding and flexure mechanisms. W1 and W2 were cracked along the openings, and a vertical flexure mechanism occurred. When the tensile strength of the material drops to 0.015 MPa, the failure mechanism of the aggregate is similar to the first test. However, when the tensile strength becomes very small ( $f_t = 0.008$  MPa), the flexure mechanism on W1 and W2 is no longer visible, and the overturning mechanism is mainly observed. The openings on the walls of W3, W4, and W5 are still relatively weak parts, but the sliding degree of the blocks above the openings is reduced.

Figure 10 shows the simulation and derivation results for aggregate at different values of masonry tensile strength.

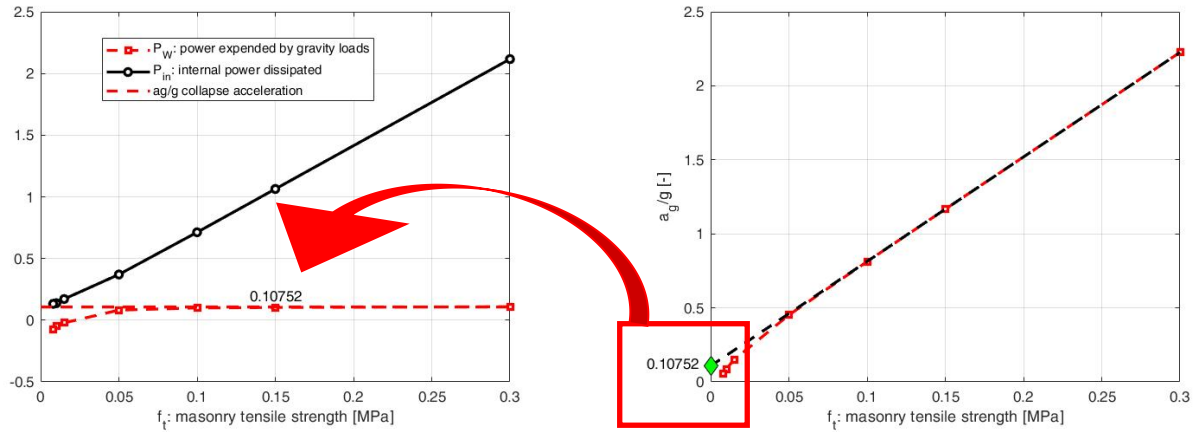
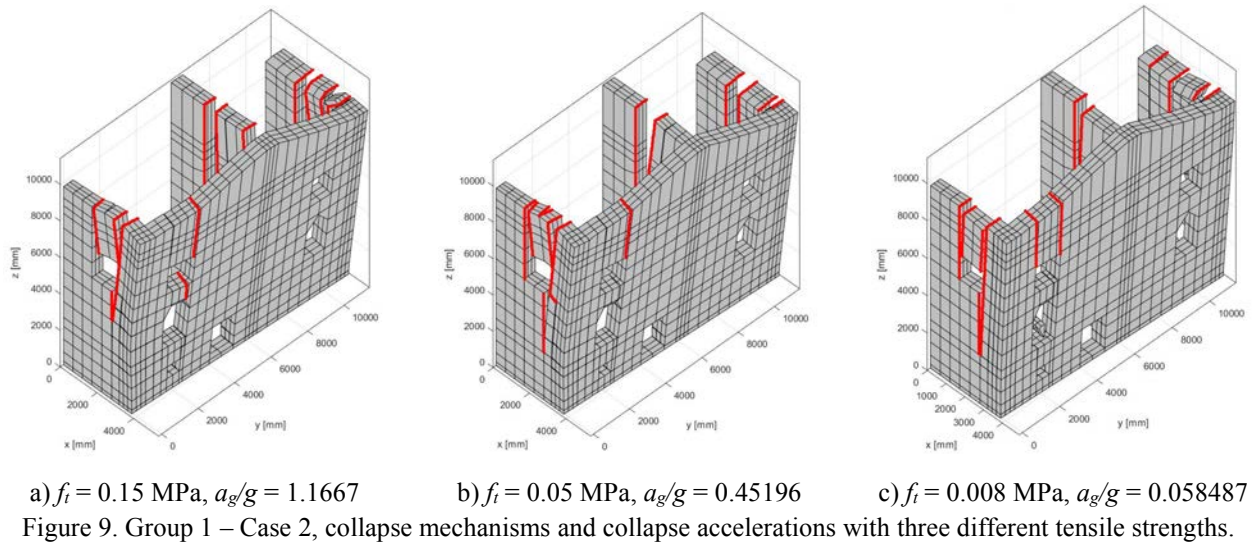


Figure 11 shows three situations when the failure mechanism of the masonry wall is changed. When the tensile strength of the masonry is higher at 0.15 MPa, the horizontal load acceleration is larger, which is 0.6925. W7, W8, and W9 are cracked along the lateral edges of the openings, and the vertical flexure mechanism on the W7 wall is activated. When the tensile strength of the material is reduced to 0.05 MPa, the overturning mechanism of the W7 along the edge of the lower opening is activated, and the vertical bending degree of the wall is ridiculous compared to the first test. When the masonry material is inferior ( $f_t = 0.008$  MPa), both W6 and W7 activate the overturning mechanism, and the overturning hinge is close to the bottom of the wall. Blocks on the W9 wall appear to slide down significantly. Notably, no obvious cracking and flexure mechanisms were observed on the W6 wall under all three material conditions. Presumably, the arches provide the W6 wall with a higher seismic capacity.

The same as before, Figure 12 shows the simulation and derivation results for aggregate at different values of masonry tensile strength.



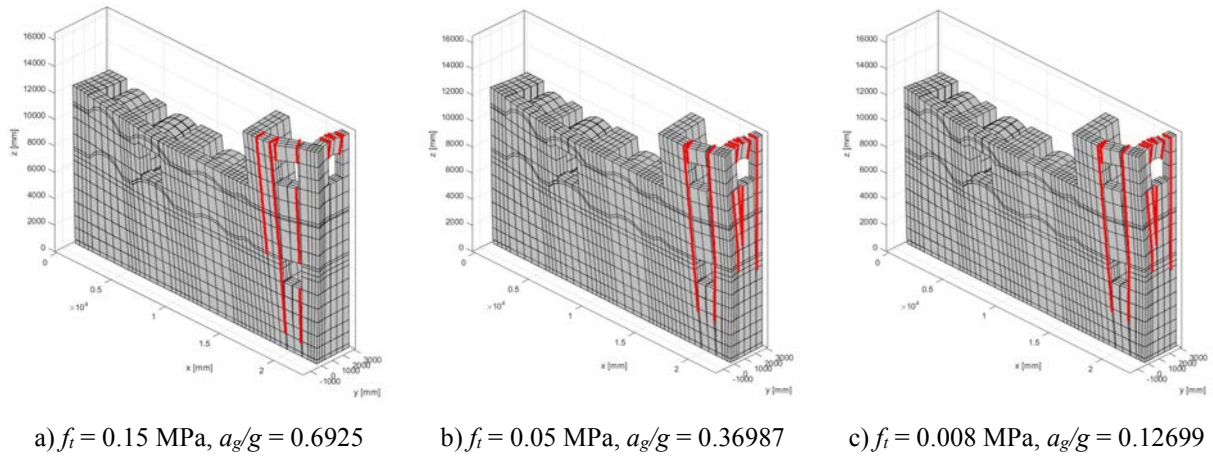
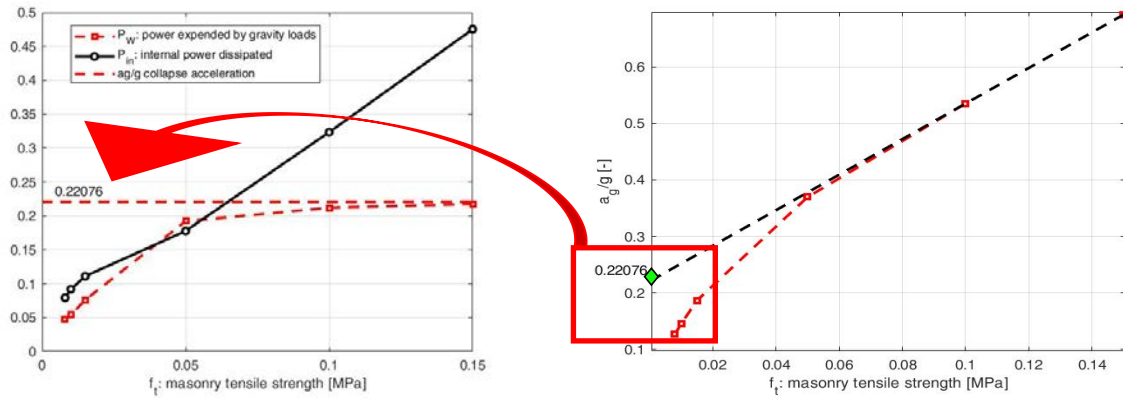


Figure 11. Group 2 – Case 2, collapse mechanisms and collapse accelerations with three different tensile strengths.



#### 4 CONCLUSION

This paper provided a novel limit analysis method that can be applied to complex masonry aggregates and chose two masonry aggregates affected by earthquakes as case studies. The research assumed that the tensile strength of the masonry material decreases gradually from a very high value to 0 (no-tension material), observing the change in the collapse mechanisms of the masonry aggregates.

Studies have shown that when the tensile strength of the material is high, the collapse acceleration is large, and the flexure mechanism is often activated. When the tensile strength of the material is reduced to a small value (close to the no-tension material), the overturning mechanism is most easily seen. At the same time, the cracks generated by the aggregate under the action of earthquakes are more common at the openings, and the blocks around the openings will slide under the action of horizontal force. The internal power dissipated by the masonry material decreases with the decrease of the tensile strength of the material, and the closer it is to the no-tension material, the more the power consumption decreases.

So, the limit analysis method proposed in this thesis can not only successfully applied to the masonry pagoda, which has a large and complex volume but also for the masonry aggregate, which has a large internal power dissipation influence.

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