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LIMIT ANALYSIS MODEL FOR A FAST EVALUATION OF CHINESE MASONRY PAGODAS VULNERABILITY

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Abstract

A masonry pagoda is usually a single independent structure. It has a large volume and complex internal space. In the event of an earthquake, the damage to the masonry material will lead to the collapse of the entire pagoda body, threatening the safety of human life and property. The limit analysis is considered to be the most appropriate technique for analyzing this kind of problem. This paper presents a novel limit analysis method to discretize a masonry pagoda through infinite resistant hexahedrons ad dissipation at the interfaces. It assumes that the interface material obeys a Mohr-Coulomb failure criterion with tension and compression cutoff. Considering the relationship between the jump of velocities and plastic multipliers on all interfaces, the normalization condition of the failure mechanism, boundary conditions, and the objective function to minimize, it uses standard linear programming to estimate the collapse multiplier and failure mechanism active. The problem obtained is self-dual, and it cannot be considered belonging to either the upper or the lower bound limit analysis families. The paper considered a case study – Zhongjiang south pagoda in China. The limit analysis assumes two cohesion conditions (0.05 MPa, 0.20 MPa), and studies the collapse mechanism under G1 and G2 distributions. The direction of the horizontal load applied comprises four angles. With the aim to study how the damage spreads within the structure and demonstrate the accuracy of the approach proposed, several simulations are also performed by a Finite Element-based software (Abaqus) for comparison. The results of the study are in good agreement with other simulation methods and the post-earthquake conditions observed.

Keywords: limit analysis, masonry pagoda, finite element simulation, seismic vulnerability.

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1 INTRODUCTION

Conservation, preservation, and rehabilitation of architectural, cultural, and historical built heritage is a current key issue for our society, especially in seismic-prone areas such as China and Italy [1-3]. Masonry structures are particularly challenging for researchers and engineers who tend to conceive appropriate structural analysis methods, computational tools, and design strategies. How to accurately assess the vulnerability of masonry constructions is an important object of architectural and cultural heritage protection projects. Through different kinds of numerical simulation methods, doing finite element analysis of these ancient masonry constructions may also bring a deep understanding of architectural dynamic characteristics, and have peculiar application value for seismic, reinforcement, and maintenance of structures. Seismic vulnerability assessment of masonry buildings in the historical centers represents a specific and very actual problem to be solved to foresee their behavior under earthquakes and, where deficiencies occur, to implement seismic protection measures.

There are various types of masonry structures around the world with different characteristics. In eastern architecture, taking China as an example, masonry materials are mainly used for two types of building structures. One is the masonry pagoda, which is usually a single building independently, with masonry material as the main load-bearing structure. It has a large volume and complex internal space. In the event of an earthquake, the damage to the masonry material will lead to the collapse of the entire pagoda body, threatening the safety of human life and property. The other type is a historical building with a wooden frame as the load-bearing structure and masonry walls as the enclosed structure. The masonry material in this type of building does not take on the structural role, and even if it collapses under the action of an earthquake, the threat of damage is less than that of the previous type. Therefore, in this paper, the research target is positioned on the more valuable ancient Chinese masonry pagodas.

Finite element analysis is the main technology in the current seismic research of masonry structures. Masonry buildings are usually made of bricks and mortar. So, making a unit model in finite element analysis will be relatively complicated. The key to solving the problem lies in how to deal with the relationship between the two bricks and mortar. There are two modeling methods now, one is a discrete model and the other is a continuous model [4-10].

The bonding relationship between bricks and mortar is the key to constructing the discrete model, but the lack of experimental data greatly reduces the calculation accuracy of the model. In order to make up for this deficiency, the homogeneous unidirectional material model has become the most used analysis method at present. The CDP model, as an isotropic elastic-plastic constitutive model with damage, is the most commonly used masonry damage simulation model in the current simulation environment. CDP modeling is based on the Drucker-Prager yield criterion, which is a continuous, plasticity-based damage model. The two main failure mechanisms assumed are tensile cracking and compression of the material. The model assumes that the uniaxial tensile and compressive responses of the material are characterized by damage plasticity, in which case the stress-strain response follows a linear elastic relationship until failure is reached. The failure stress corresponds to the occurrence of microscopic cracks in the material. Beyond the failure stress, microscopic cracking is formulated macroscopically using a softening stress-strain response, which produces strain localization in the c-material structure. This representation, although simplified, captures the main features of the material response [11-16].

Although the above-mentioned numerical simulation methods have been developed and matured, improving the simulation accuracy of masonry material properties is still the focus of current research. In view of this, the limit analysis of structures, as an important branch of plastic mechanics, becomes the focus of this paper. When the load acting on the structure increases to a certain limit value, the ideal plastic material structure will become a geometrically variable mechanism (see the geometric invariance of the structure), and its deformation will increase without limit so the structure will lose its load ability. This state is called the plastic limit state of the structure, and the load corresponding to this state is called the plastic limit load.

From the early 1950s to the present, people's advancement and development of limit analysis technology have never stagnated. In terms of meshing, Lyamin [17] realized the meshing of the calculation area based on isoparametric mapping technology and completes the "unit sector extension" process according to the requirements of stress transfer. After that, Lyami [18], Ciri [19], and Munuz, et al. [20] successively proposed adaptive meshing methods. Under the condition of the same precision, the adaptive grid technology can greatly reduce the number of elements, speed up the program convergence, and improve the calculation efficiency. In terms of planning models, Zouain and Herskovits introduced nonlinear programming models, which greatly improved the solution efficiency, saved calculation time, and greatly expanded the application range of limit analysis methods. Zouain et al. [21] used a nonlinear programming model satisfying the Kuhn-Tucker condition to optimize the solution based on the nonlinear yield function interior point iteration algorithm, which further improved the limit analysis finite element. Herskovits [22] improved based on the predecessors and proposed to use the feasible arc interior point algorithm to solve the problem. This method has the advantages of high efficiency, rapidity, easy convergence, and stable calculation, and can effectively solve large-scale constrained optimization problems. In terms of homogenization theory, in 1997, Felice first combined the static limit analysis method with the homogenization theory, simplified the ash joint as an interface, and adopted the Mohr-Coulomb criterion to obtain the ultimate load of masonry [23]. Milani et al. established a micromechanical model for the limit analysis of masonry homogenization, assuming that the masonry is in a plane stress state, and obtained the masonry failure surface by using the polynomial expansion method of the two-dimensional stress field. Combined with finite elements, the obtained failure surfaces are applied to the nonlinear analysis of masonry structures [24][25].

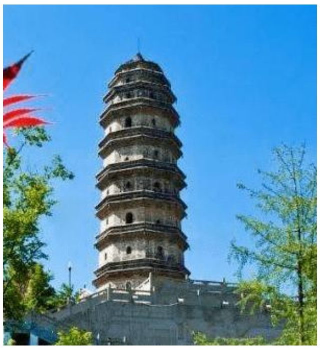
In general, although the research method of limit analysis has been greatly developed, it still has problems such as complex calculation methods, and mismatch between engineering application and theoretical analysis. Therefore, the development of a fast and efficient limit analysis method that can be applied to the vulnerability analysis of complex masonry structures can still make a long-term contribution to the development of the field of human computational mechanics.

The limit analysis method proposed in this paper for assessing the seismic vulnerability of masonry pagodas employs infinite resistance hexahedral elements and rigid-plastic interfaces. A historical Chinese masonry pagoda (Zhongjiang south pagoda) was selected as an experimental case, and combined with numerical simulation analysis in the Abaqus environment, the feasibility of the proposed limit analysis method was verified.

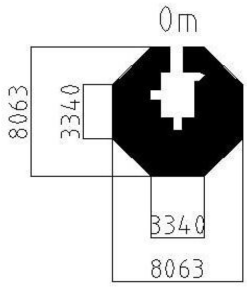
2 ZHONGJIANG SOUTH PAGODA

Zhongjiang south pagoda (Figure 1-a) is located in Zhongjiang City, Sichuan Province, China, about 30 meters above sea level. In 1610 AD, the construction of the pagoda began, and it has been preserved basically intact. The current pagoda is surrounded by farmland and roads. The Zhongjiang south pagoda has an octagonal plane (Figure 1-b), the octagonal side of the bottom plane is about 8 m, and the plane shrinks inward layer by layer. Especially on the upper two floors, the shrinkage is particularly severe. The pagoda’s interior contains 9 floors for boarding, following the traditional shape of Chinese masonry pagodas, and each floor has a central room for human activities. It is worth noting that this central room is not located in the middle of the pagoda body, but in the north of it. Of the 8 facades on each floor, 7 of them have windows and one has a door, and the positions of the doors on each floor are different. The 128-level clockwise staircase connects the inner ventricles. The total height of the Zhongjiang south pagoda is about 30.2 m.

The 2008 Wenchuan Earthquake (magnitude 8.0 on the Richter scale) strongly affected the masonry pagoda, resulting in cracks in the pagoda body, collapsed eaves, internal walls, and ground cracks. As a cultural relic protection unit in Sichuan Province, the Chinese government immediately launched active repair measures for it. Until September 16, 2009, the restoration work of the entire pagoda was completed.



a) On-site photo



b) First plan

Figure 1. Zhongjiang south pagoda.

3 ANALYSIS RESULTS

3.1 Description of the limit analysis model

The authors once proposed a lower bound limit analysis method for limit analysis of two-way bending walls in reference [26]. The research method here is developed based on it. For the sake of brevity, the specific numerical simulation method refers to another paper [27]. It is worth mentioning that this paper uses a dual Mohr-Coulomb failure criterion with tension and compression cutoffs. Assuming a discretization of an infinite resistance hexahedron, the method is based on a discretization of the structure by means of infinitely resistant hexahedron elements and quadrilateral in surfaces where all dissipation occurs, see Figure 2. The linear programming problem to be solved is derived from the kinematic approach, and then derive the formula from the dual. The model uses the element's velocity and rotation rate as raw variables, so it does not fall into either of the upper bound and lower bound methods.

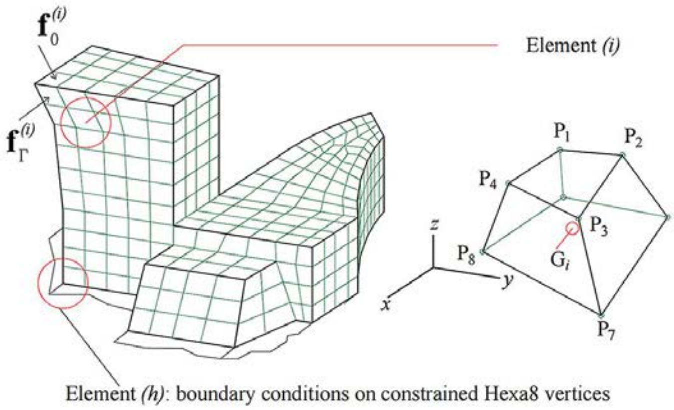


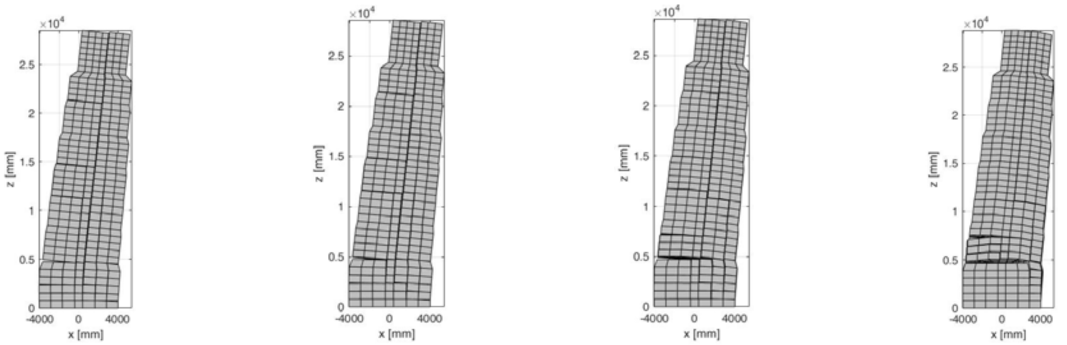
Figure 2. Discretization of an infinite resistance hexahedron for the pagoda.

3.2 Limit analysis method results

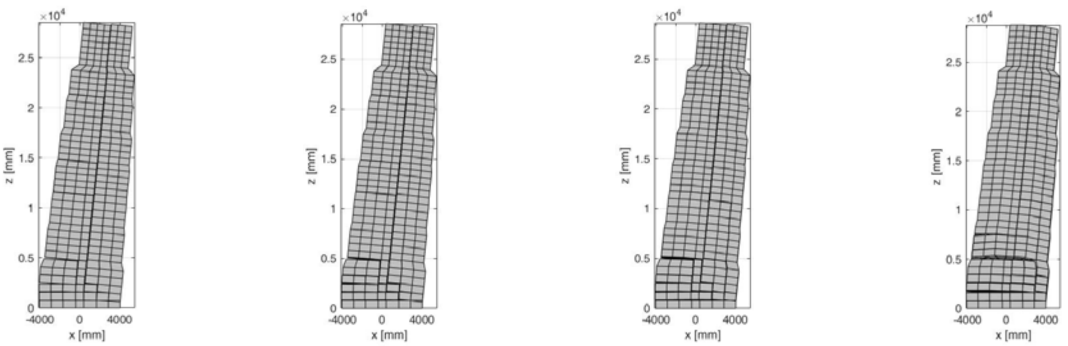
In order to carry out the established vulnerability analysis in the Matlab language environment, it is first necessary to construct a meshed 3D model outside the software. In this paper, a meshed model with a hexahedron as the basic element is constructed in general commercial software (Straus7). The model simplifies the parts that have little influence on the structural analysis, such as stairs and eaves and focuses on the central room and the openings. It contains a total of 7833 interfaces, 2907 elements, and 3776 nodes.

Although the central room is located on north of the pagoda body, the octagonal plan of Zhongjiang south pagoda still has a high degree of symmetry. Here, the research only needs to select four straight line directions at 0°, 15°, 30°, and 45° along one of the major axes of the transverse plane for limit analysis. These analyses can represent the overall seismic vulnerability level of the masonry pagoda. Two different horizontal loads (G1 and G2) are considered along the height direction. A total of two sets of tensile strength properties were tested (c = 0.05 MPa/c = 0.2 MPa), the friction angle was 30°, and the compressive strength was 2.5 MPa. The ultimate analysis results of Zhongjiang south pagoda are shown in Figure 3 – Figure 5respectively.

Direction: 0° Direction: 15° Direction: 30° Direction: 45°



G1-a_g/g = 0.1952 G1-a_g/g = 0.1969 G1-a_g/g = 0.2010 G1-a_g/g = 0.2030



G2-a_g/g = 0.2836 G2-a_g/g = 0.2847 G2-a_g/g = 0.2900 G2-a_g/g = 0.2934

Figure 3. Limit analysis result-1 of Zhongjiang south pagoda with G1 and G2 distribution (c = 0.05 MPa).

Results show that the limit analysis of a masonry pagoda activates a similar collapse mechanism under different horizontal loads. Crack along the central symmetry axis of the pagoda body with a bending deformation in the part of 1 to 2 floors at the bottom. It is obvious that the better the material properties (c = 0.2 MPa), the higher the collapse multiplier gets. The collapse multipliers of the G2 distribution are always greater than that of the G1 distribution. When the applied load is G1, the minimum and maximum values of the collapse multiplier obtained from the limit analysis always appear in the 0° direction and the 45° direction. When the applied load is G2, the maximum value of the collapse multiplier appears in the direction of 30°. In general, there is little difference in the calculated a_g/g values depending on which direction the lateral load is applied from the masonry pagoda.

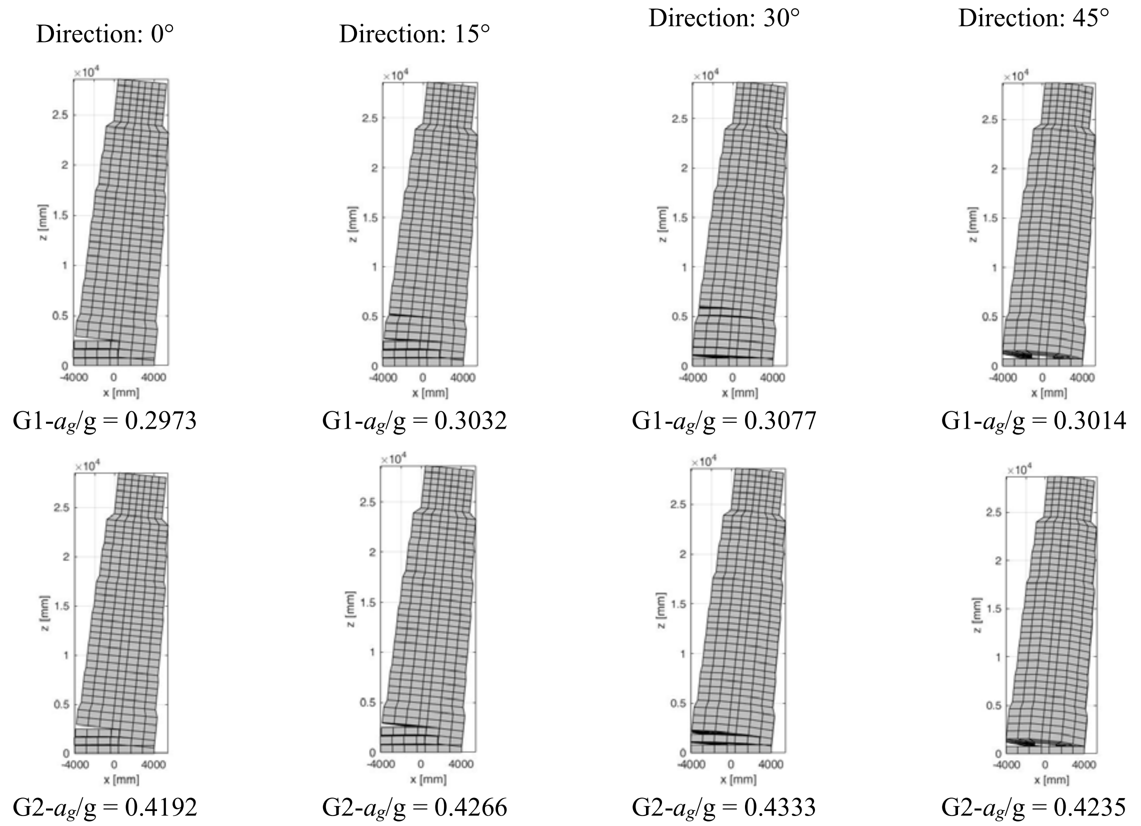
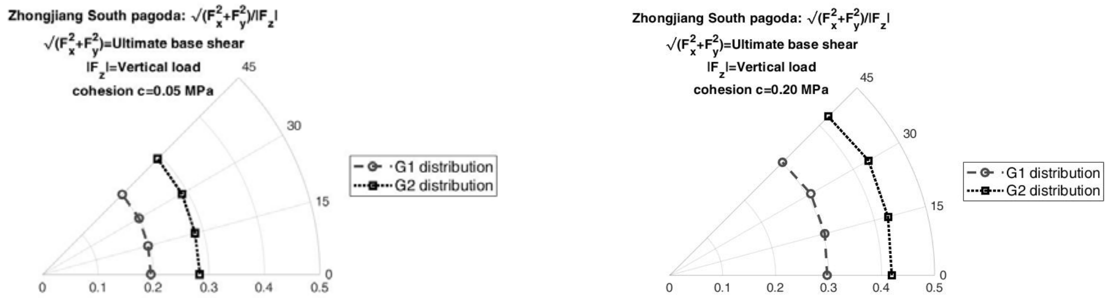


Figure 4. Limit analysis result-1 of Zhongjiang south pagoda with G1 and G2 distribution (c = 0.20 MPa).

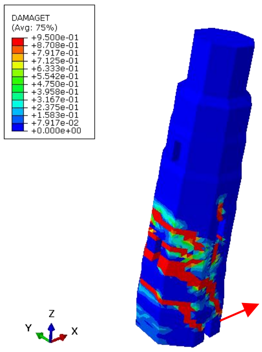


a) Tensile dam b) Tensile dam

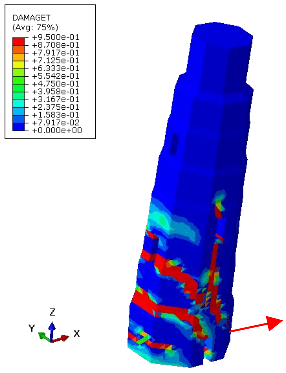
Figure 5. Collapse acceleration diagrams were obtained with the proposed limit analysis for Zhongjiang south pagoda.

3.3 Abaqus analysis results

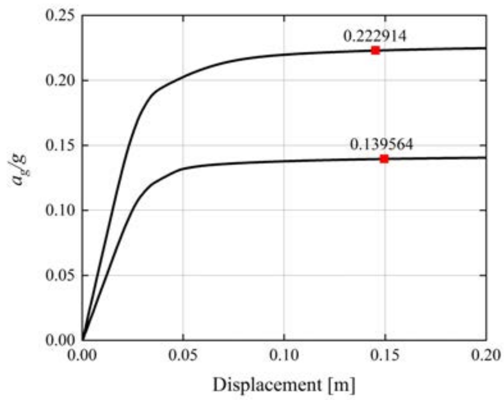
The research also did the pushover analysis along X+ (0°) direction with G1 and G2 loading conditions in Zhongjiang south pagoda. And the capacity curve a_g/g – displacement was shown. The Pushover results (Figure 6) show that under the two horizontal loading conditions, the collapse mechanism of the masonry pagoda is similar, both of which are cracking in the middle and overturning at the bottom. This is completely consistent with the results obtained in the limit analysis.



a) Tensile damage of G1 load distribution



b) Tensile damage of G2 load distribution



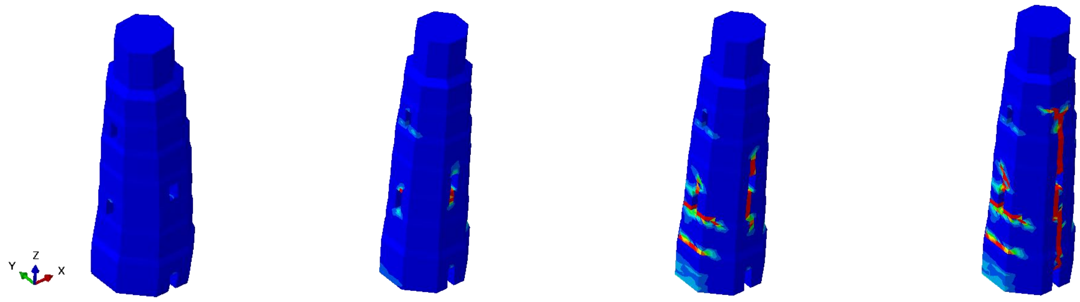
c) Capacity curve

Figure 6. Pushover analysis results of Zhongjiang south pagoda (load direction: X+).

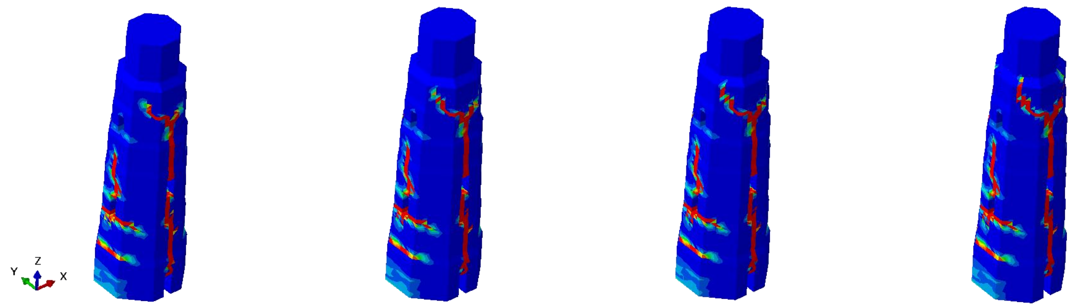
The non-linear dynamic analyses have been performed along the X+ (0°) direction too and using the same seismic wave which has been processed according to Wenchuan seismic wave.

In Figure 7, damage first appeared at the openings on the third floor of the north and south facades in 5.629 s. Subsequently, the crack extends up and down in the vertical direction. After 8.128 s, the crack reaches the sixth floor and develops in a Y-shape towards the east and west directions. After 12.330 s, the vertical crack increased, and the crack shape was basically stable. At the same time, there were overturning damages on the first and second floors of the east and west facades in 6.281 s. And above the third floor, another vertical crack appeared on the two facades. After about 9s, the masonry pagoda can not obey the seismic horizontal loading anymore.

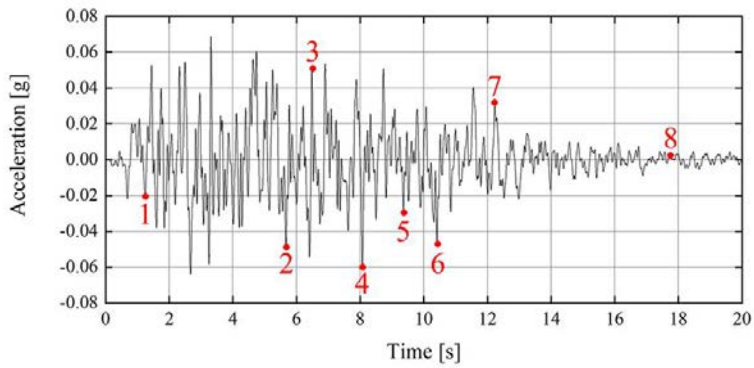
Direction	PGA [g]	Scale Factor [–]	Scaled PGA [g]
X+	0.35	0.2	0.07



Point 1: 1.226 s | Point 2: 5.629 s | Point 3: 6.281 s | Point 4: 8.128 s



Point 5: 9.423 s | Point 6: 10.380 s | Point 7: 12.330 s | Point 8: 17.660 s



Accelerogram

Figure 7. Non-linear dynamic analysis results of Zhongjiang south pagoda by X+ direction.

4 CONCLUSION

This paper proposes a novel limit analysis method for the vulnerability analysis of complex masonry pagodas. Discretize the structure as a hexahedron with rigid interfaces. The paper chooses Zhongjiang south pagoda as a research case. And through the comparison with the nonlinear static and dynamic analysis, the reliability of the method is verified.

The limit analysis assumes two cohesion conditions (0.05 MPa, 0.20 MPa), and studies the collapse mechanism of masonry pagodas under G1, and G2 load distributions. The direction of horizontal load application is carried out from four different angles.

In general, masonry pagodas are affected by horizontal loads and a vertical crack will appear, activating the ultimate bending at the bottom. Due to the symmetry of the plane, the collapse mechanism caused by the application of horizontal loads in different directions also has a certain degree of symmetry. The weak links of the pagodas are mainly reflected in the ventricle and the stairs connecting up and down. Cracks on the surface are common in the openings.

In general, the method can successfully solve the global damage simulation problem of masonry pagodas with large volumes and complex structures under seismic action. The results of the study are in good agreement with other simulation methods and the post-earthquake conditions of a real masonry pagoda.

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