

OPERATIONAL MODAL ANALYSIS OF NON-REDUNDANT WOOD FRAME BUILDING

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Abstract

For the design and assessment of structures, precise dynamic characteristics are a must. A vast array of literature regarding dynamic characteristics of RC and steel structures estimated using experimental and numerical analyses can be found in either published research as well as codal provisions. On the other hand, similar studies for structures constructed using uncommon materials are limited. This adds uncertainties in the design and assessment of structures as experimental studies prior to construction or assessment are rather unpragmatic. We address a unique issue that sheds light on the identification of dynamic characteristics of a 12.8 m tall non redundant wood frame building. The building has two bays along the longer direction and one bay along the shorter direction. The columns in the buildings are made up of Sal timber (hardwood). We conducted ambient vibration measurements in the building near the geometrical center and performed system identification using the numerical algorithm for subspace state space system identification (N4SID) approach. Using operational modal analysis, the first mode frequencies and damping ratios of the building are estimated for the shorter (X) as well as the longer (Y) directions. The fundamental vibration frequency of the building along the X direction is found to be 1.16 Hz with a damping ratio of 3.02%. Along the Y direction, the fundamental vibration frequency was obtained as 1.26 Hz with a damping ratio of 2.84%. We have compared the results from system identification with several codal formulas used to estimate fundamental period/frequency to provide a rational basis for the selection of period-height or frequency-height relations. The results could serve as a notable reference in the formulation of guidelines that are likely to be developed in the future.

Keywords: Operational modal analysis, System identification, Ambient vibration, N4SID, Wood frame building.

1 INTRODUCTION

Wood frame construction system comprises the second largest building population in Nepal after stone masonry [1]. Until a few decades ago, the abundance of lumber in the vicinity attracted people to construct wood frame buildings when reinforced concrete (RC) construction system had not gained enough momentum. Until 2011, wood frame buildings were nearly 2.5 times more than RC buildings in Nepal. Wood frame buildings are mostly found in the southern plains and Siwalik regions of Nepal. A wide variation in terms of timber species exists in Nepal and so does the wood frame construction system because such buildings are constructed using locally available timber with little to no processing. Although wood frames are one of the most dominant construction systems, studies incorporating dynamic identification, seismic and other natural hazard related vulnerabilities, and mechanical characterization are not common yet. Neither do exist any specific codal provisions for wood frame buildings so far. The building codes so far designate wood frame constructions as ‘other structures’, which incorporate all building forms except for RC and steel frame constructions. This ambiguity is particularly posing a challenge in the design and assessment of several types of construction systems including wood frame buildings. Thus, there lies a scope for the identification of dynamic characteristics to quantify the behavior of wood frame construction under dynamic actions such as earthquakes, wind, floods, etc. Also, the design of such structures can be more precise if dynamic characteristics are estimated with adequate confidence so as to use equivalent static methods.

Wood frame buildings and several sub-forms of wood frame buildings have been studied for quite some time worldwide. Some studies used ambient vibration and other dynamic measurement techniques to estimate dynamic characteristics such as frequency and damping ratio [2–5]. Some other studies performed numerical and experimental analyses to quantify the behavior of various forms of wood frame buildings (e. g. [6]). A realistic depiction of dynamic characteristics of any construction system will be the first step regarding design and assessment or even to assess health in the long run. Ambient vibration testing is one of the efficient and economic techniques to acquire the time domain response of the structure. The use of ambient vibration-based system identification covers dynamic characterization under ambient as well as excited conditions (e. g. [7–12]). In this paper, we assess the time series data collected from a wood frame building to estimate dynamic characteristics using operational modal analysis. Based on the system identification results, we also compare codal provisions and recommend a reliable fundamental period estimation formula for future use.

2 MATERIALS AND METHODS

2.1 Wood frame buildings

Wood frame buildings comprise 24.9% of the total building stock (~5.5 million) in Nepal and more than 90% of such constructions are in the southern plains and Siwalik regions. Virtually all such buildings are constructed using lumber directly providing connections at the floor level to establish floor diaphragm. A vast majority of such constructions use Sal timber (*Shorea robusta*) hardwood, meanwhile, newer constructions also use other timber species as Sal timber is not adequately available these days. Wood frame buildings are mostly two storied constructions with an open ground story, which is mostly used for non-residential purposes. The infills range from *ikra* (overlapped folded bamboo sheets) to wooden planks, whereas the flooring system comprises wooden joists and planks. The long wall direction will have a major wooden joist, which is sometimes supported by wooden pillars. The joists along the short wall direction are either throughout or sometimes slightly more than the half length

of the short wall direction and thus two joists are required along the short wall direction. In the case of the unavailability of throughout lumber for the joist along the long wall direction, lumber is joined, and post support is provided underneath.

Ambient vibration measurement was taken in a four-story wood frame building as shown in Fig. 1. Corrugated galvanized iron (CGI) sheets are used for roofing, and the rest of the components are wood elements except for nails. In this type of construction, holes are dug first, and wooden pillars are erected. Meanwhile, the ground floor is sometimes plastered later. The case study building is a non-redundant wood frame building along the shorter direction that corresponds to the X direction of the instrument.



Figure 2: Case study building (Image by: Prasanna Poudel, with permission).

2.2 Ambient vibration measurement

We took ambient vibration records in the case study building using three ETNA-2 triaxial accelerometers manufactured by Kinemetric Inc. One accelerometer was set on each story. The instruments were aligned vertically placing all of them at the tentative geometrical center of the building (Fig. 2). Several sets of 30-minute vibration records were taken at 100 Hz sampling frequency. Three GPS devices were also connected to the accelerometers to synchronize time series data. Vibration measurements were taken in the operational state of the building with the occupants and regular activities continued. However, no other excitation existed during the recording.



Figure 2: Accelerometer set up for ambient vibration recording.

2.3 Operational modal analysis

Operational modal analysis (OMA) comprises the development of mathematical models using input-output measurements of a structure, which is based on parametric identification of dynamic characteristics using autoregressive (AR), moving average (MA), autoregressive moving average (ARMA), autoregressive with extra input (ARX), among other models [13]. These models are developed using time domain modeling of the recorded response data [14]. We adopted an output-only modal identification technique to determine dynamic characteristics of the building using ambient vibration records that do not require input information. Two orthogonal components referred to as X and Y direction hereafter, of the ambient vibration records, were selected for system identification. Transients were corrected manually, and the signals were filtered using a fourth-order Butterworth filter of 20 Hz corner frequency. Then, the signal was tapered using Tukey window and finally, the preprocessed signals were processed using the Numerical Algorithm for Subspace State Space System Identification (N4SID) [15]. The algorithm constructs state matrices so that system identification to estimate frequency, damping ratio, and mode shape can be conducted [15, 16]. State space models

were developed for each direction and oblique projection of subspaces from the block Hankel matrices were estimated for the output response using N4SID. When considering q input measurements ($u_k \in \mathbb{R}^m$) and output $y_k \in \mathbb{R}^l$ from the unknown deterministic system of order n , the discrete system equations can be written as:

$$x_{k+1} = Ax_k + Bu_k \quad (1)$$

$$y_k = Cx_k + Du_k \quad (2)$$

Here, the aim is to determine n and realization of A , B , C , and D for the similarity transformation $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{l \times n}$, and $D \in \mathbb{R}^{l \times m}$. Using Hankel block, the following matrix equations can be derived from Eq. (1) and Eq. (2):

$$X_f = A^i X_p + \Delta_i U_p \quad (3)$$

$$Y_p = \Gamma_i X_p + H_i U_p \quad (4)$$

$$Y_f = \Gamma_i X_f + H_i U_f \quad (5)$$

Considering the input of order $2i$, intersection occurs between the row space of matrix U_f and one of matrix X_p , and the weighting matrix $W_1 \in \mathbb{R}^{li \times li}$ is of full rank and $W_2 \in \mathbb{R}^{j \times j}$ holds true for:

$$\text{rank}(Z_p) = \text{rank}(Z_p W_2) \quad (6)$$

If the oblique projection be O_i ,

$$O_i = Y_{f/U_f} Z_p \quad (7)$$

$$W_1 O_i W_2 = [U_1 \quad U_2] \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \quad (8)$$

$$W_1 O_i W_2 = U_1 S_1 V_1^T \quad (9)$$

$$O_i = \Gamma_i X_f \quad (10)$$

W_1 and W_2 are the weighting matrices that rectify the $\Gamma_i X_f$ estimate. The number of singular values except for zero denote the order of the model, which is given by the matrix S_1 . The extended observability matrix Γ_i becomes:

$$\Gamma_i = W_1^{-1} U_1 S_1^{1/2} T \quad (11)$$

$$X_f W_2 = T^{-1} S_1^{1/2} V_1^T \quad (12)$$

$$X_f = \Gamma_i^\dagger O_i \quad (13)$$

Where non-singular similarity transformation matrix can be written as $T \in \mathbb{R}^{n \times n}$. The matrix $Z_p = (Y_p, U_f)^T$ is formulated from the Hankel matrices $(Y_p, Y_f)^T$ and $(U_p, U_f)^T$ using Eq. (7) and Eq. (9). Thereafter, the singular value decomposition of $W_1 O_i W_2$ is performed. The order is determined as n when S_1 is identified as the diagonal matrix composed of n singular values. A and C can be estimated once $\Gamma_i = (C \ CA \ \dots \ CA^{i-1})^T$ is evaluated. C is obtained from the first l rows of Γ_i and A is obtained from the shift structure of Γ_i as follows:

$$\underline{\Gamma}_i A = \overline{\Gamma}_i \quad (14)$$

$\underline{\Gamma}_i$ denotes that the matrix Γ_i that does not include the last l rows:

$$\underline{\Gamma}_i = (C \ CA \ \dots \ CA^{i-2}) \quad (15)$$

Similarly, $\overline{\Gamma}_i$ denotes that the matrix Γ_i that does not include the first l rows:

$$\overline{\Gamma}_i = (CA \ CA^2 \ \dots \ CA^{i-1}) \quad (16)$$

Thus, A is given as:

$$A = \underline{\Gamma}_i^\dagger \overline{\Gamma}_i \quad (17)$$

Pre-multiplying Eq. (5) by $\Gamma_i^\perp U_f^\dagger$, it yields:

$$\Gamma_i^\perp Y_f U_f^\dagger = \Gamma_i^\perp \Gamma_i X_f U_f^\dagger + \Gamma_i^\perp H_i U_f U_f^\dagger \quad (18)$$

Since $\Gamma_i^\perp \Gamma_i = 0$, Eq. (18) deduces to:

$$\Gamma_i^\perp Y_f U_f^\dagger = \Gamma_i^\perp H_i \quad (19)$$

Denoting Γ_i^\perp by L and $\Gamma_i^\perp Y_f U_f^\dagger$ by M , Eq. (19) becomes:

$$M = LH_i \quad (20)$$

$L = (L_1 \ L_1 \ \dots \ L_i)$ and $M = (M_1 \ M_1 \ \dots \ M_i)$. Thereafter, B and D are estimated using linear regression. We executed the algorithm in MATLAB R2022b [17].

3 RESULTS AND DISCUSSIONS

We first assessed signal health by visually selecting and removing the transients. Fig. 3 shows the signal after removing transients manually. Both trend and seasonality affect the

characteristics of the system at various times, thus the major tasks of system identification are detrending and stationarity check. We confirmed the stationarity of the time series records plotting the cumulative power build-up of the signal. A uniform slope of the power spectral density function (Fig. 4) indicates considerable stationarity.

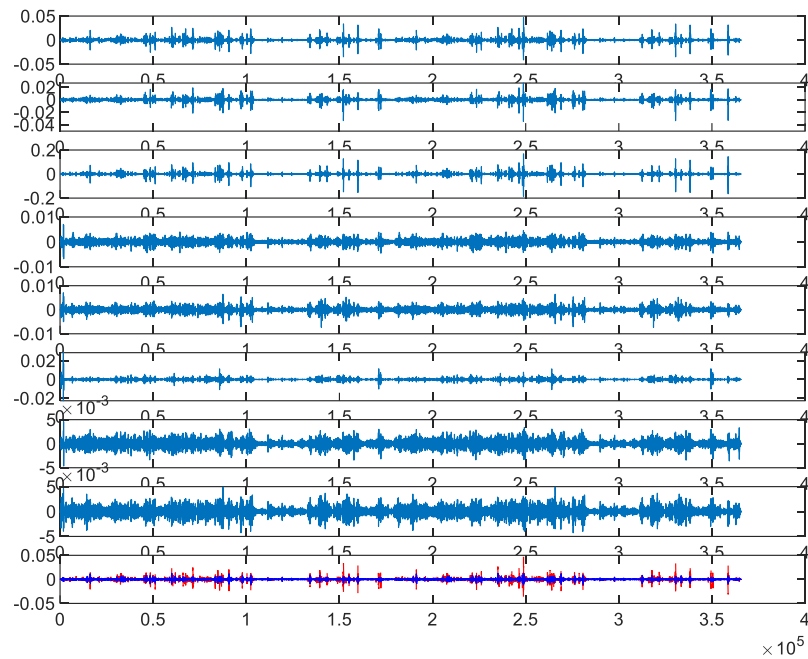


Figure 3: Synchronized time series record after removing transients.

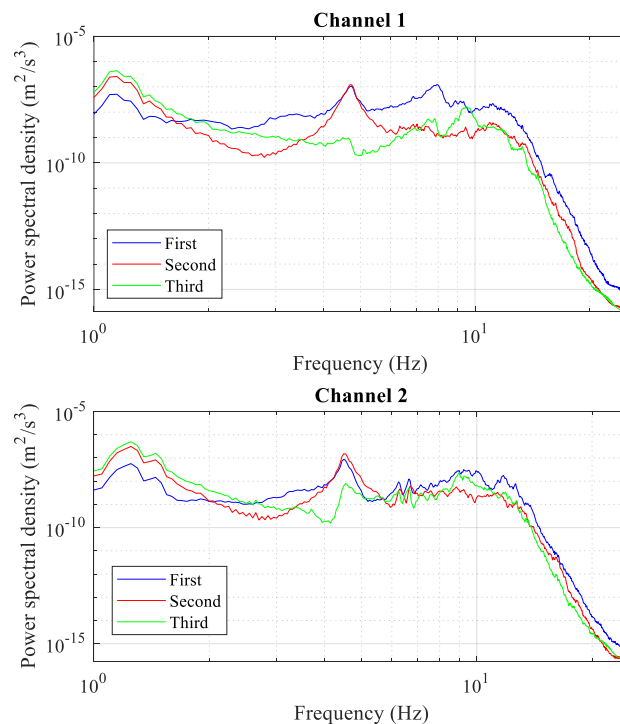


Figure 4: Power spectral density plots for two orthogonal directions X and Y.

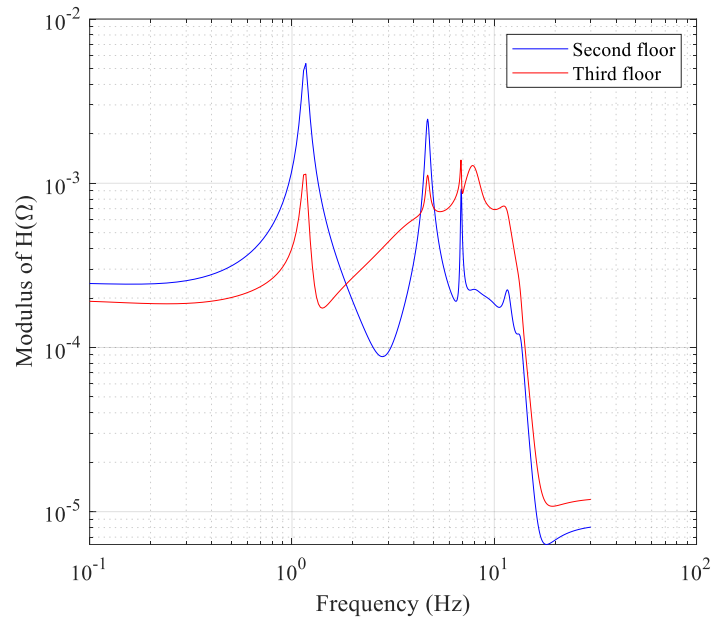


Figure 5: Frequency response function plot for second and third story.

The frequency response function, which indicates the Fourier transform of the ratio of time domain response to input that reflects dynamic flexibility, is shown in Fig. 5. We constructed two state space models for X and Y directions independently. For the unknown input excitation, the response was considered for the second and the third story. At least three translational modes are identified with confidence as shown in Fig. 5. The summary of the identified dynamic characteristics of the building is presented in Table 1. As shown in Table 1, the first translational mode of the building is in the X direction. For the first translational mode of vibration, frequencies along the X and Y directions are identified as 1.16 Hz and 1.26 Hz, respectively. Similarly, damping ratios are determined as 3.02% and 2.84%, respectively along the X and Y directions. Similarly, for the second translational mode, frequencies are identified as 4.68 Hz and 4.29 Hz, respectively for the X and Y directions. For the second mode, damping ratios are estimated as 2.27% and 5.24%, respectively along the X and Y directions.

Mode	Frequency (Hz)		Damping ratio (%)	
	X	Y	X	Y
1	1.16	1.26	3.02	2.84
2	4.68	4.29	2.27	5.24
3	6.86	4.49	0.66	0.69

Table 1: Summary of dynamic characteristics of the building.

The past and current building codes in Nepal do not incorporate any provisions regarding the design of wood frame buildings; however, any building form except for reinforced concrete and steel frame buildings are classified under the same category. For other structures category, the fundamental period of vibration (T) recommended by the Nepal Building Code-1994 (NBC 1994) [18] is as follows:

$$T = \frac{0.09H}{\sqrt{D'}} \quad (21)$$

Where H is the height of the building and D' is the overall length of the building at the base in the direction under consideration. The recently formulated building code in Nepal (NBC 2020) [19] recommends a more conservative fundamental period of vibration formula that does not account for the length of the building as:

$$T = 0.05H^{3/4} \quad (22)$$

As neither of these regulations considers wood frame buildings, we assessed codal formulas from several building codes to demarcate the most representative empirical formula that could be used for wood frame buildings in Nepal. Based on the comparison between NBC (1994) [18], NBC (2020) [19], Indian Standard (IS) Code-1893 (2002) [20], Indian Standard (IS) Code-1893 (2016) [21], National Building Code of Canada (NBCC 2015) [22], Uniform Building Code (UBC 1997) [23], Camelo (2003) [5], and Eurocode 1-4 (2010) [24] as shown in Fig. 6, we conclude that the frequency estimate of similar wood frame buildings would be best represented by the NBCC (2015). Similarly, the earlier version of the Nepali building code (NBC-1994) is more representative than the recent NBC-2020 for wood frame buildings as shown in Fig. 6. Thus, the NBCC (2015) can be used to design or assess similar wood frame buildings in Nepal.

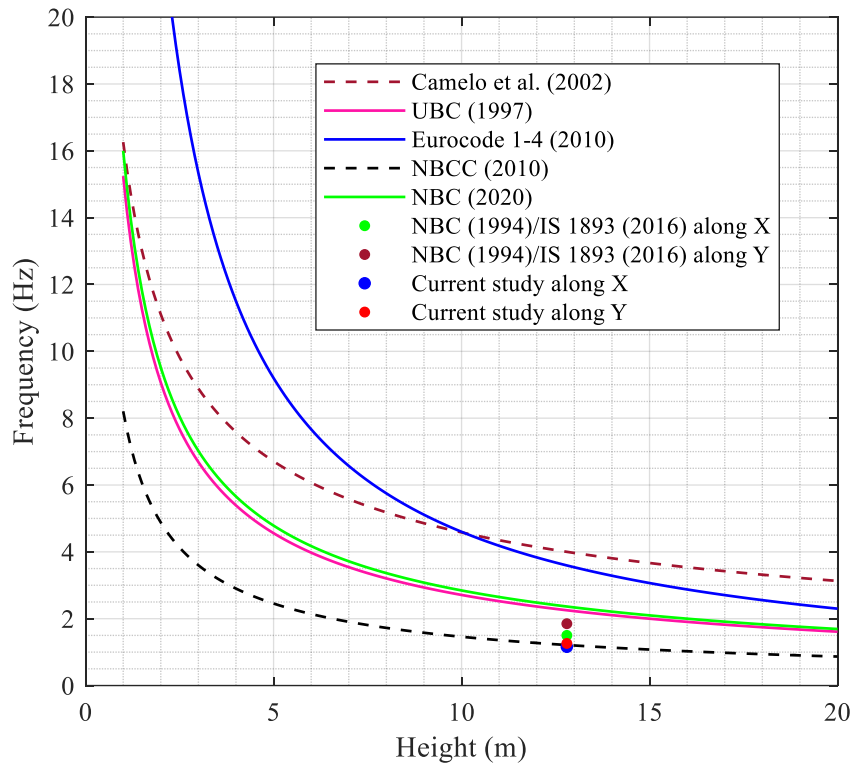


Figure 6: Frequency-height plots based on various building regulations together with the experimentally identified frequency characteristics of the case study building.

4 CONCLUSIONS

Using operational modal analysis, we estimated vibration frequencies and damping ratios for some translational modes of vibration of a four-story wood frame building. We used N4SID algorithm to estimate the dynamic characteristics based on ambient vibration records on the first, second, and third stories of the building. The fundamental period of vibration, which is one of the major design/assessment parameters, is estimated as 0.86 sec. The estimated fundamental period of vibration depicts that wood frame buildings possess a higher vibration period than other construction systems such as reinforced concrete and masonry buildings of the same height. In one of the first attempts to characterize wood frame constructions in Nepal, we conclude that the damping ratio of wood frame buildings is around 3%. In comparison with the current and past building regulations, we conclude that the recently updated building code recommendation strays more than the preceding one. Thus, we recommend using the past building code, NBC-1994, to estimate the fundamental period of vibration for wood frame buildings when required. We also compared several building code recommendations and found that the NBCC (2015) recommendation would be the most representative to be used in the estimation of the fundamental period of vibration for similar buildings. This study uses single building measurement, future studies can conduct ambient vibration measurement in many sample buildings of the same class and develop frequency-height relation, which could be more representative to be used instead of using the formula recommended for undefined building category. We are also testing other wood frame buildings and we plan to develop a refined finite element model for wood frame construction to assess seismic performance.

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