

## **OPTIMAL DESIGN OF A TUNED MASS DAMPER INERTER USING A WHALE OPTIMIZATION ALGORITHM FOR THE CONTROL OF BUILDINGS SUBJECTED TO GROUND ACCELERATIONS**

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### **Abstract**

*Control devices, also known as protection systems, have been proposed to reduce undesirable vibrations, dissipate the input energy, and preserve the integrity of structures subjected to ground motion. The Tuned Mass Damper Inerter (TMDI) is a passive control device that integrates a classical Tuned Mass Damper (TMD) with an inerter mechanism that induces inertial resisting forces to the controlled system and consequently provides a mass equivalent amplification effect, also known as inertance, with the advantage that it demands less attached mass to achieve suitable vibration control of structures subjected to dynamic loads. This paper presents the implementation and optimal design of a TMDI on a mid-rise building in which the structural model is simplified as a two-dimensional shear frame subjected to different ground motion-induced vibrations, intending to reduce the lateral displacements of the structure. Moreover, for the tuning process of the device, the design parameters have been optimized using a natural-based metaheuristic algorithm known as the whale optimization algorithm (WOA). The objective function to be optimized is a linear combination of the maximum peak displacement and the root mean square (RMS) response for the displacement. The obtained results show remarkable reductions in the dynamic response of the structure, thus demonstrating the effectiveness of TMDI for structural control and damage reduction.*

**Keywords:** Structural control, tuned mass damper inerter, Whale Optimization Algorithm, ground motion-induced vibrations.

## 1 INTRODUCTION

In recent years, there has been a significant focus on developing and researching control devices implemented in structures. The purpose of these devices is to minimize damage and dissipate energy caused by dynamic environmental loads in civil structures. These control methods can be classified into four groups: passive control, active control, semi-active control, and hybrid control [1-3].

Passive devices are designed to modify the stiffness or the structural damping of a system without requiring an external power source to develop forces that counteract the motion of the structure. These devices present an attractive alternative to other types of controllers due to their mechanical simplicity, lower cost, ease of operation, and maintenance [4, 5].

The Tuned Mass Damper (TMD) is a passive system proposed by Frahm in 1909 [6] to attenuate vibrations in single-degree-of-freedom (SDOF) systems subjected to harmonic motion. Subsequent applications [7-9] have demonstrated that the device is capable of preventing damage, discomfort, and structural failure by employing inertial forces to dissipate energy and suppress undesired vibrations during seismic events. The TMD incorporates a secondary mass, a damper, and a spring linked to the structure in some strategic points, to improve the dynamic response [10]. TMDs have proven efficiency in damage reduction when are optimally tuned to one of the fundamental frequencies of the structure to be protected [11, 12]. These devices operate by transferring vibration energy from the main structure to the secondary mass, which then dissipates the energy by vibrating out of phase with the structural motion [4], resulting in a reduction of the motion amplitude.

Considering the limitations of TMDs in achieving optimal seismic performance, the Tuned Mass Damper Inerter (TMDI) was proposed by Marian and Giaralis [13]. This device consists of a classical linear TMD attached to the structure with the spring-damper system and linked to the floor via an inerter [14]. The inerter, which is a mechanical two-node device, induces an inertial force of resistance that is proportional to the relative acceleration of the two nodes, resulting in a mass-equivalent amplification effect, also known as inertance [15]. The addition of the inerter in the TMDs offers a lightweight passive vibration alternative that significantly improves the damping properties of the device. Numerous investigations have been carried out to optimally design the TMDIs. For instance, Brzeski *et al.* [16] investigated a TMDI featuring a specialized inerter that allows for precise inertance modifications, thus facilitating a wider range of applications. Pietrosanti *et al.* [17] proposed a methodology for achieving optimal TMDI performance in multi-degree-of-freedom (MDOF) structures, considering two TMDI configurations: grounded and ungrounded. Additionally, Lara-Valencia *et al.* [18] explored the optimal tuning and design of TMDIs employing various performance indices to minimize structural response.

One of the most effective methods for tuning control devices is through the use of meta-heuristic optimization algorithms because of their simplicity, flexibility, and local optima avoidance [19]. To obtain the optimal design parameters that ensure the best performance of the TMDI it will be used the Whale Optimization Algorithm (WOA) [20] which mimics the behavior of humpback whales in nature and the bubble-net hunting strategy, first encircling the prey, and then creating bubble nets for the final attack.

The current study presents a procedure for the design of TMDIs located on the upper storey of a mid-rise building subject to seismic excitations. The tuning process aims to minimize the dynamic response of the structure using a linear combination of two parameters which are the maximum peak displacement and the root mean square (RMS) value for displacements. The design parameters are the critical damping ratio,  $\zeta_{TMDI}$ , and frequency ratio,  $\nu_{TMDI}$ , which are computed via WOA. Finally, the seismic performance is evaluated considering the attached mass of 5%, and inertance added of

5% with respect to the total mass of the structure. The present study demonstrates that the TMDI can achieve substantial reductions in the structural response, covering key parameters such as peak displacements, RMS value of displacement, peak acceleration, RMS value of acceleration, and interstorey drift. This improved performance can be attributed to the inerter device, which amplifies the mass and stiffness of the structure, thereby enhancing its behavior while preserving its stability and functionality.

## 2 TUNED MASS DAMPER INERTER MODEL

### 2.1 TMDI configuration for a multi-degree-of-freedom (MDOF) structure subjected to ground acceleration

To obtain a mathematical model that describes the performance of the TMDI, it is considered a linear damped multi-degree-of-freedom (MDOF) dynamical system as shown in Figure 1, which is a two-dimensional frame modeled as a shear beam building with  $n$  degrees of freedom associated to the horizontal displacements on each storey and subjected to a ground acceleration  $\ddot{u}_g(t)$ . The structure is supplied with a TMDI attached to the last two storeys. Considering the structure shown in Figure 1, the properties of each storey  $i$  ( $i = 1, 2, \dots, n$ ) are represented with a spring of stiffness constant  $k_i$ , viscous damper with damping constant  $c_i$ , and concentrated mass  $m_i$ . Moreover, the TMDI is modeled as the combination of a classical TMD located in the upper level of the frame and an inerter that connects the conventional TMD and the level  $n - 1$  of the structure. The properties of the idealized device are  $m_{TMDI}$ ,  $k_{TMDI}$ ,  $c_{TMDI}$ , and  $b$  that refers to the mass, stiffness, damping, and inertance coefficients respectively.

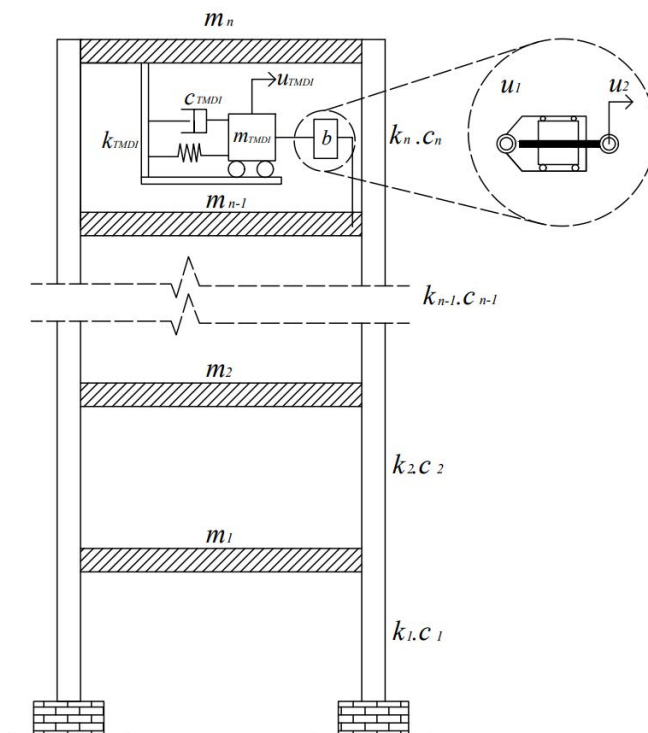


Figure 1: MDOF system integrated with TMDI.

## 2.2 The inerter device

Several analogies between electrical and mechanical systems have led to the development of the inerter [14] which is an element linked to two nodes and free to move independently as shown in Figure 2. This device has a small mass in comparison to the structure and has the property that the force that it generates is proportional to the relative acceleration between its nodes [21]. Its proportionality constant,  $b$ , is the inertance that characterizes the behavior of the device and is measured in terms of mass, furthermore,  $b$  can be two hundred or more orders of magnitude greater than the real inerter mass. [15]. The resisting force produced by an inerter is given by the equation (1) as follows:

$$F = b(\ddot{u}_j - \ddot{u}_i) \quad (1)$$

Where  $\ddot{u}_i - \ddot{u}_j$  represents the relative acceleration of the nodes  $i$  and  $j$  of the device.

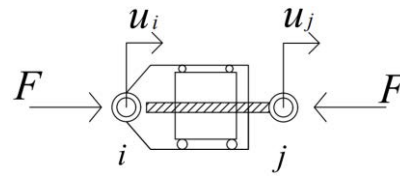


Figure 2: Inerter device.

## 2.3 Governing equations of motion for MDOF structure equipped with a TMDI

The  $n + 1$  governing equations of motion that describe the behavior of the structure showed in Figure 1 subjected to ground acceleration, can be modeled in matrix form with the equation (2):

$$\mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{C}\dot{\mathbf{U}}(t) + \mathbf{K}\mathbf{U}(t) = -\mathbf{M}\{\mathbf{1}\}\ddot{u}_g(t) \quad (2)$$

Where  $\mathbf{U}(t)$ ,  $\dot{\mathbf{U}}(t)$  and  $\ddot{\mathbf{U}}(t)$  are respectively the relative displacement, velocity, and acceleration system vectors as a function of time with dimensions  $(n + 1) \times 1$ . On the other hand,  $\ddot{u}_g(t)$  is a scalar representing the induced ground acceleration which also depends on the time, and  $\{\mathbf{1}\}$  is a unitary vector of order  $(n + 1) \times 1$ .

The matrices  $\mathbf{M}$ ,  $\mathbf{K}$ , and  $\mathbf{C}$  with dimensions  $(n + 1) \times (n + 1)$  respectively associated with the mass, stiffness, and damping of the structure, are proposed to include the effect of the TMDI. The mentioned matrices can be calculated with the expressions (3 – 5).

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & m_2 & 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & m_3 & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & m_4 & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & m_{n-1} + b & 0 & -b \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 & m_n & 0 \\ 0 & 0 & \dots & \dots & \dots & -b & 0 & m_{TMDI} + b \end{bmatrix} \quad (3)$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & \dots & \dots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 & \dots & \dots & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & 0 & \dots & \dots & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 & -k_5 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & k_{n-1} & -k_n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & -k_n & k_n & -k_{TMDI} \\ 0 & 0 & \dots & \dots & \dots & 0 & -k_{TMDI} & k_{TMDI} \end{bmatrix} \quad (4)$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 & \dots & \dots & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 & 0 & \dots & \dots & 0 \\ 0 & -c_3 & c_3 + c_4 & -c_4 & 0 & \dots & \dots & 0 \\ 0 & 0 & -c_4 & c_4 + c_5 & -c_5 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & c_{n-1} & -c_n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & -c_n & c_n & -c_{TMDI} \\ 0 & 0 & \dots & \dots & \dots & 0 & -c_{TMDI} & c_{TMDI} \end{bmatrix} \quad (5)$$

Further, the natural frequency and the critical damping ratio of the TMDI can be expressed as:

$$\omega_{TMDI} = \sqrt{\frac{k_{TMDI}}{m_{TMDI} + b}} \quad (6)$$

$$\zeta_{TMDI} = \frac{c_{TMDI}}{2(m_{TMDI} + b)\omega_{TMDI}} \quad (7)$$

It is convenient for the analysis and design of systems equipped with TMDIs to consider the mass ratio ( $\mu$ ), inertance ratio ( $\beta$ ), and frequency ratio ( $\nu$ ):

$$\mu = \frac{m_{TMDI}}{M_s} \quad (8)$$

$$\beta = \frac{b}{M_s} \quad (9)$$

$$\nu = \frac{\omega_{TMDI}}{\omega_s} = \frac{\sqrt{\frac{k_{TMDI}}{m_{TMDI} + b}}}{\omega_s} \quad (10)$$

Where  $M_s$  is the total mass of the structure and  $\omega_s$  will be assumed as the angular frequency of the structural first mode of vibration.

### 3 TMDI OPTIMIZATION METHODOLOGY

#### 3.1 Parameters to be optimized

The optimization process is based on finding the optimal values of  $k_{TMDI}$  and  $c_{TMDI}$ , equivalent to the dimensionless parameters  $\nu$  and  $\zeta_{TMDI}$  calculated with equations (7) and (10). The objective

of the tuning process is to optimize the variables under consideration, in such a way as to minimize the response of the structure under the induced ground acceleration.

The domain of the search is defined between:

$$0.50 \leq v \leq 2.00 \quad (11)$$

$$0.10 \leq \zeta_{\text{TMDI}} \leq 0.50 \quad (12)$$

The limits mentioned above were established following recommendations found in the literature and common practice mainly in TMD devices, which have been further investigated [22, 23].

### 3.2 Objective function

The objective function ( $OF$ ) proposed in the present research seeks to minimize the dynamic response of the structure when it is subjected to seismic excitations, this is measured using the minimum values of two response parameters: the maximum peak displacements  $U_{i\text{peak}}$ , and the RMS response of the displacements  $RMS(U_i)$ . The final  $OF$  is set up as a linear combination, and the weight of each term is established according to the desired reduction for each parameter.

$$OF = 0.6 \frac{\max(|U_{i\text{peak}}(\text{controlled})|)}{\max(|U_{i\text{peak}}(\text{uncontrolled})|)} + 0.4 \frac{\max(|RMS(U_{i(\text{controlled})})|)}{\max(|RMS(U_{i(\text{uncontrolled})})|)} \quad (13)$$

### 3.3 Whale optimization algorithm (WOA)

The Whale Optimization Algorithm (WOA) is a nature-inspired meta-heuristic algorithm proposed by Mirjalili and Lewis in 2016 [20]. They were inspired by the social behavior of humpback whales and their special hunting method known as the bubble-net feeding strategy. In recent investigations, other researchers have used this algorithm for the optimization of different engineering problems for example Kaveh and Ghazaan [24, 25] studied the sizing optimization of skeletal structures, Azizi *et al.* [26] upgraded the WOA for fuzzy logic-based vibration control of nonlinear steel structures, and Huang M *et al.* [27] developed a damage identification method based on the WOA.

For the application of the optimization process, the authors' first strategy is to define a population of whales which is the equivalent of a set of random solutions where each solution is thought to be a whale that tries to occupy a new place in the searching space, taking as a reference the best whale of the group. Two strategies are used for the hunting process of the prey (exploration) and the attack (exploitation).

#### 3.3.1 Bubble-net attacking method (exploitation phase)

Two approaches are used for the mathematical model that simulates the bubble-net behavior of humpback whales, the first one is encircling and the other one is the bubble-net creation via spiral updating. After the selection of the initial best search agent of the set of random solutions, the whales update their position for either a random agent or the best solution obtained at the moment.

The hunting mechanism of humpback whales is a combination of swimming around the prey within a shrinking circle and along a spiral-shaped path [29], thus, the probability of using one of the two mentioned mechanisms is the same.

### 3.3.2 Search for prey (exploration phase)

In this phase, the whale position is upgraded based on a randomly selected search agent and not on the best solution obtained so far, to make it possible, the current whale can search far away from the best solution and the WOA can achieve a global search avoiding local optima [29]. Figure 3 shows the flowchart of the WOA with the steps that this method requires.

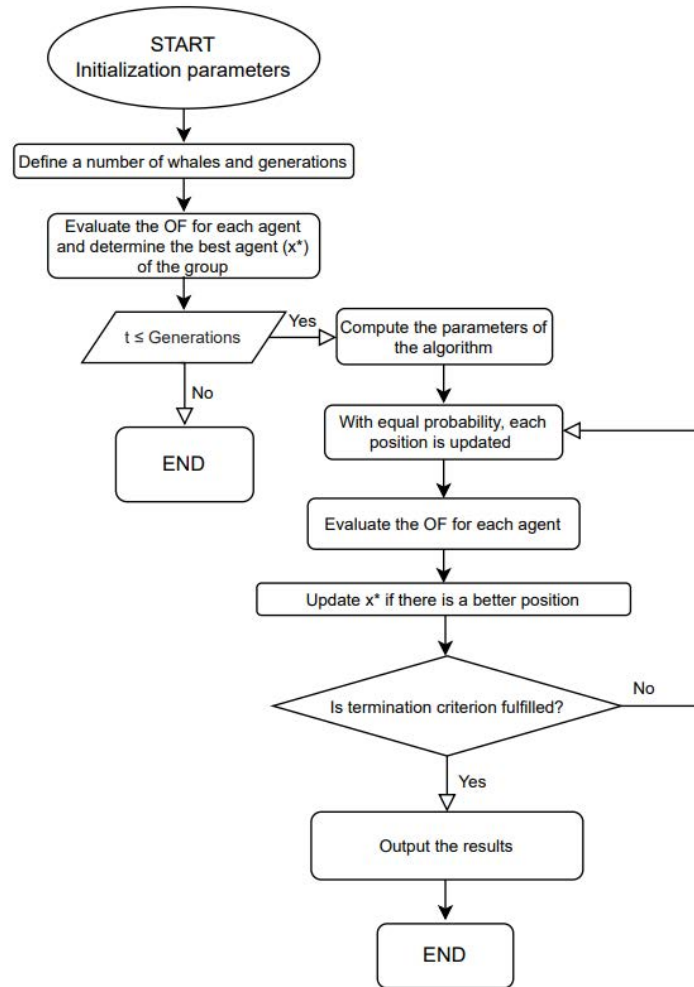


Figure 3: Flow chart of the WOA.

## 4 NUMERICAL EXAMPLE

The suggested methodology is applied to a 12-storey reinforced concrete (RC) residential building in Medellín, Colombia. The structure has a total height of 30 m, and the lateral force-resisting system of the structure consists of resistant moment frames combined with structural walls. Figure 4 shows an elevation view of the building and the selected frame for the study.

The mass matrix  $\mathbf{M}$  and stiffness matrix  $\mathbf{K}$ , have dimensions of  $12 \times 12$  considering only the horizontal degrees of freedom of the structure (one at each level), this is derived by assuming rigid floor diaphragms and applying static condensation of the rotational degrees of freedom and

elimination of the vertical degrees of freedom. On the other hand, the resulting damping matrix  $\mathbf{C}$  of dimensions  $12 \times 12$  was calculated using Rayleigh's method for 5% damping in the first and second vibration modes. The calculation of the modal analysis was performed with Matlab [32] and the results are shown in Table 1.

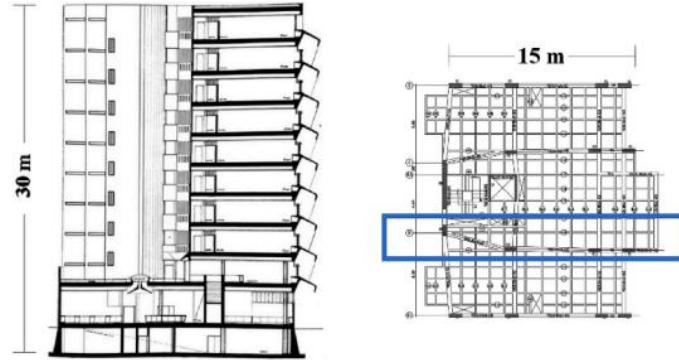


Figure 4: Building and frame used for the study.

Mode	1	2	3	4	5	6	7	8	9	10	11	12
T [s]	1.68	0.51	0.27	0.17	0.11	0.08	0.06	0.05	0.04	0.03	0.03	0.03
F [Hz]	0.59	1.96	3.69	5.85	8.70	12.11	16.06	20.63	25.52	29.46	31.89	34.94
$\omega$ [rad/s]	3.73	12.35	23.19	36.76	54.65	76.09	100.93	129.60	160.37	185.11	200.39	219.56

Table 1: Modal properties of the 2D frame.

The seismic-induced accelerations are represented by four ground motion records with different dynamic characteristics between them. The accelerograms were taken from the Pacific Earthquake Engineering Research (PEER) Centre database [30] and the Center for engineering strong-motion data [31], the information related to these events is summarized in Table 2.

Seismic record	Event name	Year	Station	Component	Magnitude	PGA [g]	Duration
1	Mexico	1985	La Unión	S00E	8.1	0.169	62.90
2	Chile	1985	Melipilla	0	7.8	0.686	79.36
3	Petrolia	1992	Cape Mendocino, CA	90	7.0	1.039	59.98
4	Christchurch	2011	Christchurch Resthaven	S88E	6.3	0.710	81.90

Table 2: Acceleration records.

The purpose of the numerical example is to test and prove the effectiveness of the WOA in determining the best parameters for the design of TMDIs, which improve the performance of structures subjected to ground motion, this is achieved using actual records of accelerograms as input seismic excitations and only elastic analysis were carried out.



## 5 RESULTS

The design parameters of the TMDI are obtained through the optimization of equation (13) via WOA. The objective function aims to minimize the structural response, specifically in terms of displacements. This is why it is formulated as a linear combination of peak displacements and RMS value of displacements, rather than utilizing other variables such as accelerations or interstorey drifts to optimize the dynamic response. This section presents a comparison between the controlled and uncontrolled structural response of a mid-rise building subjected to ground motion to evaluate the effectiveness of the TMDI in enhancing the dynamic behavior of the system. The analysis is conducted in the linear range, where the displacements are assumed to be part of the elastic behavior of the structure. As a result, plastic deformations and any potential damage are not taken into consideration. The primary goal of the controller device is to keep the structural response within the linear range. If that is not possible, the controller device aims to minimize the displacements in the non-linear range to prevent structural damage.

### 5.1 Uncontrolled response

Table 3 summarizes the response of the last four storeys of the uncontrolled frame subjected to the four different seismic records.

Seismic record	Storey	Peak displacement [m]	RMS of the displacement [m]	Peak acceleration [m/s <sup>2</sup> ]	RMS of the acceleration [m/s <sup>2</sup> ]	Peak interstorey drift [m]
1	9	0.1118	0.0285	4.8654	0.7895	0.0168
	10	0.1206	0.0315	3.7083	0.7715	0.0165
	11	0.1311	0.0341	5.6530	1.0354	0.0150
	12	0.1430	0.0363	8.5393	1.4855	0.0127
2	9	0.1346	0.0305	10.3952	1.7635	0.0307
	10	0.1496	0.0339	10.4644	1.5027	0.0347
	11	0.1640	0.0372	14.4649	2.1241	0.0338
	12	0.1767	0.0403	24.6003	3.4506	0.0286
3	9	0.1956	0.0420	12.4658	1.2861	0.0264
	10	0.2134	0.0462	14.2340	1.0170	0.0257
	11	0.2299	0.0497	14.9989	1.3822	0.0250
	12	0.2500	0.0528	17.6687	2.2355	0.0218
4	9	0.5600	0.0916	15.8605	1.6193	0.0868
	10	0.6322	0.1008	17.9111	1.8869	0.0776
	11	0.6956	0.1087	20.4108	2.3096	0.0657
	12	0.7490	0.1154	22.9834	2.8371	0.0547

Table 3: Dynamic response of the last four storeys of the uncontrolled structure under the seismic records.

According to the numerical results presented in Table 3, the uncontrolled frame exhibited a maximum peak displacement of 0.75 m in the 12<sup>th</sup> storey during the "Christchurch" earthquake, which is 5.24 times higher than the peak displacement observed in the same storey during the

"Mexico" earthquake. Furthermore, the 12<sup>th</sup> storey also had the highest RMS value for displacement during the "Christchurch" earthquake with a value of 0.12 m. The maximum peak acceleration and the maximum RMS value for acceleration occurred under the "Chile" seismic record in the top storey of the building, with values of 24.60 m/s<sup>2</sup> and 3.45 m/s<sup>2</sup>, respectively.

The maximum displacement of 0.75 m may be enough for the structure to reach the inelastic range. However, the focus of this study is solely on the linear behavior of the structure, as stated earlier. Additionally, it is crucial to acknowledge that the building was modeled as a 2D shear frame, and if the total stiffness of the structure were considered, the displacements of both the controlled and uncontrolled structures would likely be reduced.

## 5.2 TMDI optimization process

For the calibration of the algorithm, values of 5% are set for both mass ratio ( $\mu$ ) and inertance ratio ( $\beta$ ), since recent investigations [2, 18] have shown that the results obtained with these values achieve a significant response reduction. Besides, considering that one of the advantages of implementing a TMDI is to reduce the amount of mass used on a classic TMD, it is reasonable to use this percentage which allows the device to be physically viable. Consequently, the values of  $m_{TMDI}$  and  $b$  are 54.20 Mg.

During the execution of the WOA, different combinations of whales and generations were tested, including 30, 50, 100, and 300 whales and 20, 30, 50, and 100 generations. The results showed that using a higher number of whales led to more precise optimization of the TMDI design parameters, as it helped to avoid the limits of the searching space and local optima. Moreover, it was found that the controlled response could be significantly reduced by using a combination of 100 whales and 50 generations. Table 4 presents the optimized critical damping ratio ( $\zeta_{TMDI}$ ) and frequency ratio ( $\nu$ ) founded using the WOA for each seismic record.

Seismic record	Number of whales	Generation	$\zeta_{TMDI}$	$\nu$
1	100	50	0.2683	0.7690
2	100	50	0.2396	1.1286
3	100	50	0.1368	0.7794
4	100	50	0.1000	1.0872

Table 4: WOA combinations and design parameters of the TMDI for each seismic record.

## 5.3 TMDI-controlled response

Table 5 presents the response of the last four storeys of the controlled structure subjected to each earthquake. First, the maximum peak displacement and the maximum RMS value of the displacement were obtained on the last storey under the "Christchurch" seismic record with values of 0.5094 m and 0.0799 m respectively, showing reductions of 31.99% and 30.78% for each response parameter. Then, the maximum peak acceleration and the maximum RMS value of the acceleration occurs on the 12<sup>th</sup> storey of the structure when the structure was subjected to the "Chile" earthquake, decreasing from 24.60 m/s<sup>2</sup> to 18.05 m/s<sup>2</sup> and from 3.45 m/s<sup>2</sup> to 2.68 m/s<sup>2</sup>, these decreases represent reduction percentages of 26.62% and 22.24% respectively. Finally, the maximum interstorey drift was observed in the 9<sup>th</sup> storey of the structure under the "Christchurch"

earthquake, with a value of 0.0635 m and a reduction of 26.86%. However, the structure exhibits its highest reduction in the interstorey drift on the 10<sup>th</sup> storey under the “Chile” earthquake, with a value of 30.15%.

Seismic record	Storey	Peak controlled displacement [m]	Controlled RMS of the displacement [m]	Peak controlled acceleration [m/s <sup>2</sup> ]	Controlled RMS of the acceleration [m/s <sup>2</sup> ]	Controlled peak interstorey drift [m]
1	9	0.0854	0.0209	4.8141	0.6932	0.0163
	10	0.0973	0.0233	3.5152	0.6602	0.0160
	11	0.1096	0.0254	4.7987	0.9064	0.0145
	12	0.1200	0.0273	8.1623	1.3042	0.0119
2	9	0.0904	0.0230	10.1056	1.5612	0.0229
	10	0.1027	0.0257	9.3688	1.2544	0.0243
	11	0.1131	0.0285	10.3111	1.6458	0.0242
	12	0.1223	0.0311	18.0521	2.6832	0.0202
3	9	0.1984	0.0316	11.8323	1.1061	0.0248
	10	0.2126	0.0348	13.4717	0.8679	0.0269
	11	0.2344	0.0376	14.2771	1.1084	0.0268
	12	0.2567	0.0400	16.6028	1.8020	0.0231
4	9	0.4088	0.0611	15.9374	1.1962	0.0635
	10	0.4453	0.0681	15.6906	1.3648	0.0572
	11	0.4780	0.0743	16.1852	1.7247	0.0481
	12	0.5094	0.0799	17.4830	2.2064	0.0389

Table 5: Dynamic response of the last four storeys of the TMDI-controlled structure under the seismic records.

Figure 5 to Figure 8 presents simultaneously the controlled response with the blue line, and the uncontrolled response with the orange one. The structural response is plotted in terms of peak displacements in the 12<sup>th</sup> storey, RMS values of the displacement per storey, peak accelerations in the 12<sup>th</sup> storey, RMS values of the acceleration per storey, and peak interstorey drift for the structure subjected to the four ground motions. The figures offer a clear representation of the dynamic behavior of the system.

Table 5 shows the results for the “Petrolia” earthquake, indicating that the controlled maximum peak displacements increase for the last four storeys in comparison to the uncontrolled results. This increase occurs during the first peak of the earthquake at 2.98 seconds, which has a relatively high PGA value of 1.039 g and is outside the trend observed in this seismic record as it is shown in Figure 7. If the algorithm attempts to estimate parameters for the TMDI that reduces this peak, it may not result in proper behavior for the rest of the time, as it is presented in Figure 7. Therefore, the parameters are established to achieve an optimal overall response. Taking into consideration that earthquakes are an unpredictable phenomenon, this type of condition can easily occur. Additionally, due to the increase in displacements, the maximum peak interstorey drifts are not

successfully reduced. However, for future research, it may be possible to optimize the objective function by including the peak interstorey drift as a parameter of the linear combination.

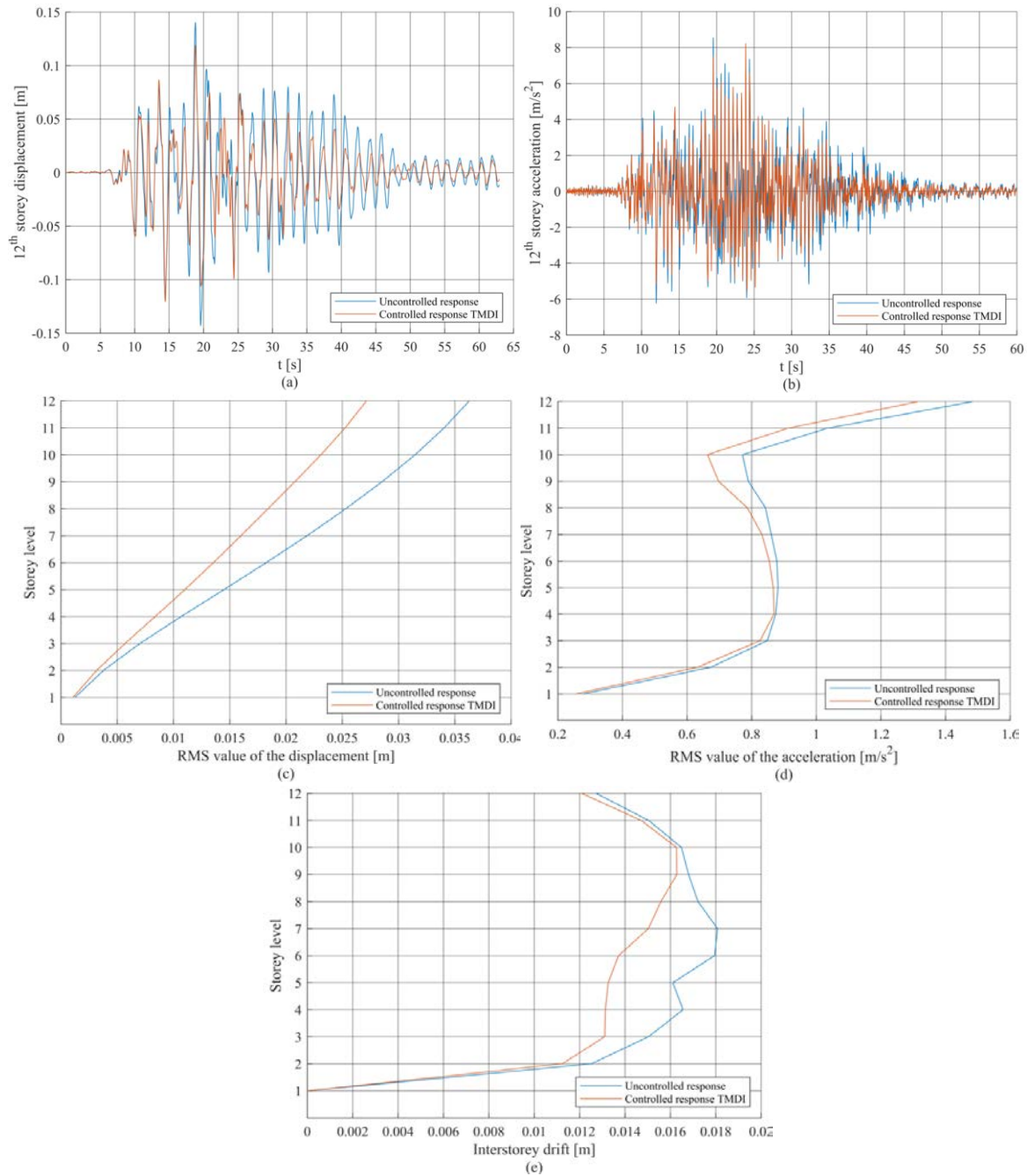


Figure 5: Response of the structure subjected to “Mexico” Seismic record. (a) 12<sup>th</sup>-storey maximum displacements. (b) 12<sup>th</sup>-storey maximum accelerations. (c) RMS value of the displacements per storey. (d) RMS value of the accelerations per storey. (e) Interstorey drift.

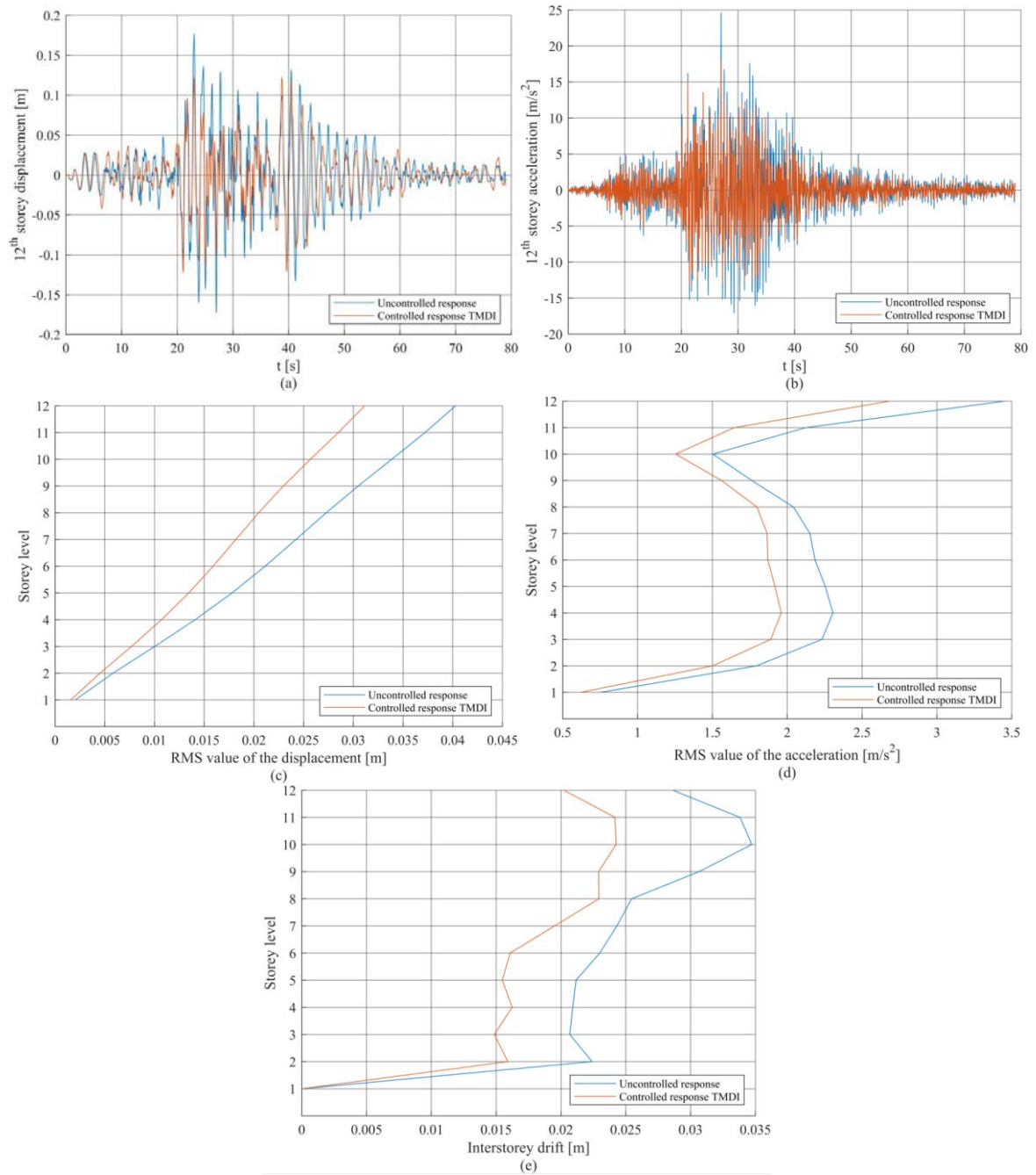


Figure 6: Response of the structure subjected to “Chile” Seismic record. (a) 12<sup>th</sup>-storey maximum displacements. (b) 12<sup>th</sup>-storey maximum accelerations. (c) RMS value of the displacements per storey. (d) RMS value of the accelerations per storey. (e) Interstorey drift.

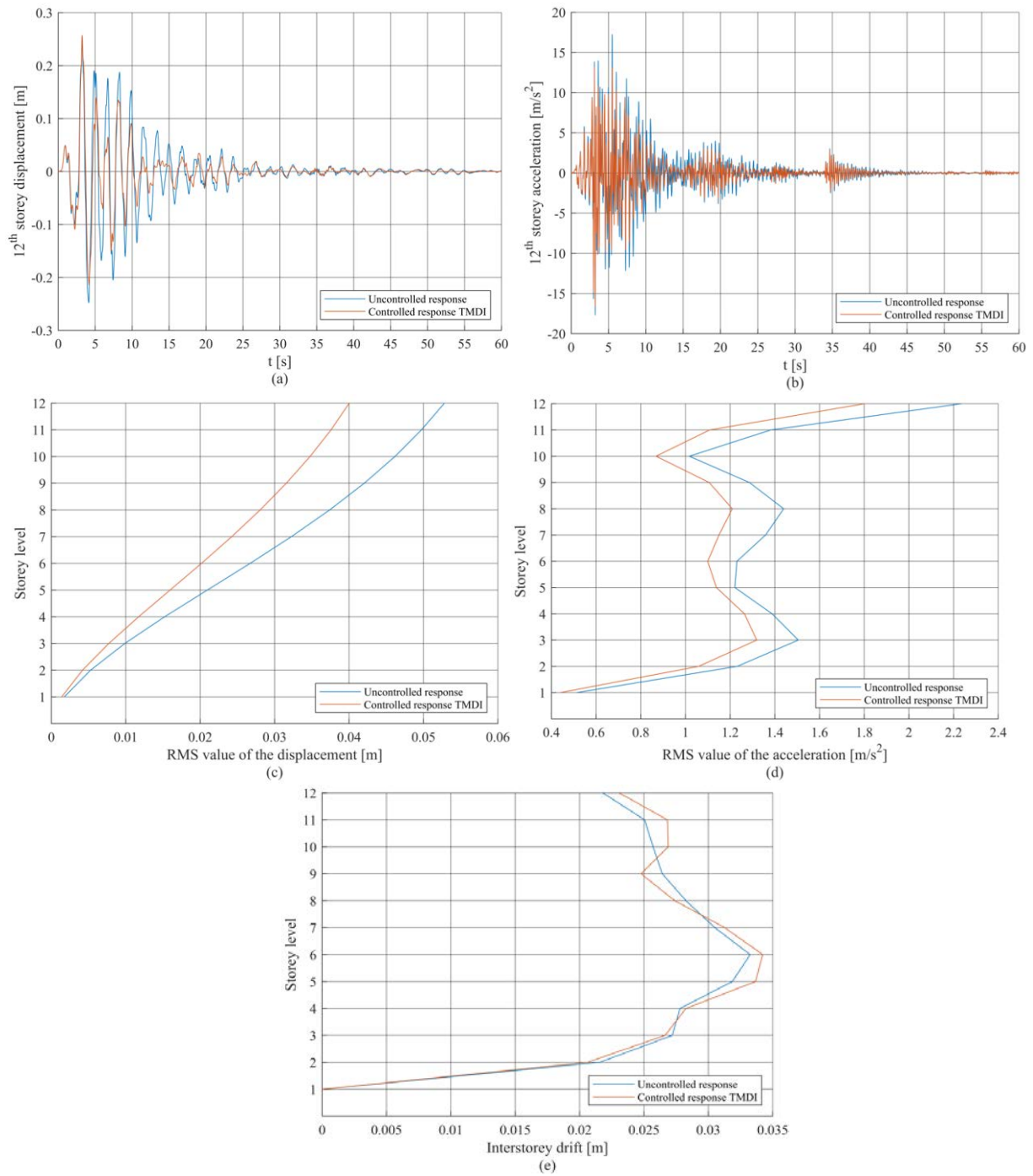


Figure 7: Response of the structure subjected to “Petrolia” Seismic record. (a) 12<sup>th</sup>-storey maximum displacements. (b) 12<sup>th</sup>-storey maximum accelerations. (c) RMS value of the displacements per storey. (d) RMS value of the accelerations per storey. (e) Interstorey drift.

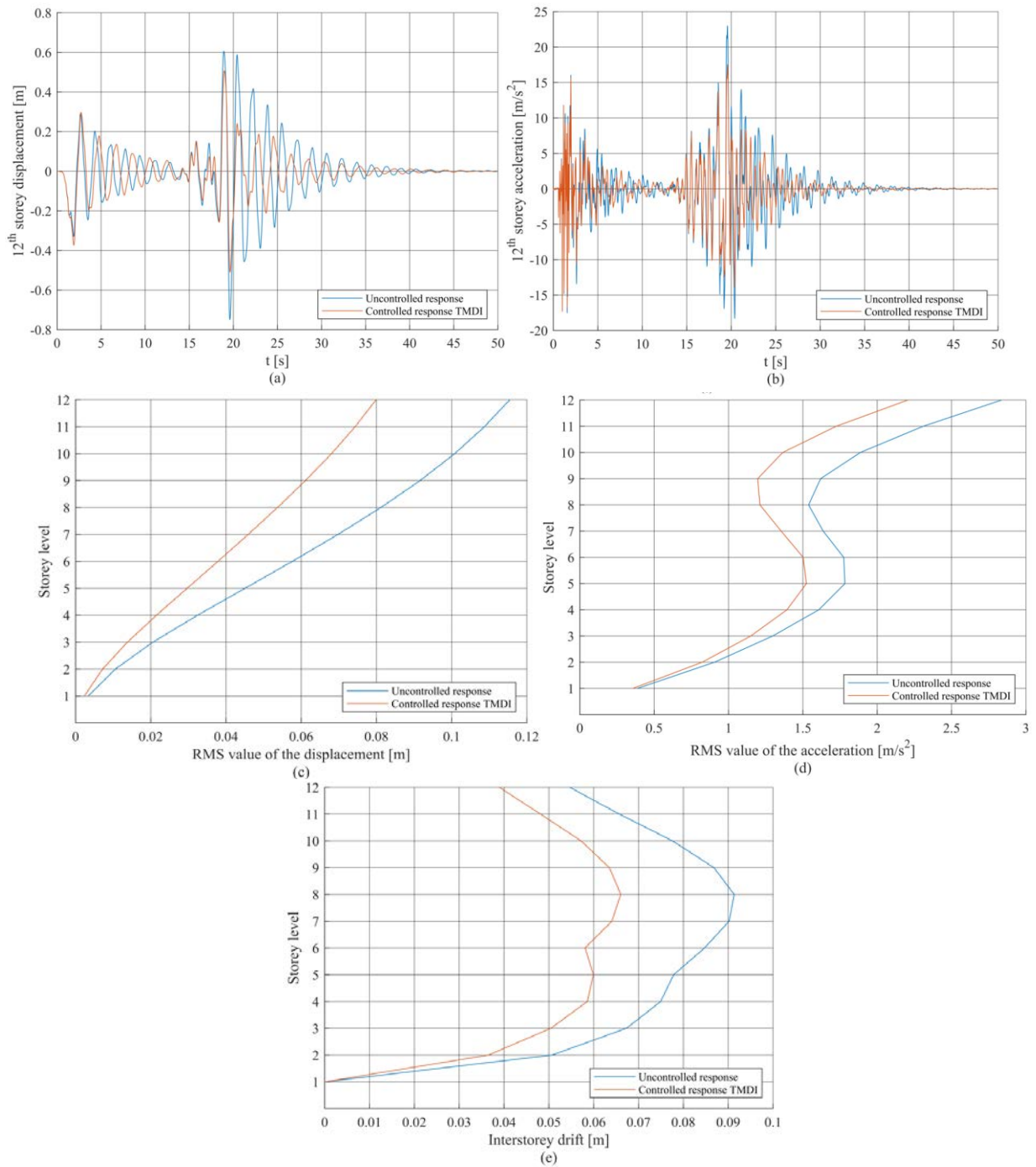


Figure 8: Response of the structure subjected to “Christchurch” Seismic record. (a) 12<sup>th</sup>-storey maximum displacements. (b) 12<sup>th</sup>-storey maximum accelerations. (c) RMS value of the displacements per storey. (d) RMS value of the accelerations per storey. (e) Interstorey drift.

## 6 CONCLUSIONS

- This study demonstrates the effectiveness of the TMDI device for the structural control and reduction of the dynamical response of a 12-storey building subjected to four different seismic records. The parameters of the devices were optimized via WOA, with the objective function proposed as a linear combination that aims to reduce both peak displacements and the RMS value of displacements. The maximum reduction percentages obtained for these parameters are 31.99% and 26.60% respectively proving the effectiveness of the objective function in the optimization process. Besides, reductions for peak accelerations, RMS value for acceleration, and peak interstorey drift are achieved for most of the analyzed earthquakes. The success of the controller in regulating the behavior of the structure is evident from the reduction achieved in its response.
- The TMDI is a modified version of the classical TMD that includes an inerter device. This addition induces a mass amplification effect that enables the device to dissipate the energy caused by an earthquake instead of the structure. TMDI is an attractive alternative for improving structural behavior while preserving its stability and functionality, thus, the use of this control device can reduce the risk of collapse under extreme seismic conditions.
- The WOA algorithm is a novel meta-heuristic technique that mimics the behavior of humpback whales in nature and showed to be a compelling alternative among the other state-of-art meta-heuristic methods available. Notably, the WOA is distinguished by its simplicity, efficiency, and fast-converging properties, making it a robust and versatile optimization algorithm that has shown to be highly effective in finding the optimal parameters of the TMDI that ensures to improve in the dynamic response. To guarantee optimal outcomes in the control of the structure is important to optimize both dynamic response and computational power, therefore the optimization process focused on obtaining the maximum reduction of the response and enhancing computational cost.

## REFERENCES

- [1] J. Y. Xu, J. Tang, and Q. S. Li, "Semi-active control devices in structural control implementation," *Structural Design of Tall and Special Buildings*, vol. 14, no. 2, pp. 165–174, 2005, doi: 10.1002/tal.270.
- [2] P. Cristian and L. Septimiu, "Structural Control Systems Implemented in Civil Engineering," no. January, 2005.
- [3] D. Caicedo, L. Lara-Valencia, J. Blandon, and C. Graciano, "Seismic response of high-rise buildings through metaheuristic-based optimization using tuned mass dampers and tuned mass dampers inerter," *Journal of Building Engineering*, vol. 34, no. September, 2021, doi: 10.1016/j.jobbe.2020.101927.
- [4] D. Pietrosanti, "Optimal design and performance evaluation of systems with Tuned Mass Damper Inerter ( TMDI )," no. January, pp. 1367–1388, 2017, doi: 10.1002/eqe.2861.
- [5] M. C. Constantinou, T. T. Soong, and G. F. Dargush, *Design and Retrofit*. 1986.
- [6] Hermann Frahm, "Device for damping vibrations of bodies," 1911.



- [7] C. Lee, Y. Chen, L. Chung, and Y. Wang, "Optimal design theories and applications of tuned mass dampers," vol. 28, pp. 43–53, 2006, doi: 10.1016/j.engstruct.2005.06.023.
- [8] F. Rahimi, R. Aghayari, and B. Samali, "Application of Tuned Mass Dampers for Structural Vibration Control : A State-of-the-art Review," vol. 6, no. 8, pp. 1622–1651, 2020.
- [9] T. E. Saaed, G. Nikolakopoulos, J. E. Jonasson, and H. Hedlund, "A state-of-the-art review of structural control systems," *JVC/Journal of Vibration and Control*, vol. 21, no. 5, pp. 919–937, 2015, doi: 10.1177/1077546313478294.
- [10] G. P. Cimellaro and S. Marasco, *Introduction to Dynamics of Structures and Earthquake Engineering*, vol. 45. 2018. [Online]. Available: [https://doi.org/10.1007/978-3-319-72541-3\\_20%0Ahttp://link.springer.com/10.1007/978-3-319-72541-3](https://doi.org/10.1007/978-3-319-72541-3_20%0Ahttp://link.springer.com/10.1007/978-3-319-72541-3)
- [11] John R. Sladek and Richard E. Klingner, "EFFECT OF TUNED-MASS DAMPERS ON SEISMIC RESPONSE By John R. Sladek 1 and Richard E. Klingner, 2 M. ASCE," vol. 109, no. 8, pp. 2004–2009, 2009.
- [12] D. Ambrosini, "Single and multiple TMD optimization to control seismic response of nonlinear structures," vol. 252, no. September 2021, 2022, doi: 10.1016/j.engstruct.2021.113667.
- [13] L. Marian and A. Giaralis, "Optimal design of a novel tuned mass-damper – inerter ( TMDI ) passive vibration control configuration for stochastically support-excited structural systems," *Probabilistic Engineering Mechanics*, pp. 1–9, 2014, doi: 10.1016/j.probengmech.2014.03.007.
- [14] M. C. Smith, "Synthesis of Mechanical Networks : The Inerter," vol. 47, no. 10, pp. 1648–1662, 2002.
- [15] C. Papageorgiou and M. C. Smith, "Laboratory experimental testing of inerters," pp. 3351–3356, 2005.
- [16] P. Brzeski, T. Kapitaniak, and P. Perlikowski, "Novel type of tuned mass damper with inerter which enables changes of inertance," *J Sound Vib*, vol. 349, pp. 56–66, 2015, doi: 10.1016/j.jsv.2015.03.035.
- [17] D. Pietrosanti, M. de Angelis, and M. Basili, "A generalized 2-DOF model for optimal design of MDOF structures controlled by Tuned Mass Damper Inerter (TMDI)," *Int J Mech Sci*, p. 105849, 2020, doi: 10.1016/j.ijmecsci.2020.105849.
- [18] L. A. Lara-Valencia, Y. Farbiarz-Farbiarz, and Y. Valencia-González, "Design of a Tuned Mass Damper Inerter (TMDI) Based on an Exhaustive Search Optimization for Structural Control of Buildings under Seismic Excitations," *Shock and Vibration*, vol. 2020, 2020, doi: 10.1155/2020/8875268.
- [19] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey Wolf Optimizer," *Advances in Engineering Software*, vol. 69, pp. 46–61, 2014, doi: 10.1016/j.advengsoft.2013.12.007.
- [20] S. Mirjalili and A. Lewis, "The Whale Optimization Algorithm," *Advances in Engineering Software*, vol. 95, pp. 51–67, 2016, doi: 10.1016/j.advengsoft.2016.01.008.
- [21] I. F. Lazar, "Using an inerter-based device for structural vibration suppression," no. November 2013, pp. 1129–1147, 2014, doi: 10.1002/eqe.

- [22] S. Pal, D. Singh, and V. Kumar, “Hybrid SOMA: A tool for optimizing TMD parameters,” in *Advances in Intelligent Systems and Computing*, 2017, vol. 546, pp. 35–41. doi: 10.1007/978-981-10-3322-3\_4.
- [23] A. Y. T. Leung, H. Zhang, C. C. Cheng, and Y. Y. Lee, “Particle swarm optimization of TMD by non-stationary base excitation during earthquake,” *Earthq Eng Struct Dyn*, vol. 37, no. 9, pp. 1223–1246, Jul. 2008, doi: 10.1002/eqe.811.
- [24] A. Kaveh, “Sizing Optimization of Skeletal Structures Using the Enhanced Whale Optimization Algorithm,” in *Applications of Metaheuristic Optimization Algorithms in Civil Engineering*, Springer International Publishing, 2017, pp. 47–69. doi: 10.1007/978-3-319-48012-1\_4.
- [25] A. Kaveh and M. I. Ghazaan, “Enhanced whale optimization algorithm for sizing optimization of skeletal structures,” *Mechanics Based Design of Structures and Machines*, vol. 45, no. 3, pp. 345–362, Jul. 2017, doi: 10.1080/15397734.2016.1213639.
- [26] M. Azizi, R. G. Ejlali, S. A. Mousavi Ghasemi, and S. Talatahari, “Upgraded Whale Optimization Algorithm for fuzzy logic based vibration control of nonlinear steel structure,” *Eng Struct*, vol. 192, pp. 53–70, Aug. 2019, doi: 10.1016/j.engstruct.2019.05.007.
- [27] M. Huang, Z. Wan, X. Cheng, Z. Xu, Y. Lei, and D. Pan, “Two-stage damage identification method based on fractal theory and whale optimization algorithm,” *Advances in Structural Engineering*, vol. 25, no. 11, pp. 2364–2381, Aug. 2022, doi: 10.1177/13694332221095629.
- [28] H. Chen, Y. Xu, M. Wang, and X. Zhao, “A balanced whale optimization algorithm for constrained engineering design problems,” *Appl Math Model*, vol. 71, pp. 45–59, Jul. 2019, doi: 10.1016/j.apm.2019.02.004.
- [29] F. S. Gharehchopogh and H. Gholizadeh, “A comprehensive survey: Whale Optimization Algorithm and its applications,” *Swarm Evol Comput*, vol. 48, pp. 1–24, Aug. 2019, doi: 10.1016/j.swevo.2019.03.004.
- [30] Pacific Earthquake Engineering Research Center: Ground motion database. Available: [http://peer.berkeley.edu/peer\\_ground\\_motion\\_database](http://peer.berkeley.edu/peer_ground_motion_database).
- [31] Center for engineering strong-motion data (CESMD) [www.strongmotioncenter.org](http://www.strongmotioncenter.org).
- [32] The MathWorks Inc, MATLAB R2019a, Natick, MA, USA, 2019.