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TEST OF AN IDEA FOR SETTING THE NONLINEARITY TOLERANCE IN NONLINEAR RESPONSE HISTORY ANALYSES ACCORDING TO PROCEDURES ORIGINATED IN THE SEISMIC CODE OF NEW ZEALAND NZS 1170.5:2004

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Abstract

Nonlinear response history analysis using an integration scheme and an iterative nonlinear solution method is a versatile tool for analyzing structural systems subjected to earthquakes. Nevertheless, the resulting responses are inexact and the analyses are generally complicated and computationally expensive. In a recent study, a slight change is applied to the continuation/end of the nonlinear solutions' iterations and the probable stop of the analysis. As a result, the simplicity and efficiency of the analysis according to the seismic code of New Zealand NZS 1170.5:2004 is enhanced, without negative effects on the response accuracy. In this paper, in order to simplify the analysis further, and under the assumption of piecewise linear behavior, three simple suggestions for setting the nonlinearity tolerance are tested. Consequently, when the response history analysis is based on repetition of time integration analysis, as it is in the seismic code of New Zealand, NZS 1170.5:2004, the nonlinearity tolerance can be simply set and reduced with each new repetition. Such selections not only simplify the analysis by eliminating the concerns on the nonlinearity tolerance, but also may reduce the computational effort. The reduction in computational effort may be considerable specifically when the behavior is highly nonlinear. The added simplicity can increase the interest in using response history analysis in earthquake engineering practice, as well.

Keywords: Seismic Nonlinear Response History Analysis, Nonlinearity Tolerance, Seismic Code of New Zealand NZS 1170.5:2004, Response Accuracy, Analysis Simplicity, Analysis efficiency.

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1 INTRODUCTION

Nonlinear response history analysis is one of the most versatile analysis tools for studying the behaviors of structural systems, irreplaceable in many real analyses [1, 2]. The computed responses are however approximations and the analysis is generally time-consuming [2-4]. The probability of stop of analysis because of failure of nonlinear solutions iteration exists, as well [5-7]. In addition, for computing the response, nonlinear response history analysis generally uses a time integration method combined with an iterative nonlinear solution method [2]; accordingly, various analysis parameters should be set prior to the analysis. Considering these, there is little interest in using nonlinear response history analysis in the earthquake engineering practice especially in regions where advanced computational facilities are hardly available. Many researchers have reported studies on the above-mentioned problems, e.g. see [2, 8-15], and also on the efforts to reduce these problems, e.g. see [15-25]. Unresolved problems still exist, e.g. see [2, 7, 26, 27]. One of these problems, in view of an established legalized analysis procedure introduced in the seismic code of New Zealand, NZS 1170.5:2004 [28, 29], is the selections of the integration method, the iterative nonlinear solution method, and the nonlinearity tolerance. The concentration here is on the nonlinearity tolerance.

Analysis of structures' seismic nonlinear dynamic behaviors typically starts with discretizating the mathematical models in space and defining the behavior in the framework of the solution of the resulting ordinary initial value problem, stated below (see [2]):

$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{f}_{\text{int}}(t) = -\mathbf{M}\boldsymbol{\Gamma}\ddot{u}_g(t) \qquad 0 \le t \le t_{\text{end}}$$

$$\text{Initial Conditions :} \quad \left| \begin{array}{l} \mathbf{u}(t=0) = \mathbf{u}_0 \\ \dot{\mathbf{u}}(t=0) = \dot{\mathbf{u}}_0 \\ \mathbf{f}_{\text{int}}(t=0) = \mathbf{f}_{\text{int}_0} \end{array} \right. \quad . \qquad (1)$$

$$\text{Additional Constraints :} \quad \mathbf{Q}$$

In Eq. (1), which is in some cases directly available under assumptions, like shear frame behavior [30-32], t and t_{end} imply the time and the analysis time interval; $\mathbf{M}$ is the mass matrix; $\mathbf{f}_{\text{int}}$ is the vector of the internal force, because of the elastic forces and damping (in linear problems, generally $\mathbf{f}_{\text{int}} = \mathbf{K}\mathbf{u} + \mathbf{C}\dot{\mathbf{u}}$, where $\mathbf{K}$ and $\mathbf{C}$ stand for the matrices of stiffness and viscous damping, respectively); $\ddot{u}_g(t)$ implies the single-component ground acceleration, and Γ is a vector with the size of the degrees of freedom, needed for matrix multiplication and considering spatial changes of $\ddot{u}_g(t)$; $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$, denote the vectors of displacement, velocity, and acceleration, relative to the ground; $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}_0}$, define the initial status, and $\mathbf{Q}$ implies the limiting conditions due to nonlinearity. (When the ground motion is multi-component, Γ and $\ddot{u}_g(t)$ will change to a matrix and a vector, respectively.) The typical approach for analysis of Eq. (1) is to use a time integration method and a method for nonlinear solution; see Fig. 1. Accordingly, the integration method, the nonlinear solution method, and the related parameters, e.g. the nonlinearity tolerance should be set in advance. Focusing on nonlinearity tolerance, 10^{-4} is a conventional value for the tolerance to control the errors occurring in modeling the nonlinearities; see [2, 33]. Meanwhile, although it is conventional to use a single value for the tolerance, the tolerance can change for the nonlinearities at different parts of the structural system and at different time instants. Even more, for the analyses that are based on repetition of time integration (recommended in the literature, [30, 34], and legalized in the seismic code of New Zealand NZS 1170.5:2004 [28, 29]), the

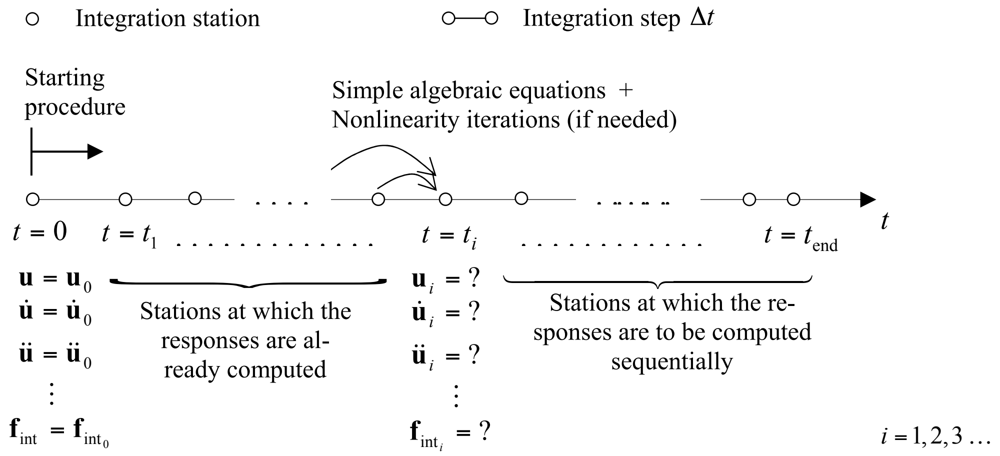

Figure 1: Brief description of the process of time history analysis using a time integration method [2].

tolerance can change with each repetition of the time integration. Theories and suggestions exist for reduction of the nonlinearity tolerance with each new time integration analysis, e.g. see [35, 36]. Since the procedure in the seismic code of New Zealand, NZS 1170.5:2004 [28, 29], is the only legalized procedure for seismic nonlinear response history analysis [7, 37], it is reasonable to consider this procedure as a basis for studying the analysis performance. Accordingly, attention is paid to sequential time integration analyses. Furthermore, the theories defining changes of the tolerance in sequential time integration analyses are based on the most important necessity of numerical computations, i.e. convergence of the approximate response to the exact response [38, 39]. Considering these issues, it is reasonable to pay attention to the convergence-based theories in changing the tolerance from an analysis to the next analysis, while, for the sake of simplicity, keep the tolerance constant throughout the structure and total time interval.

Returning to the analysis procedure, a slight change is recently proposed [7, 40, 41] in the analysis procedure of NZS 1170.5:2004 [28, 29]. The change is very effective in the analysis simplicity and efficiency [7], and hence is in complete consistence with the objectives of this paper, stated in the first paragraph. Accordingly, the procedure introduced in [7, 41] will be the analysis procedure to which the suggestions in this paper will be applied. For the change of the nonlinearity tolerance, though the suggestion based on responses convergence [35, 36] is considerably effective [42-45], it is hardly practical in view of its complexity and dependence to some new parameters. In this paper, three simplifications of this suggestion is introduced for the analysis of structural systems with piecewise linear behavior (a conventional nonlinearity in earthquake engineering [30, 31, 46]) and their performances are tested in seismic analysis of a structural system considering different earthquake records and different severities of nonlinear behavior. In continuation, after reviewing the main concepts on nonlinear time integration analysis, the existing suggestion on reducing the nonlinearity tolerance [35, 36] is explained. The new simplified suggestions are stated discussed and tested numerically, and after brief discussions, the paper is concluded with a set of the conclusions.

2 THEORY

2.1 The main Concepts

Regardless of the static or dynamic behavior of structural systems, and even in applications different from structural and earthquake engineering, a broadly accepted relation governing the iterations of nonlinear solutions is as follows; see [5, 6, 25, 26, 35, 47-50]:

$$\begin{array}{l} \|\delta_k\| > \bar{\delta} \text{ and } k < \bar{k}: \text{ Continue the iterations} \\ \|\delta_k\| \leq \bar{\delta} \text{ and } k \leq \bar{k}: \text{ Stop the iterations and continue the analysis} \\ \|\delta_k\| > \bar{\delta} \text{ and } k = \bar{k}: \text{ Stop the iterations and stop the analysis} \\ \qquad k = 1,2,3,\ldots k' \quad , \quad k' \leq \bar{k} \quad , \quad \bar{k} < \infty \end{array} . \tag{2}$$

In Eq. (2), $\|\delta_k\|$ denotes the inaccuracy in modeling the nonlinearity after the k th iteration of the nonlinear solution, $\bar{\delta}$ implies the tolerance (maximum acceptable inaccuracy in modeling the nonlinearity), and $\bar{k}$ is the maximum acceptable number of the iterations representing the available computational facility. The definition of $\|\delta_k\|$ and $\bar{\delta}$ can be in terms of displacement, force, energy, etc., and there are many methods for iterative nonlinear solution; see [6, 25, 47-56]. It is also notable that the case $k = \bar{k}$ may be unavailable due to the divergence of the response and very large values of the response, e.g. when using the Newton Raphson at values of the dynamics stiffness close to zero (see [30, 31, 51]), at which case the analysis is automatically stopped. This case is rarely probable in dynamic problems due to the difference between static and dynamic stiffness in time integration analyses (see [30, 31]), and when using iterative methods like modified Newton Raphson [51] and fractional time stepping [56, 57].

Recently, the following simple change is proposed in the above formulation [7, 40, 41, 58]:

$$\begin{array}{l} \|\delta_k\| > \bar{\delta} \text{ and } k < \bar{k}: \text{ Continue the iterations} \\ \|\delta_k\| \leq \bar{\delta} \text{ or } k = \bar{k}: \text{ Stop the iterations and continue the analysis} \\ \qquad k = 1,2,3,\ldots k' \quad , \quad k' \leq \bar{k} \quad , \quad \bar{k} < \infty \end{array} \tag{3}$$

Application of this change in the analysis procedure of NZS 1170.5:2004 results in a procedure for nonlinear response history analysis that is simpler and more efficient compared to using Eq. (2) in the procedure of NZS 1170.5:2004 [28, 29]. The brief reason is the no stop of analysis when the iterations of nonlinear solution fails, which can cause less time integration analyses and more analysis efficiency; see also [7]. The resulting procedure is considered as the analysis procedure in this paper, and the nonlinearity tolerances are to be set for use in this procedure.

In view of the definition of piecewise linear systems [36], the nonlinear behaviors of structural systems in earthquake engineering are mostly of the piecewise linear type; see [28, 29, 46]. Besides, the details of the existing suggestion on changes of nonlinearity tolerance in sequential time integration analyses consider nonlinear behaviors of piecewise linear type [35, 36]. Considering these, in this paper, the nonlinear behaviors are of the piecewise linear type, and for simplicity fractional time stepping [56, 57] is used for nonlinear solution.

2.2 The existing suggestion on changes of nonlinearity tolerance

The suggestion for the nonlinearity tolerance in sequential analyses [35, 36] is set such that the response obtained from the time integration analysis converges properly [2, 59] with respect to the integration step (see Fig. 2, where E and Δt represent the response error [60] and the integration step [2-4], respectively). The main relation is as follows:

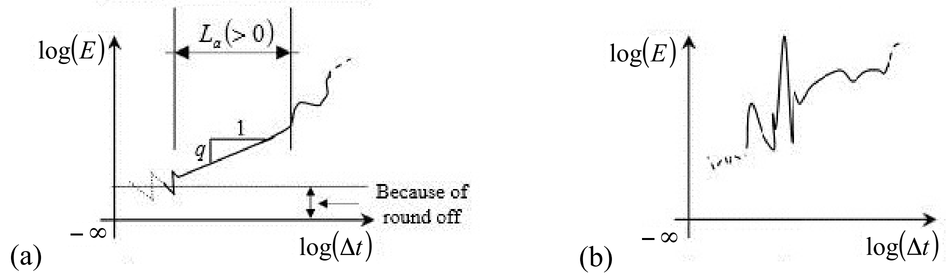

Figure 2: Typical changes of computational errors with respect to the integration step: (a) proper convergence, improper convergence [59].

$$
{}_{l+1}\bar{\delta}_{\Omega_p} = \left[\underset{j=1,2,\ldots\,{}_lJ_p}{\mathrm{Min}} \left({}_l\delta_j \right) \right] \left(\frac{{}_{l+1}\Delta t}{{}_l\Delta t} \right)^q \quad , \quad {}_l\delta_j \text{ occurs in } \Omega_p \qquad p = 1,2,\ldots P \quad , \quad l = 1,2,\ldots \tag{4}
$$

where, q denotes the rate of convergence with respect to the integration step in absence of nonlinearity (generally equal to the integration method's order of accuracy and equal to two [3, 4, 61, 62]), l is an indicator for the sequence of time integration analysis, ${}_l\Delta t$ stands for the integration step of the l th time integration analysis, Ω_P implies the total space of probable nonlinearity, briefly defined in Fig. 3, p is an indicator for the subspace of probable nonlinearity under consideration (see Fig. 3), ${}_l\delta_j$ represents the final $\|\delta_k\|$ in Eqs. (2) or (3) at the j th nonlinearity detected in the l th time integration analysis in Ω_p, ${}_l\bar{\delta}_{\Omega_p}$ denotes the nonlinearity tolerance to be used in Ω_p in the l th time integration analysis, and ${}_lJ_p$ stands for the number of nonlinearities detected in Ω_p in the l th time integration analysis. Evidently, Eq. (4) increases the complicatedness of nonlinear response history analysis; specifically, the initial study needed to determine Ω_P and to select the subspaces Ω_p, and the computation of Eq. (4) itself are notable. Therefore, Eq. (4) can hardly be acceptable in the practice of nonlinear response history analysis. Simplification of Eq. (4) is studied in the next section.

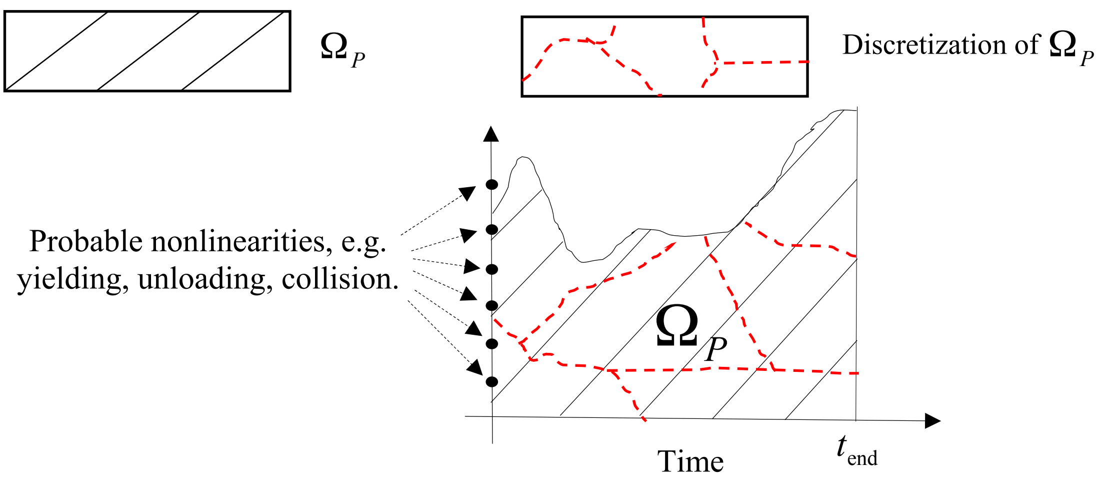

Figure 3: Typical space of probable nonlinearity in response history analysis of piecewise linear systems and the typical discretization.

2.3 Three new suggestions for the nonlinearity tolerances

From Eqs. (2)-(4), we can deduce that nonlinearity tolerance is to decrease when repeating the time integration analysis with smaller steps, i.e. when l in Eq. (4) increases. Besides, generally and as recommended in [4, 61],

$$q = 2 \tag{5}$$

and as stated in [28, 29]:

$$\frac{{}_{l+1}\Delta t}{{}_{l}\Delta t} = 0.5 \tag{6}$$

and finally the discretization of Ω_P is arbitrary, and we can select:

$$P = 1 \tag{7}$$

Considering these, we can simplify Eq. (4) to

$$ {}_{l+1}\bar{\delta} = \frac{1}{4}\left[\underset{j=1,2,\ldots,{}_{l}J}{\mathrm{Min}}\left({}_{l}\delta_j\right)\right] \quad , \quad {}_{l}\delta_j \text{ occurs in } \Omega_P \qquad l = 1,2,\ldots \tag{8}$$

and practically eliminate the need to determine Ω_P, by using the following equation instead of Eq. (8):

$$ {}_{l+1}\bar{\delta} = \frac{1}{4}\left[\underset{j=1,2,\ldots,{}_{l}J}{\mathrm{Min}}\left({}_{l}\delta_j\right)\right] \quad , \quad l = 1,2,\ldots \tag{9}$$

In simplification of Eq. (9), attention is paid to:

1. In view of Eqs. (2) and (3), it is reasonable to expect that in most of the nonlinearity detections:

$$ {}_{l}\delta_j \cong\leq {}_{l}\bar{\delta} \tag{10}$$

2. In view of the theory leading to Eq. (4) (see [35, 36]), the minimization in Eq. (4) causes the resulting tolerances to be conservative from the viewpoint of the responses proper convergence.
3. The selection in Eq. (7) highlights the previous point and makes the resulting tolerance even more conservative.
4. According to the nature of sequential time integration analyses, the number of the analyses is two or more, and hence in view of the conventional value of the tolerance, i.e. 1.0E-4 [33], it is reasonable to consider the nonlinearity tolerance of the second analysis around the conventional tolerance.
5. Both as conventional in practice and academia and tolerable by structural analysts the maximum number of sequential analyses would rather be not more than around six.
6. In view of hardware capabilities and round off errors, it sounds reasonable to consider lower bounds for the nonlinearity tolerance around 1.0E-12.
7. In view of Eqs. (2), (3) and (8), the reduction of nonlinearity tolerance from a time integration analysis to its next analysis should be more than four times. With this in mind, for simplicity, the nonlinearity tolerance in a time integration analysis will be obtained by dividing the tolerance in the previous analysis by a power of ten.

Considering Eq. (9) and the above points, three suggestions are stated in Table 1. The next section is dedicated to testing the performance of analyses using these three suggestions in comparison to the procedure introduced in [7, 41].

Analysis Sequence	Suggestion 1	Suggestion 2	Suggestion 3
1	1.0E-1	1.0E-2	1.0E-1
2	1.0E-3	1.0E-4	1.0E-4
3	1.0E-5	1.0E-6	1.0E-7
4	1.0E-7	1.0E-8	1.0E-10
5	1.0E-9	1.0E-10	1.0E-13
6	1.0E-11	1.0E-12	1.0E-13
7 and more	1.0E-13	1.0E-12	1.0E-13

Table 1: Suggestions for nonlinearity tolerance in sequential nonlinear time integration analyses.

3 NUMERICAL INVESTIGATION

3.1 Preliminary notes

Since this is a first test on the suggested nonlinearity tolerances (see Table 1), in order to better understand and interpret the changes, in this section, one structural system with piece-wise linear behavior is taken into account. Two different earthquake records are considered as the excitations in two separate studies (see Fig. 4; where, ${}_f\Delta t$ is the step of digitization), and three severities of nonlinear behavior [7, 63-65] are considered in three separate studies, by applying the following three scales to the two excitations:

$$S = 1, 2, 5 \tag{11}$$

Therefore, totally six tests are carried out for each of the suggestions in Table 1. Using either of the suggestions in Table 1 eliminates the selection of appropriate values to be assigned to the nonlinearity tolerance. Meanwhile, similar to the analysis procedure in NZS 1170.5:2004 [28, 29], the analysis procedure proposed in [7, 41] is equipped with some control on the accuracy of the target response in the last steps of the analysis procedures. Therefore, the main purpose of the above-mentioned tests is to study the computational effort needed to implement the suggestions in Table 1 in the procedure introduced in [7, 41]. In view of the fact that the procedure in [7, 41] is consisted of sequential time integration computations, using or not using the suggestion in Table 1 does not affect the computer in-core memory, and the study of computational effort can be based on the analysis run-time.

With regard to the integration methods, in view of the suggestions existing for nonlinear analysis [5], the Newmark's average acceleration method [66] is used for all of the analyses. For the nonlinearity solution, in view of the system's piecewise linear behavior, and for the sake of simplicity, the selection is to use the fractional time stepping method with the conventional value for maximum number of iteration, i.e. $\bar{k} = 5$ [56, 57]. Based on this selection, the analysis run-time can be simple studied in view of the total number of integration steps; see [2].

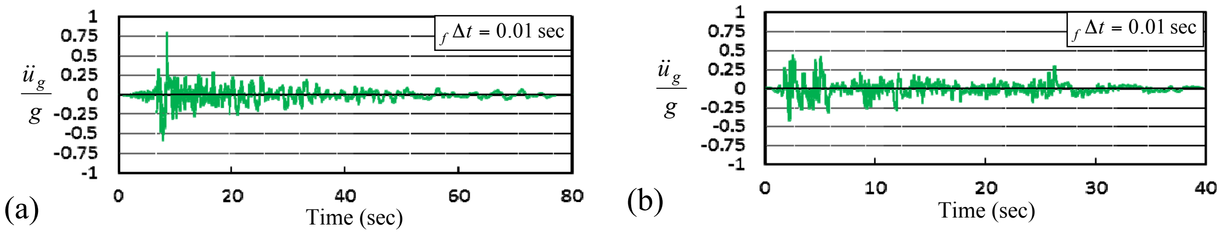

Figure 4: The two earthquake records under consideration in the numerical investigation.

3.2 The example

Taken from [7], the structural system is a preliminary model of a tall building; see Fig. 5 and Table 2. The target response is the top displacement, for which the exact responses are reported in Fig. 6, in correspondence to the six models defined by Fig. 4, and Eq. (11) (Num stands for the total number of stiffness change representing the severity of nonlinear behavior).

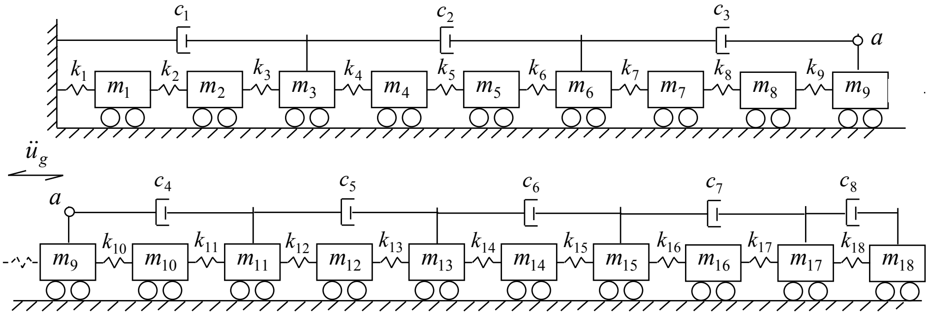

Figure 5: The structural model in the numerical investigation [7].

Property	i																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$10^{-9} \times m_i$ (Kg)	3	3	3	3	3	3	3	3	1.5	1.5	1.5	1.5	1.5	1.5	0.5	0.5	0.5	0.5
$10^{-12} \times k_i$ (N/m)	2	2	2	2	1.2	1.2	1.2	1.2	0.6	0.6	0.6	0.6	0.6	0.6	0.1	0.1	0.1	0.1
$10^{-8} \times c_i$ (N sec/m)	12	8	6	2.5	2.5	1.5	0.5	0.2										
$10^{2} \times u_{y_i}$ (m)	8	8	8	8	8	8	5	5	5	5	5	5	5	5	3	3	3	3

Table 2: Main properties of the structural model in Fig. 5 [7, 32].

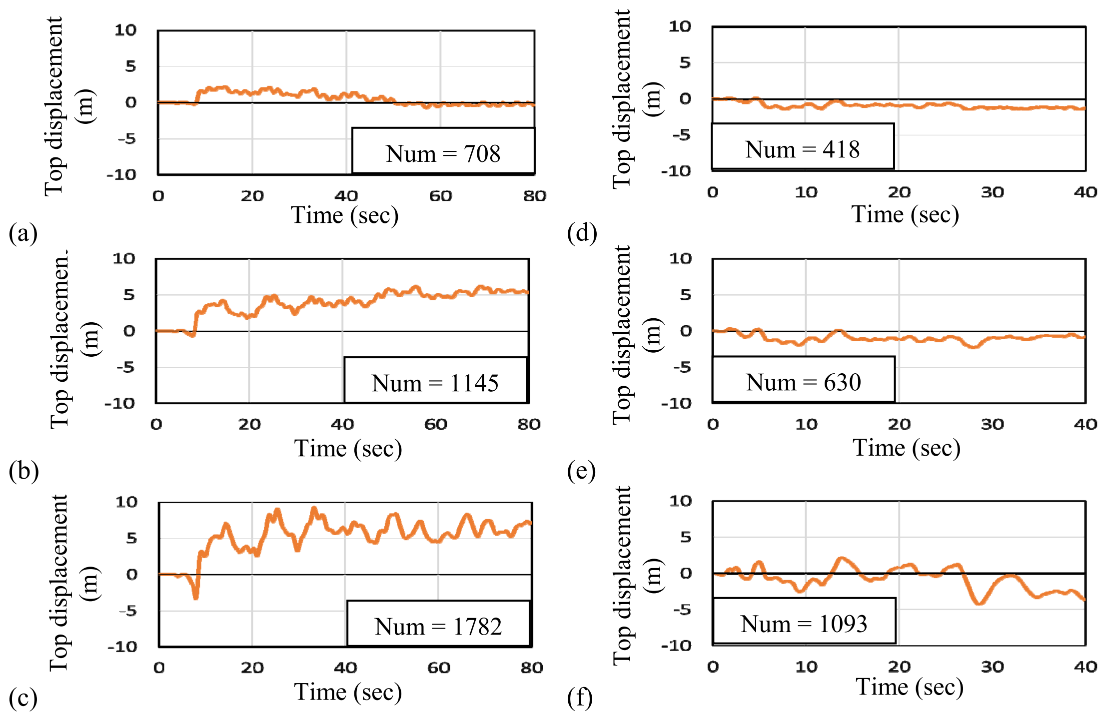

Figure 6: Exact top displacements for the structural system introduced in Figs. 4 and 5, Table 2, and Eq. (11), when the excitation is represented with: (a) Fig. 4(a), (b) Fig. 4(a) scaled by $S = 2$, (c) Fig. 4(a) scaled by $S = 5$, (d) Fig. 4(b), (e) Fig. 4(b) scaled by $S = 2$, (f) Fig. 4(b) scaled by $S = 5$.

In view of the three suggestions in Table 1, for arriving at approximations of the target responses displayed in Fig. 6, the procedure introduced in [7, 41] is carried out 18 times; all starting with time integration analyses with steps equal to 0.01 sec (according to the analysis procedure). The approximations are achieved in the price of the total number of steps and total number of time integration analyses reported in Tables 3-8. As expected, with more severe nonlinear behavior the computational effort is more. However the interesting point is that the differences between the computational efforts corresponding to the three suggestions decrease when the severity of nonlinear behavior increases. To have a better study on the computational efforts, the analyses are carried out without implementing the suggestions in Table 1 several times (with constant nonlinearity tolerances equal to 1.E-2, 1.E-4, and 1.E-6). The average total number of integration steps and the average total number of analyses are reported in Table 9. In view of Table 9, the suggestions in Table 1 not only simplify the nonlinear response

Feature	Suggestion 1	Suggestion 2	Suggestion 3
Number of Analyses	3	3	3
Number of steps	68851	76032	76726

Table 3: Total numbers of the analyses and integration steps when using the suggestions in Table 1, for Fig. 6(a).

Feature	Suggestion 1	Suggestion 2	Suggestion 3
Number of Analyses	4	3	3
Number of steps	161176	88243	90106

Table 4: Total numbers of the analyses and integration steps when using the suggestions in Table 1, for Fig. 6(b).

Feature	Suggestion 1	Suggestion 2	Suggestion 3
Number of Analyses	5	5	5
Number of steps	337808	365998	380398

Table 5: Total numbers of the analyses and integration steps when using the suggestions in Table 1, for Fig. 6(c).

Feature	Suggestion 1	Suggestion 2	Suggestion 3
Number of Analyses	3	3	3
Number of steps	35519	39859	40207

Table 6: Total numbers of the analyses and integration steps when using the suggestions in Table 1, for Fig. 6(d).

Feature	Suggestion 1	Suggestion 2	Suggestion 3
Number of Analyses	3	3	3
Number of steps	39597	45945	47148

Table 7: Total numbers of the analyses and integration steps when using the suggestions in Table 1, for Fig. 6(e).

Feature	Suggestion 1	Suggestion 2	Suggestion 3
Number of Analyses	3	3	3
Number of steps	47930	59728	60576

Table 8: Total numbers of the analyses and integration steps when using the suggestions in Table 1, for Fig. 6(f).

Feature	Table 3	Table 4	Table 5	Table 6	Table 7	Table 8
Number of analyses	2.3333	3	6	2.3333	3.3333	4
Number of steps	49168.3	90964	5604098	25236.67	57624.67	370546.33

Table 9: Average numbers of the analyses and integration steps when not using the suggestions in Table 1.

history analysis but also may reduce the computational effort especially when the nonlinearity is more severe. And more that: (1) the three suggestions seem not very different in analysis efficiency especially when the severity of nonlinear behavior is more, (2) the enhancement in analysis efficiency may be considerable; in view of Table 9, see the 94% and 87% reduction in computational effort for Suggestion 1 in Tables 5 and 8, respectively. Though, the number of the tests is small, the resulting analysis simplicity, with no negative effect on accuracy in the sense of the analysis procedure [7, 28, 29, 41], and besides the observed effects on the efficiency, explains much further study on suggestions like those presented in Table 1. The next section briefly presents an effort to explain the observed effects on analysis efficiency.

3.3 Brief explanation of the observed analysis efficiency

In view of the analysis procedure stated in [7, 41], the minimum number of time integration analyses is two (essential for the error control in the end of the procedure). Besides, in view of the discussions presented in [35, 36] and as reasonable, the changes of the computational errors of nonlinear analyses when using/not using the suggestions in Table 1 is as typically displayed in Fig. 7. As an evident result, when using similar integration steps and close nonlinearity tolerances in the first time integration analyses of the procedure introduced in [7, 41], without and with applying the reductions proposed in Table 1, the total number of time integration analyses is equal or more when not using the suggestions in Table 1 (this is also observed in the studied example and details of Table 9, not reported for the sake of brevity). Considering this along with the fact that in analysis of piecewise linear systems the contribution of nonlinearity solutions in the computational effort reduces when using smaller integration steps (see [2]), the higher number of time integration analyses when not using the suggestions in Table 1 implies the more computational effort when not applying the suggestions compared to when applying the suggestions. The case when the total numbers of time integration analyses are equal in applying/not applying the suggestions mostly occurs when the total number of the analyses is small, i.e. two, three, or four, the nonlinearity tolerances of the first analysis in applying/not applying the suggestions are not equal, and the two graphs in Fig. 7 are not completely separated. As implied in Tables 3-8, this happens when the severity of nonlinear behavior is low. It is interesting that in these cases the computational effort is generally low and the additional efforts because of the suggestions in Table 1 worth the achieved simplicity. In cases with more severity of nonlinear behavior however the computational efforts are high and the suggestions in Table 1 reduce these high computational effort during a simple analysis, as desired.

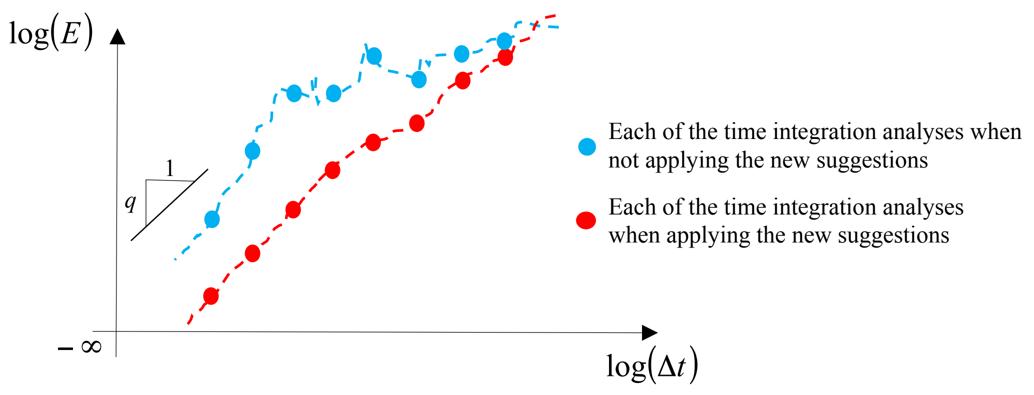

Figure 7: Typical trend of computational errors in the analyses according to the procedure introduced in [7, 41].

4 DISCUSSION

The observation and explanation presented in the previous sections, implies considerable simplification in seismic nonlinear response history analysis of piecewise linear systems, along with the possibility of more analysis efficiency when the severity of nonlinear behavior is considerable. Much further study is essential before counting on these statements. However, the simplicity and the considerable probable reduction in the computational effort can explain the further study. Even more, the following observations:

1. Elimination of the concern on nonlinearity tolerance is a considerable simplification of nonlinear response history analysis.
2. The simplicity and analysis efficiency because of implementation of the suggestions in Table 1 depend on the selected suggestion, only slightly (the first suggestion seems slightly better). The dependence is less when the severity of nonlinear behavior is more.
3. The enhancement in analysis efficiency is more when the severity of nonlinear behavior is more.

can simply cause considerable increase in the structural analysts' interest in nonlinear response history analysis. This can be true, especially when large and fast computer systems are not simply available and reducing the analysis effort is attractive, e.g. in less developed countries with low income and large structural projects. The secondary consequence can be better seismic structural analysis and safer buildings and infrastructures. Consequently, more effort to establish some idea about the nonlinearity tolerances in nonlinear response history analysis of piecewise linear systems is reasonably essential, both from a pure academic point of view, as well as considering the practical aspects. Some of the issues that should be included in an extended study are:

1. Many examples should be studied.
2. The sizes of the examples from the point of view of the numbers of degrees of freedom would rather be completely different.
3. The effect of the computational facility (the $\bar{k}$ in Eqs. (2) and (3)) should be studied.
4. The effect of the integration method, and specifically the numerical damping, would rather be studied.
5. Considering the important role of pounding in seismic damages [67-69], different types of nonlinearities should also be studied.
6. More detailed attention would rather be paid to the concept of severity of nonlinear behavior.

5 CONCLUSUION

In order to simplify the procedure of seismic nonlinear response history analysis, according to a recent procedure [7, 41] (slightly different from that in [28, 29]), three suggestions on nonlinearity tolerance are tested in analysis of six piecewise linear structural cases. The observations are briefly explained, as well.

Because of the simplicity of the rules suggested for selection of the nonlinearity tolerance (see Table 1), the simplification of the analysis is evident. Besides, it is observed that:

1. There is no specific difference between the analyses considering the three suggestions. It is only needed to take into account the seven points addressed in Section 2.3. Still, the performance of the first suggestion (see Table 1), i.e.

$$ {}_{l}\bar{\delta} = \begin{cases} 10^{-(2l+1)} & l \leq 6 \\ 10^{-13} & l > 6 \end{cases} \tag{12} $$

was in overall slightly better (l is the number of sequential time integration analysis).

2. The reduction in computation effort because of the suggestions in Table 1 appears when the nonlinear behavior is sufficiently severe.
3. The amount of the reductions in computational effort can be considerable, e.g. 94% (see Section 3.2)

Because of the significance of the observations and the probable influence of the enhancements in analysis simplicity and efficiency on practical seismic response history analysis, further studies specifically on issues discussed in Section 4 are strongly recommended.

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