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SUPPLEMENTARY DAMPING DEVICES TO MITIGATE THE NEAR-FAULT SEISMIC EFFECTS ON THE UFREI-ISOLATED STRUCTURES

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Abstract

The efficiency of Unbonded Fiber-reinforced Elastomeric Isolators (UFREI) in isolating the structure from near-fault ground motions with the help of various supplementary damping devices is investigated. The improved efficiency of UFREI in mitigating the more frequent far-fault ground motions as compared to its steel-reinforced counterparts has already been observed in various literature before. However, the near-fault ground motions generally contain pulses of high amplitude and longer time periods, which thereby results in an excessive structural response like displacements at isolation level in base-isolated structures. Therefore, various supplemental damping devices are incorporated alongside the isolation system to mitigate these excessive responses. The introduction of supplementary devices to the base-isolated structure also introduces some additional stiffness; thus, an extensive study of the effect of the various supplementary damping devices on the various response parameters of the isolated structural system is carried out.
Four different types of supplementary damping devices have been considered in this study based on various damping mechanisms: Fluid Viscous Damper (FVD), Tuned Damper Inerter (TMDI), Magnetorheological Damper (MRD), and Negative Stiffness Damper (NSD). These dampers are introduced alongside the UFREIs in an isolated benchmark structure, and the whole assembly is subjected to a wide range of input ground motions consisting of both near-fault and far-fault characteristics. A comparative response analysis is carried out for the UFREI-isolated structure under the set of input ground motions with various considered supplementary damping devices. A significant improvement in the UFREI isolated structure's response is observed with all the supplementary damping devices. The best response improvement to UFREI-isolated structure is achieved with TMDI and MRD (semi-active) damper systems.

Keywords: Base-isolation, Unbonded Fiber-reinforced Elastomeric Isolators, Supplementary Dampers, Near-fault, Seismic Analysis.

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1 INTRODUCTION

Seismic hazards are one of the major reasons for the loss of life and property in developing countries, which elevates the need for accessible and efficient mitigation techniques. This is achieved mainly with the use of several types of control schemes, i.e., Active control, Semi-active control, and Passive control. Base isolation (BI) is a cost-effective and efficient passive control mechanism based on low horizontal stiffness and high damping characteristics, which limits and controls the displacement to the isolation level only. Elastomeric bearings are the most commonly used base-isolation device which uses the alternate layers of reinforced elastomers to achieve an improved structural response. Conventionally, steel shims were used as reinforcement in the elastomeric bearings and it was called Steel-reinforced Elastomeric Isolators (SREI). Later on, several fiber layers were introduced as reinforcement which has several benefits over the conventional steel counterparts[1]. The benefits of these isolation systems included lesser weight, lesser manufacturing cost and ease, and improved isolation efficiency. The improvement in isolation efficiency was achieved due to a unique rollover phenomenon where the edges of the unbonded isolator rolled off the supports and thus decreasing the horizontal stiffness even further[2], [3]. These isolators are called Unbonded Fiber-reinforced Elastomeric Isolators (UFREI). The rollover phenomenon is followed by a Full-rollover state where the edges start touching the opposite supports, which starts restricting the further horizontal displacement and thus preventing overturning. The improved isolation efficiency of UFREI has already been witnessed earlier in several studies[4], [5]. Despite its improved isolation efficiency under more prevalent far-fault ground motions, the behavior of the isolated structure is still unfavorable under near-fault ground motions. Near-fault ground motions are often characterized by high amplitude and long-period pulses, which amplify the isolated structure's responses; thus, supplementary damping devices are often used in conjugation for an improved response[6], [7].

Various types of supplementary damping devices (i.e., active, semi-active, and passive) can be used for this purpose. Four supplementary devices have been used in this study which comprises one semi-active device, MR Damper (MRD), and three passive devices, i.e., Tuned Mass Damper Inerter (TMDI), Negative Stiffness Damper (NSD), and Fluid Viscous Damper (FVD). MRD is a semi-active device based on magnetorheological fluid, which changes its rheological property in real-time to absorb the incident seismic energy. A semi-active device like MRD is selected for its stability during operation and its improved efficiency in seismic response mitigation which has been investigated earlier[6], [8], [9]. TMDI is a passive device that consists of a combination of a conventional tuned mass damper (TMD) and an inerter. The Inerter is a two-terminal mass-augmentation device that has been used standalone or alongside classical TMD for improved seismic mitigation capabilities[10]–[12]. Recently, closed-form expressions of the optimal TMDI parameters for a damped structure under seismic excitation were also proposed[7]. NSD is a unique type of damper based on a compressed/negative spring and a viscous damper placed parallelly. The compressed spring stiffness concept, known as Adaptive Negative Stiffness Systems, was proposed by Nagarajaiah et al.[13], and Pasala et al.[14]. The effectiveness of NSD in various configurations for seismic hazard mitigation has already been observed in various studies[15], [16]. FVD is another conventional type of passive damper that use the rheological properties of the fluid to dampen seismic vibrations.

This study introduces different types of supplemental damping devices alongside the UFREI isolation system to mitigate the seismic hazard oncoming to a benchmark structure. A wide range of ground motions consisting of both near-fault and far-fault characteristics are considered as the input ground motion. Major response parameters, i.e., Top Floor Acceleration, UFREI Displacement, and Inter-story Drift Ratios, are investigated, and a comparative study is

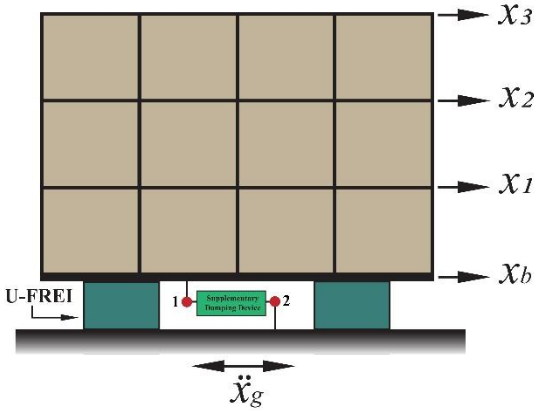


Figure 1: Base-isolated benchmark structure.

carried out for all the configurations. The supplementary damping devices are optimized, too, based on the desired response objective of the isolated benchmark structure.

2 BENCHMARK STRUCTURE AND ANALYSIS

A three-storied benchmark structure is considered from the literature, and it is isolated using UFREI in conjugation with supplemental damping devices. The vertical and rotational degrees of freedom (DOF) of the benchmark structure are condensed out, and only the horizontal degrees of freedom are retained. The first three natural frequencies of the fixed base structure are 9.95, 28.13, and 41.04 rad/s, respectively, and the first-mode damping ratio is 0.02. The isolators were designed using the design procedure proposed by Ehsani and Toopchi-Nezhad[17]. The supplementary damping device is also installed in the isolation basement only. The UFREI specifications are given in Table 1, and the isolated structure with supplementary device alongside is shown in Figure 1. The UFREI are modeled using the improved shear behavior formulation proposed by Prakash and Jangid[6].

Property	Value
Dimension	313 mm × 313 mm × 125.2 mm
Elastomer layer thickness	C23
Fiber layer thickness	C33
Number of elastomer layers	C43
Shear Modulus, G	C53
Quantity	50

Table 1: UFREI specifications

The equation of motion of the base-isolated structure with a supplemental damping device under an input ground motion is written as

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X}+\boldsymbol{\Gamma}_{\mathbf{n}}F_{\mathrm{n}} = -\mathbf{M}\boldsymbol{\Gamma}_{\mathbf{g}}\ddot{x}_{\mathrm{g}} \tag{1}$$

Here, **M**, **C,** and **K** are mass, damping, and stiffness matrices of the isolated structure, respectively; **X** is the displacement along the DOFs; F_{n} and $\boldsymbol{\Gamma}_{\mathbf{n}}$ are the combined nonlinear isolator (F_{ni}) and damper forces (F_{d}) and their positional vector respectively; $\boldsymbol{\Gamma}_{\mathbf{g}}$ is the influence vector of the input ground motion. The isolator force is divided into two components, i.e., linear (F_{li}) and nonlinear (F_{ni}). The linear component is combined into structural matrices, and the nonlinear component (F_{ni}) is clubbed with damper force to obtain F_{n}. Eq. (1) is further written into the state-space format, and the resulting system is solved using the incremental Newmark average acceleration method. Eight ground motions are taken as input excitation for the isolated benchmark building, which comprises a wide range of far-fault (FF) and near-fault (NF) ground motions. The high-amplitude-long-period pulses of NF ground motions often amplify the isolated structure's response, thus, creating the need for supplemental damping devices. The list of input ground motions is given in Table 2.

Earthquake	Station	Component	PGA, g
Northridge 1994-NF	Rinaldi	229	0.87
Northridge 1994-FF	Mulholland Drive	35	0.59
Loma Prieta 1989-NF	Gilroy Array 5	90	0.37
Loma Prieta 1989-FF	Presidio	00	0.1
Kocaeli 1999-NF	Yarimca	NS	0.23
Kocaeli 1999-FF	Ambarli	EW	0.19
El Centro 1940-FF	Terminal Substation	NS	0.35
Chi-Chi 1999-NF	TCU052	NS	0.45

Table 2: List of earthquake ground motions

3 SUPPLEMENTARY DAMPING DEVICES

Four different types of supplemental damping devices have been investigated in this study. All of the devices are two-terminal devices (Figure 5) that have been placed in the isolation basement and connected to the ground and the ground floor with two ends.

3.1 MR Damper

The MRD derives its damping capabilities from the rheological property of the MR fluid, which changes under the application of a magnetic field. The MR fluid consists of micron-sized magnetically polarizable material dispersed in a silicone oil medium. The MRD has been applied in three modes: MRD(SA), MRD(P-ON), and MRD(P-OFF). The voltage is fixed at the maximum value in MRD(P-ON) mode while it's fixed at the minimum value in MRD(P-OFF) mode. The analytical model proposed by Spencer et al.[18] is used to model the MRD (Figure 5), and the H_2/LQG control algorithm is used to obtain an optimal objective response for semi-active mode/MRD(SA). For a system defined in Eq. (2), the cost function, **J,** is defined as shown in Eq. (3) based on the control force and the objective response parameter.

$$\dot{\mathbf{Z}} = \mathbf{AZ}+\mathbf{B}F_n+\mathbf{E}\ddot{x}_g \qquad \mathbf{Y} = \mathbf{CZ}+\mathbf{D}F_n+v \tag{2}$$

$$\mathrm{J} = \lim_{\tau\to\infty}\frac{1}{\tau}\mathrm{E}\left[\int_0^{\tau}\left((\mathbf{CZ})^{\mathrm{T}}\mathbf{Q}(\mathbf{CZ})+\mathrm{F_u}^{\mathrm{T}}\mathbf{R}\mathrm{F_u}\right)d\tau\right] \tag{3}$$

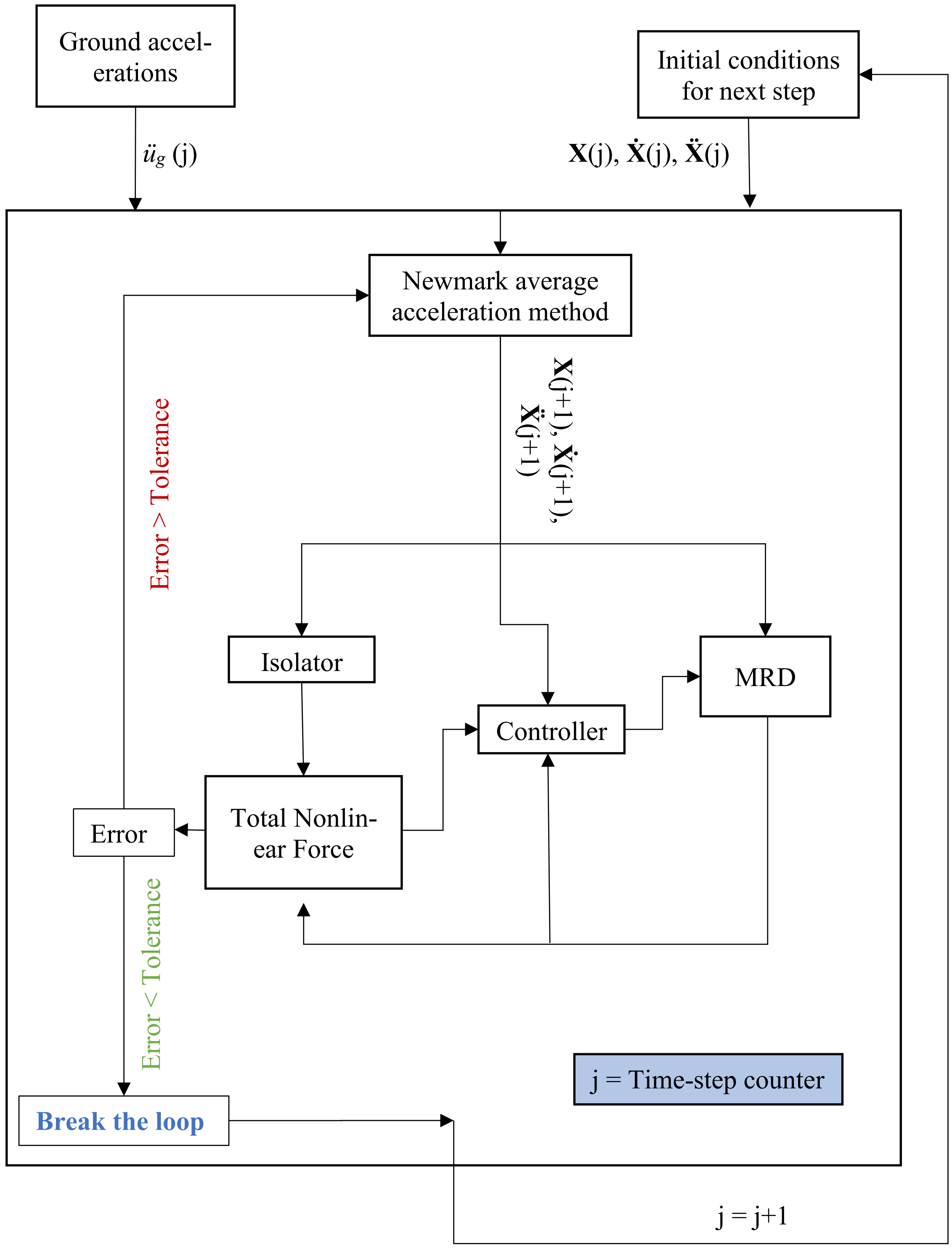


Figure 2: Nonlinear time-history analysis flowchart

Here, **Z** and **Y** are the state vector and output vector, respectively; **A**, **B**, **C**, **D**, and **E** are system matrices; **Q** and **R** are the cost parameters for the objective response and the control force, respectively. A Kalman filter is used to obtain the optimal observer due to the presence of noise.

The required control force (F_u) is calculated using the gain matrices ($\mathbf{K} = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{S}$ and $\mathbf{L} = \mathbf{S}\mathbf{C}^T$) as shown in Eq. (5).

$$\mathbf{A}^T\mathbf{S}+\mathbf{SA}-\mathbf{SBR}^{-1}\mathbf{B}^T\mathbf{S}+\mathbf{C}^T\mathbf{QC} = \mathbf{0}$$
$$\mathbf{AT}+\mathbf{TA}^T-\mathbf{TC}^T\mathbf{CT}+\gamma\mathbf{EE}^T = \mathbf{0} \quad (4)$$

$$F_u = \mathbf{K}\bar{\mathbf{X}} \quad (5)$$

$$\dot{\bar{\mathbf{Z}}} = \mathbf{A}\bar{\mathbf{Z}}+\mathbf{B}F_n+\mathrm{L}(\mathbf{Y}-\mathbf{C}\bar{\mathbf{Z}}-\mathbf{D}F_n) \quad (6)$$

The force applied by MRD is controlled by varying the voltage as per the required control force (F_u), nonlinear force (F_n), and damper force (F_d) at any instance.

$$\mathrm{V} = \mathrm{V}_{max}\mathrm{H}\{(F_u-F_n)F_d\} = \mathrm{V}_{max}\mathrm{H}\{(F_u-F_{ni}-F_d)F_d\} \quad (7)$$

Here, H{•} is the Heaviside step function. An iterative analysis is carried out to solve Eq. (**1**) at each time step, and the corresponding flowchart is shown in Figure 2. A large-scale 20-ton capacity MRD with parameters adequately scaled is adopted[19].

3.2 Tuned Mass Damper Inerter

The TMDI is a combination of a conventional TMD and an inerter in series. The TMD portion consists of an auxiliary mass (m_t), spring (k_t), and damper (c_t), while the inerter provides an inertial mass, b. A TMDI connected to a generalized single degree of freedom structure (Figure 5) can be characterized using four parameters, i.e., the auxiliary mass ratio (μ_t), the total mass ratio (μ), damping (ξ_t), and frequency ratio (f). The corresponding parameters are defined as:

$$\mu_t = \frac{m_t}{m_s},\ \mu = \frac{m_t+b}{m_s},\ \xi_t = \frac{c_t}{2(m_t+b)\omega_t},\ \text{and}\ f = \frac{\omega_t}{\omega_s} \quad (8)$$

Here, m_s and ω_s are the mass and natural frequency of the single degree of freedom system. The value of μ_t is taken as 0.01, and the optimal value of the rest of the parameters is calculated using the closed-form equations proposed by Prakash and Jangid[7]. The TMDI is tuned to the first mode of the isolated structure to control the UFREI displacement response primarily.

3.3 Negative Stiffness Damper

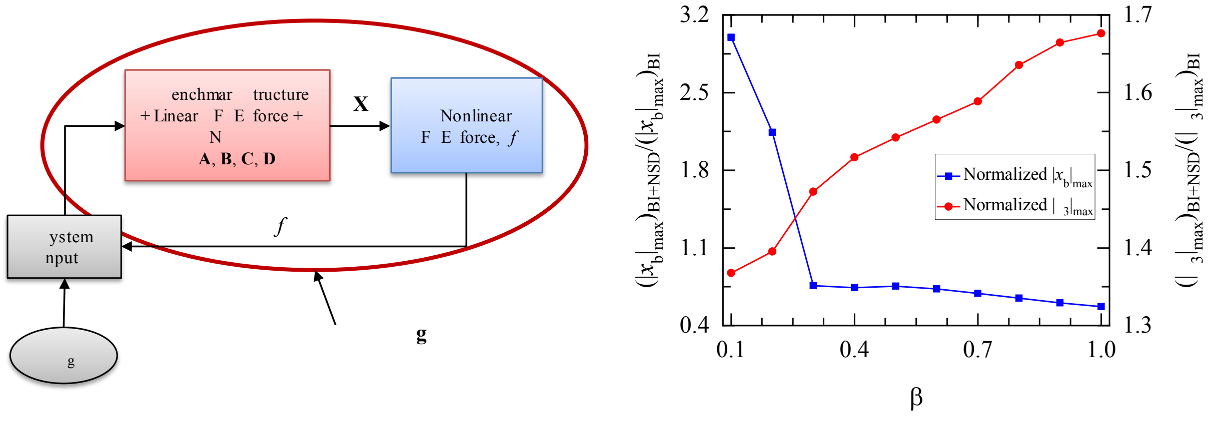
NSD is a combination of a viscous damper (c_d) and a negative stiffness spring (k_n) in parallel, working in series with a positive spring (k_p) to dampen out the incoming vibrations (Figure 5). The force generated by NSD is given as

$$f_d = k_n(y_2 - y_3) + c_d(\dot{y}_2 - \dot{y}_3) = k_p(y_1 - y_2) \quad (9)$$

Here, y_1 and y_3 are displacements at damper ends, while y_2 is the intermediate node displacement. The NSD applied to a generalized single degree of freedom structure can be characterized using four parameters, i.e., $\alpha = k_n/k_p$, $\beta = k_p/k$, $\lambda_P = \omega/\omega_n$, and $\xi_d = c_d/2m\omega_n$. The optimal NSD is designed using the expressions proposed by Islam and Jangid[16], and Eq. (9) is written in a state-space format to include f_d as a state of the system.

$$A_n\dot{f}_d = B_n(\dot{y}_1 - \dot{y}_3) + C_n(y_1 - y_3) - f_d \quad (10)$$

Here, $A_n = c_d/(k_p+k_n)$, $B_n = c_dk_p/(k_p+k_n)$, and $C_n = k_nk_p/(k_p+k_n)$. The new state is combined into the structural matrices, and the new system is iteratively solved, as shown in Figure 3(a).



(a) (b)

Figure 3: (a) Nonlinear time-history analysis flowchart; (b) Normalized response variation with β for NSD under Kocaeli 1999-NF ground motion

Optimal NSD is also dependent on a user-defined parameter, β. An increase in β increases the stiffness of the positive spring, thus increasing the acceleration response and decreasing the displacement response. The variation of normalized response parameters with β is shown in Figure 3(b). A cost function, **C,** is defined and minimized further to obtain an optimum β value.

$$\mathbf{CF} = \frac{1}{4}\text{Max}(|\ddot{x}_3|) + \frac{3}{4}\text{Max}(|x_\text{b}|) \tag{11}$$

Here, Max(P) denotes the maximum value of variable P throughout its time history. The UFREI displacement response has been given more weightage in the cost function as it is a more pronounced response in base-isolated structures, especially under near-fault ground motions. An optimal β value is calculated for each ground motion in the same way before commencing the final analysis.

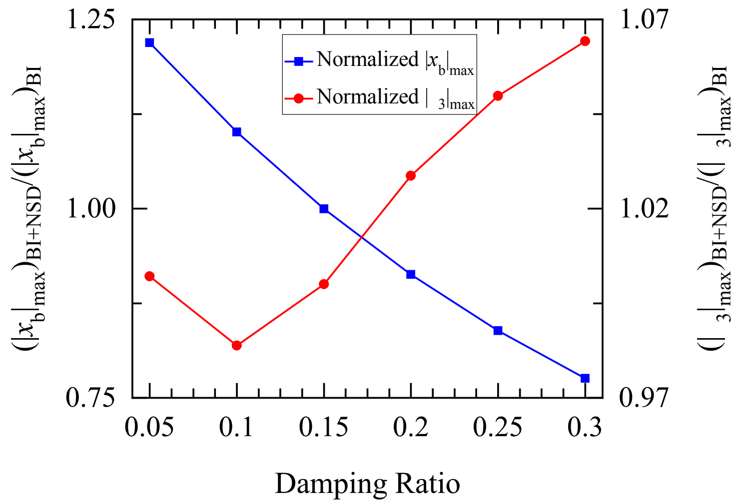
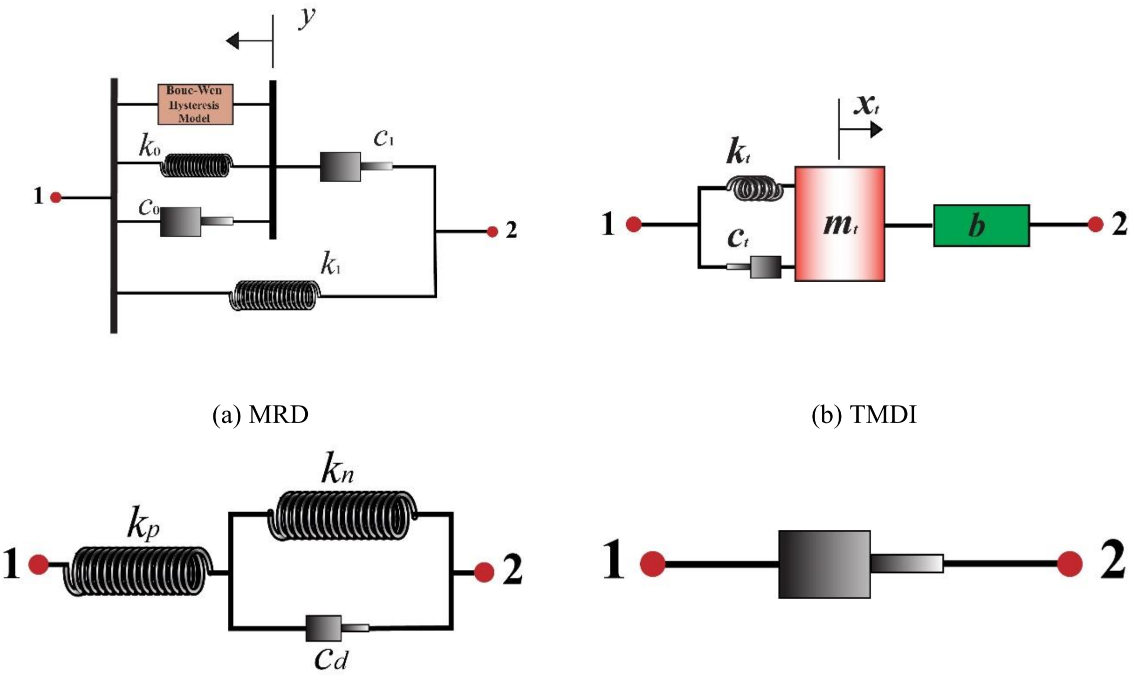


Figure 4: Normalized response variation with damping ratio for FVD under Loma Prieta 1989-NF ground motion

3.4 Fluid Viscous Damper

The FVD is a conventional damping device that derives its damping capabilities from the piston moving through a viscous fluid. There are two types of FVD models, i.e., linear and nonlinear. A linear damper force modeling approach is used in this study where the force applied by the FVD is directly proportional to the relative velocity of its two ends. Since the FVD is placed alongside the UFREI isolation system, its damping coefficient is added to that of the isolation system. The optimal damping of the FVD is again obtained by minimizing the cost function defined in Eq. (11). The variation of the normalized response parameters with the damping ratio of the FVD is shown in Figure 4. The optimal damping ratio of the FVD is calculated similarly for each ground motion before commencing the final analysis.



(a) MRD (b) TMDI

(c) NSD (d) FVD

Figure 5: Supplementary damper models

4 RESULTS AND DISCUSSION

The three major response parameters, i.e., Top Floor Acceleration, UFREI Displacement, and Inter-story Drift Ratio (IDR) of the isolated structure with various supplementary damper devices under the considered input ground motions, are investigated. The response histories of a pair of FF (Loma Prieta 1989-FF) and NF (Loma Prieta 1989-NF) ground motions are given for reference, and the maximum absolute response parameters for the rest of other ground motions are plotted together for brevity. The variation of each response parameter under different supplemental damper devices is discussed below.

4.1 Top Floor Acceleration

The significant improvement of response over uncontrolled structure is observed in every isolated configuration, irrespective of the damper type. The higher acceleration response under the NF counterpart of any of the considered ground motions over its FF counterpart can also be easily observed. The top-floor acceleration history in Figure 6 shows that almost all of the considered dampers increase the acceleration response of the isolated structure except for FVD, TMDI, and MRD(P-OFF). As the FVD only adds a linear damper to the isolated structure, and the MRD(P-OFF) also behaves passively with the voltage set to a minimum; thus, the acceleration response amplification is minimum with these two devices. The top floor acceleration response absolute amplitudes under other ground motions are also plotted in Figure 7. Since the NSD and MRD(P-ON) both stiffen the isolated structure significantly, the maximum top floor acceleration response for the isolated configuration is generally obtained with NSD and MRD(P-ON) dampers installed with isolators in conjugation. MRD(P-ON) is the stiffest configuration of the MRD, with the voltage set to the maximum throughout the input history, while NSD is also designed to give more weightage to minimize the UFREI displacement. BI+MRD(SA) generally succeeds these two configurations in generating higher acceleration responses of the isolated structure when applied alongside the UFREI isolation system. Despite these fluctuations, all of the configurations significantly improve the top-floor acceleration response of the uncontrolled structure.

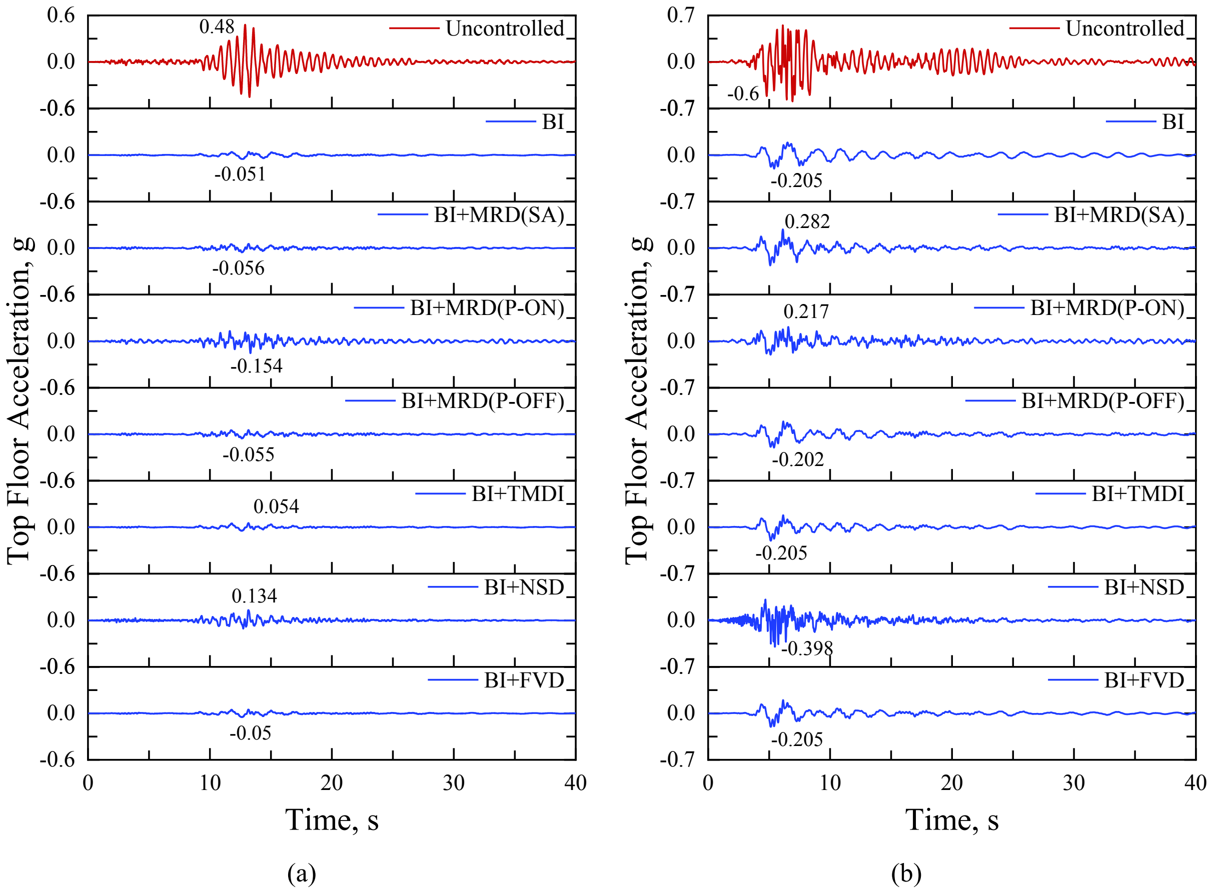


Figure 6: Top floor acceleration history under Loma Prieta 1989-FF (a), and Loma Prieta 1989-NF (b) ground motions

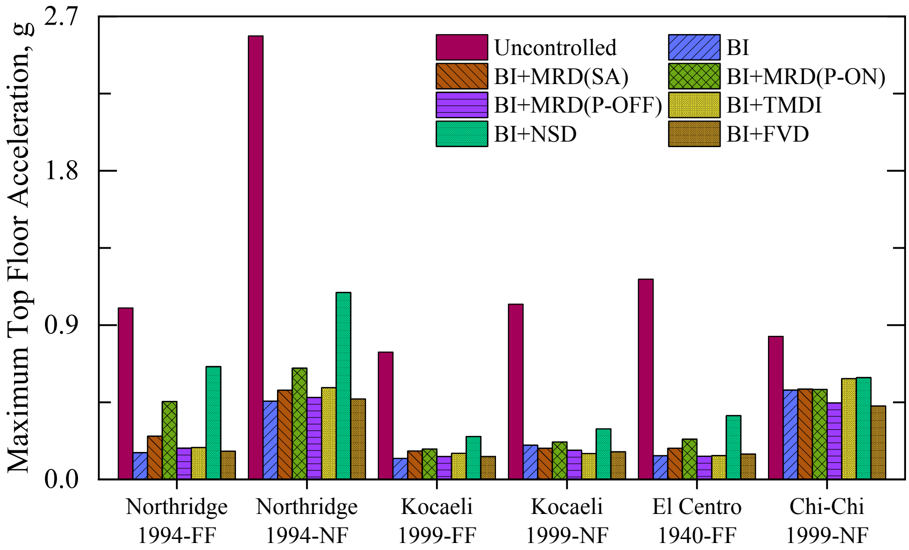


Figure 7: Maximum absolute top floor acceleration responses

4.2 UFREI Displacement

The isolator response history for different configurations of the isolated structure under the FF and NF ground motions of the Loma Prieta 1989 earthquake is shown in Figure 8. The absolute UFREI displacement amplitudes under other ground motions are also plotted in Figure 9. The extraordinary difference between the UFREI displacement response under the NF and FF ground motions of any considered earthquake event can easily be observed. It is also observed that the introduction of a damper improves the UFREI displacement response irrespective of the damper type/mechanism. The minimum UFREI displacement is achieved with the BI+MRD(P-ON) configuration as it is the stiffest. The BI+MRD(P-OFF) and BI+FVD damper configurations, which had an excellent acceleration response, have the highest UFREI displacement among all configurations. The BI+TMDI, and BI+MRD(SA) configuration has maintained a better response characteristic for UFREI displacement response too. It is also to be

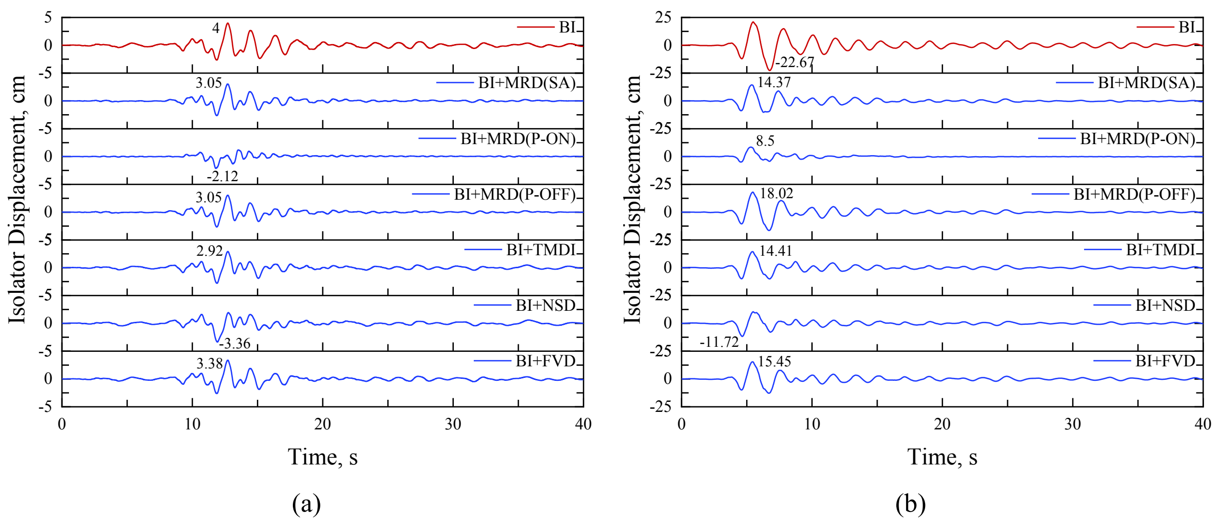


Figure 8: Isolator displacement history under Loma Prieta 1989-FF (a), and Loma Prieta 1989-NF (b) ground motions

noted that MRD(P-ON) is the costliest among all MRD configurations due to high damper forces. MRD(SA) configuration strikes a balance between the damping cost and response improvement.

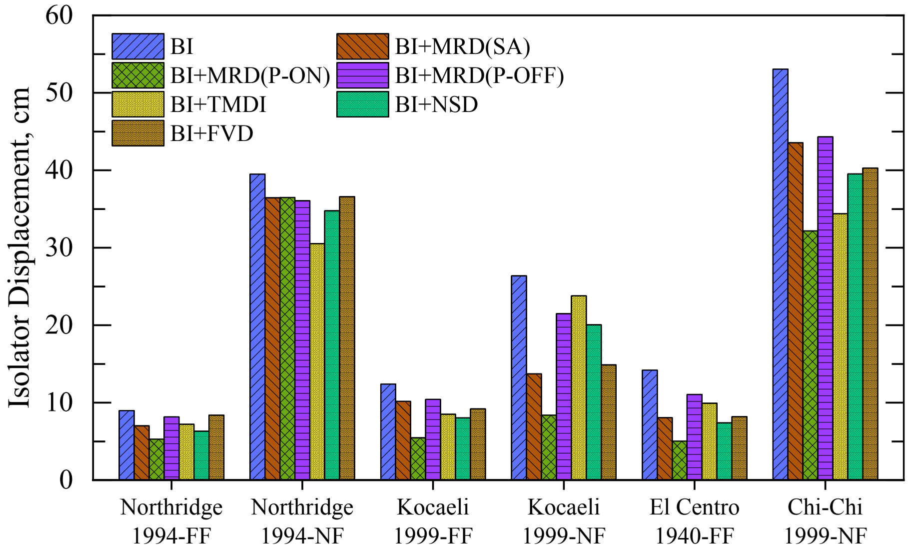


Figure 9: Maximum absolute UFREI displacement responses

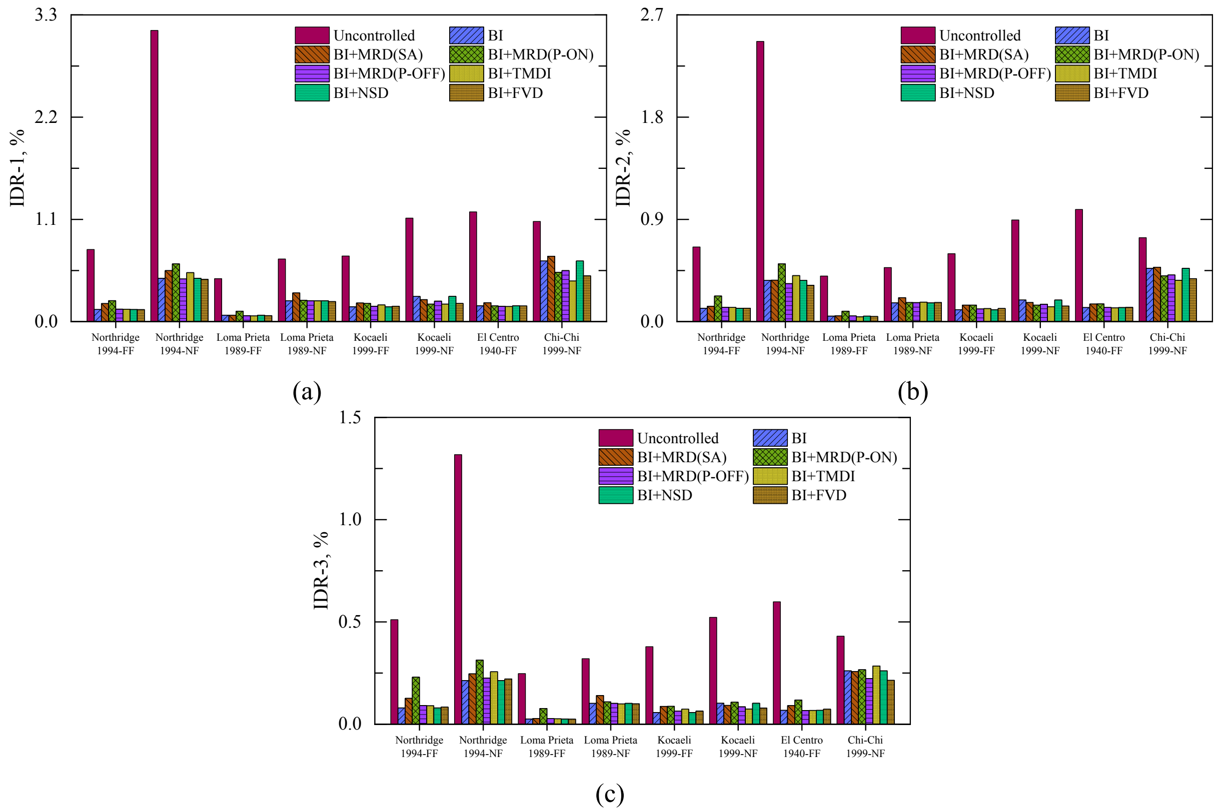


Figure 10: Maximum absolute inter-story drift responses

4.3 Inter-story Drift Ratio

The maximum absolute IDR for all three floors (IDR-1, IDR-2, and IDR-3) for all the configurations under the considered ground motions is shown in Figure 10. It can easily be observed that the IDR for the uncontrolled configuration often crosses the safe habitable limits of 0.4%, and it decreases by a large margin to safe habitable limits with the use of an isolation system. The IDR generally improves with the use of supplementary damper systems by a small margin, except for some configurations like BI+MRD(P-ON) and BI+MRD(SA), where the IDR response increases minimally.

5 CONCLUSIONS

This study investigates the effect of the introduction of supplementary damper devices on the response of structures isolated using UFREI. The UFREI isolation system is designed for isolating an existing benchmark structure, and the resultant isolated structure is further fitted with supplementary damper systems. Four different types of dampers are considered in this study, out of which one is semi-active type (MRD) and rest three are passive types (TMDI, FVD, and NSD). MRD is also used in two extra passive modes, i.e., MRD (P-ON) and MRD(P-OFF). The major conclusions from the study can be summarized as follows:

1. Passive dampers like NSD and FVD impact the acceleration and displacement responses in the opposite manner; thus, the optimal damper properties are decided using a cost function optimization approach based on the weights given to different objective response parameters.
2. The responses under NF ground motions are much higher than that under FF ground motions, especially for UFREI displacement response. While the UFREI improved the uncontrolled structure's response significantly, almost all the dampers increased the acceleration response of the isolated structure except for FVD, TMDI, and MRD(P-OFF). The minimum top floor acceleration amplitude with an isolator-damper combination is achieved with BI+MRD(P-OFF) and BI+FVD configuration, while the maximum top floor acceleration amplitude is achieved with BI+MRD(P-ON) and BI+NSD configuration.
3. The extremely high UFREI displacement response of the isolated structure under NF ground motions is significantly reduced with the use of supplementary damper devices. The minimum UFREI displacement response is achieved with BI+MRD(P-ON) configuration, while the maximum UFREI displacement response is generally achieved with BI+MRD(P-OFF) and BI+FVD configuration. While the other damper devices exchange the improvement in one response with degradation in another response, BI+MRD(SA) and BI+TMDI maintain better responses throughout.
4. The introduction of the UFREI isolation system to the uncontrolled structure brings the IDR to safe habitable limits (<0.4%). The introduction of supplementary dampers alongside the isolation system reduces the IDR minimally even further, except for the BI+MRD(P-ON) and BI+MRD(SA) configuration, where the IDR response increases minimally.

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