

THE DOME OF THE SAN FRANCESCO DI PAOLA BASIL: A REAL GEOMETRY BASED ASSESSMENT

Claudia Cennamo¹, Concetta Cusano¹, Arsenio Cutolo², Federico Guarracino², Ida Mascolo^{*2}

¹ *Department of Architecture and Industrial Design, University of Campania Luigi Vanvitelli, Aversa (CE), Italy, claudia.cennamo@unicampania.it*

² *Department of Structures for Engineering and Architecture, University of Naples "Federico II", Naples, Italy, arsenio.cutolo@unina.it, fguarrac@unina.it, ida.mascolo@unina.it*

Abstract

This study presents an assessment of the state of stress of the dome of San Francesco di Paola Basil Naples. The number of Finite Analyses of masonry structures proposed over the years is now uncountable, but most of the studies make reference to ideal geometries, even if based on actual measurements. The present work, on the contrary, does not rely upon any regular geometrical model but, instead, on what could be defined as a scan of the actual structure which accounts for both the original plan and on the inevitable construction imperfections.

It is found that these apparently negligible, but diffuse imperfection from an ideal modelling lead to a substantially different state of stress from what could be expected from a regular geometry.

The findings are discussed and it is also shown that they give reason of the actual cracking patterns on the dome.

1. Introduction

The evaluation of the stress state of masonry dome is a crucial but challenging structural engineering topic that is both very essential and practical [1-4]. The difficulty arises from the interaction between biaxial stress on curved geometries and the strongly nonlinear mechanical behavior of masonry materials. The geometry of the dome, which can migrate the loads according to a specified position, and the construction details both have a significant impact on the issue [5-15].

Therefore, it might easily result detrimental to have to assess both the material's mechanical properties and the actual geometry of the building in numerical modeling.

A nonlinear FE model based on an accurate representation of the material and of the actual shape of the dome is proposed in the current work. The well-known Willam and Warnke formulation [16], which is frequently utilized to predict failure in cohesive-frictional materials, was applied to represent the characteristic non-linear behavior of the masonry. Both cracking and crushing failure modes were accounted for.

The precise replication of the dome's shape was achieved using a 3D laser scanning survey of the structure.

In fact, the use of laser scanning technology to assess the actual construction details and the current as-built conditions has become increasingly practical, especially for historical buildings as it allows to increase considerable both the accuracy and the quantity of data.

2. History of architectural sciences and techniques

2.1 The dome of the San Francesco di Paola Basil

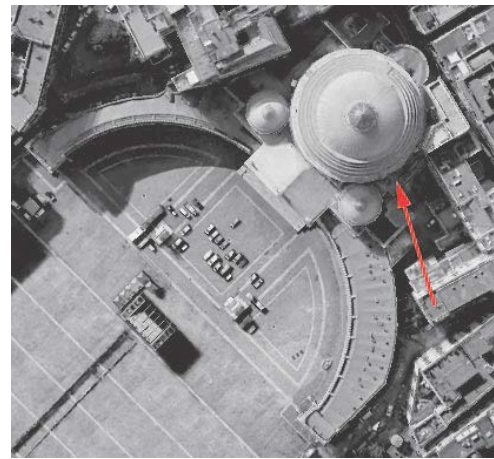
The dome of the San Francesco di Paola Basil (Figs. 1 and 2), also known as *Tempio dei Borboni*, represents the largest and most significant of the examples of Neoclassical Neapolitan architecture.

Located in Piazza del Plebiscito, Naples' historic center, it was built as a completion of the square in front of the Royal Palace in 1816. The Basilica is most notable for its imposing hemispherical dome roof that emerges from the entire complex.

The imposing roof, 53 meters high and with a diameter of 36 meters, is the only example of a hemispherical dome in Neoclassical Neapolitan architecture. It is related to the Pantheon in Rome not only in terms of geometric and proportional ratios, despite the fact that its less than perfect hemisphericity detracts from the usability of the interior spaces, but also in formal and compositional aspects. In addition, originally the dome ended with a central closing ring (oculus) in which structural thrusts still converge and balance; later the oculus was closed with the present lantern, which is actually a conical superstructure of about 9 meters in height completely glazed, allowing the filtering of light.



Figure 1. The dome of the Basilica of San Francesco di Paola



The Basilica of San Francesco di Paola was built by the will of King Ferdinand IV (1816) and designed by the architect Pietro Bianchi. Its construction is related to a series of vicissitudes and political circumstances that have contributed to determine and, above all, to affect its architectural value [17].

The present church arises in the place where, originally, in the thirteenth century, the Angevin kings had built the chapel in honour of St. Louis, King of France. In the 15th century, Ferdinando I of Aragon dedicated this chapel to Francesco di Paola as a welcome sign to the city of Naples. The Temple, in brief time, was enlarged becoming a convent. During the period of French domination (1804-1815), in the frame of the urbanization project of the city, a series of works for the modification of Largo di Palazzo (now Piazza del Plebiscito) began with the aim of providing a large public space, with a centralized structure within a semicircular perimeter (still visible today) with a colonnade (to contain the demonstrations or popular assemblies).

The constructive program underwent a radical change in 1815, during the Bourbon Restoration. King Ferdinand IV ordered to build a temple in honour of Francesco di Paola rather than a public building. After noticing the high progress degree of the construction and, therefore, the impossibility of demolishing it, the King decided to transform it to amend its elevated political significance. For the realization of the temple, a contest was organized in which the pattern of the project is evident: the Pantheon of Rome. This can be seen in the advanced body, a hexastyle peripteral temple with two pillars of Ionic order on which a triangular pediment sets. The inner

space also reflects the Roman model, especially its extraordinary caisson ceiling hemispherical dome [17].

2.1.2 Geometric and constructive details

The configuration of the Church represents the highest expression of Neapolitan neoclassical language [18]. The hemispherical dome is typically used to cover spaces with central plant, in which the character of spatial continuity predominates rather than the perception of verticality.

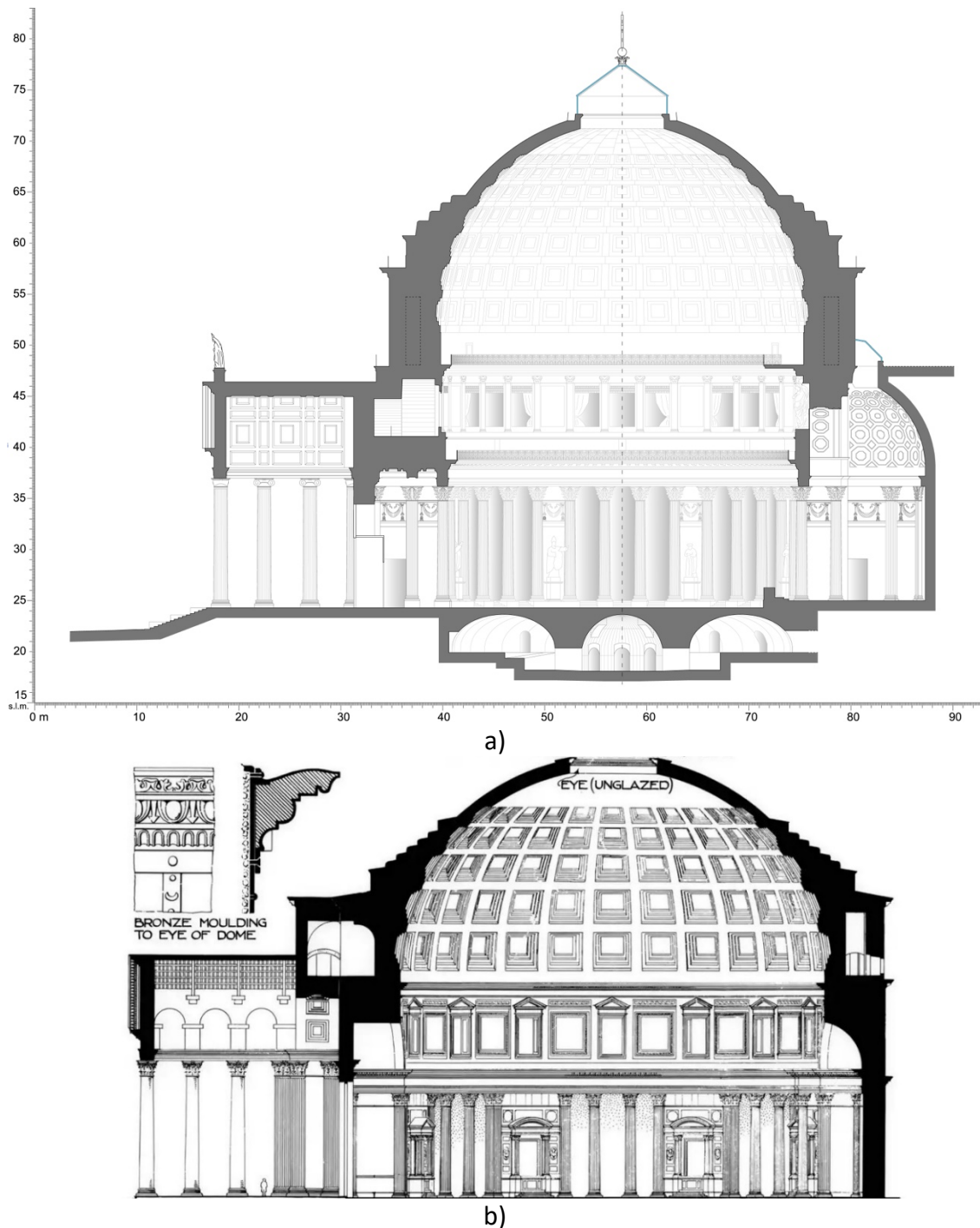


Fig. 3. Comparison between the two domes: (a) Re-drawing of the section of the dome of the Basilica of San Francesco di Paola (reworking of the original drawing by Tecno IN Geosolutions); (b) Section of the Pantheon (©Sir Bannister Fletcher [20]).

In Naples, the system of coverage with hemispheric domes has not been widely applied, not even in the case of religious buildings, except for a few examples, among which the Basilica of San Francesco di Paola that stands out either in size and notoriety. This is probably due to the fact that the typology of the hemispherical dome was born to be the right element to cover temples configured with a single room, corresponding to the typology of pagan architecture, distant from the cross plan proper to the Christian religion [17].

Undoubtedly, an eloquent example of this type of architecture is the Pantheon of Rome, which King Ferdinand IV proposed as a model of reference, either as regards to geometry and proportions as for the strictly compositional and formal configuration [17].

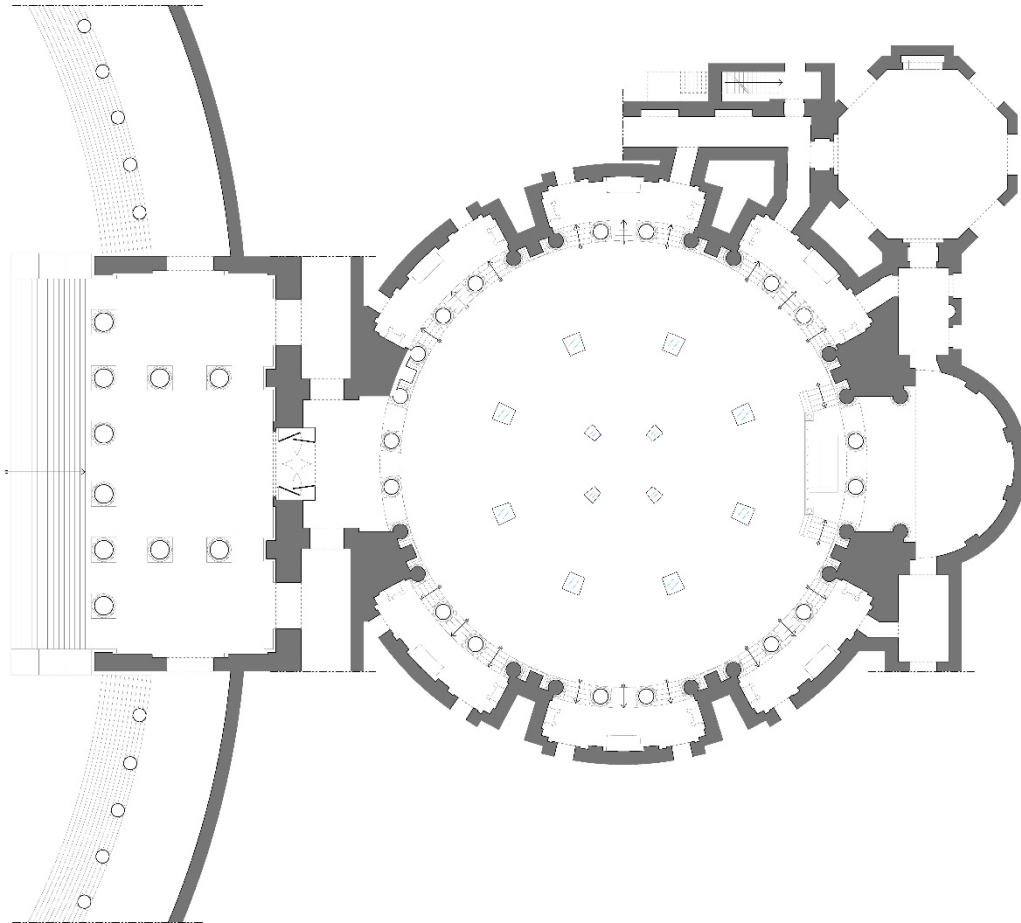


Fig. 4. The general plant of the ground floor of the Basilica of San Francesco di Paola Basil (reworking of the original drawing by Tecno IN Geosolutions).

Figure 3 shows a comparison between the Basilica of San Francesco di Paola and the Pantheon, displaying different geometric relationships. The Basilica of San Francesco di Paola has a height of 53 meters and a diameter of 36 meters approximately, in contrast the Pantheon is high 43,30 meters with an exactly corresponding diameter. This fact implies a different spatial perception, in which the dome of the Bianchi envelops a wider usable space. The illumination is attained, as in the Pantheon, through a central oculus, approximately 9 meters high, with an embossed glass cone. The dome loads on a circular base consisting of 8 pillars. To the outside the dome is covered by lead and it articulates with the vertical structure of the tambour by means of four annular steps. The dome ends with a lantern whose access is outside through the four stairs situated on the extrados of the dome.

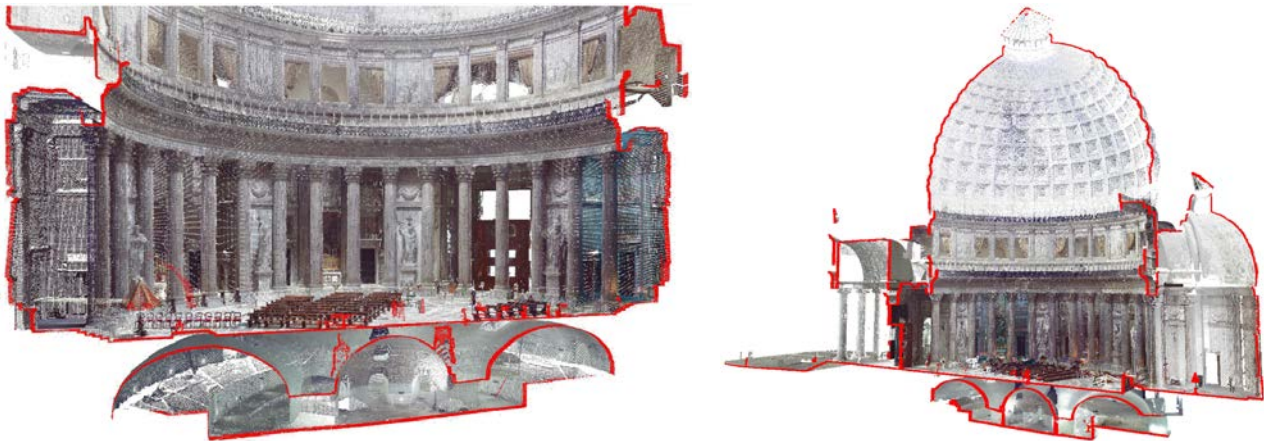


Fig. 5. Survey. Cloud of points of the inner space (courtesy of Tecno IN Geosolutions [20])

3. Numerical modelling

Many treatments have been carried out on large domes by researchers, including applying the FEM method, so there is a large bibliography on this subject of which only a few examples [22, 23] here can be given.

The finite element analysis of the dome of the San Francesco di Paola Basil was carried out by means of the commercial FE package Ansys® 2023 R1 and an ad hoc algorithm was implemented using the ANSYS Parametric Design Language (APDL) to recreate its real geometry.

The geometrical model used in the numerical simulations (Fig.6, left) was built on the basis of the survey conducted by Tecno IN S.p.A. Geosolutions (San Donato Milanese, Italy) with laser scanning technology.

In fact, laser scanning technology has already been successfully demonstrated on numerous projects related to civil engineering such as building drawings, deformation monitoring and 3D representations. Points and linear feature extraction of objects is attained using a variety of methods, which heavily depends on object characteristics.

In this framework laser scanning can be employed to scan any desired objects and virtually any target. Several scans were captured at constant grid size because more than two scan locations are necessary to build a full stereo model of any object without obscured areas.

The used cloud of points is shown in Fig.5, while Table 5 presents the basic information for point clouds measured.

Linear Tetrahedral Elements, i.e. SOLID 185, were used to discretize the physical model of the dome. The element is capable of plasticity, hyperelasticity, stress stiffening, creep, large deflection, and large strain. To determine the optimal mesh size, a mesh-convergence study was initially carried out. The degrees of the freedom at the bottom edges of the dome model were prevented in all directions (Fig. 6) and the dome was assumed subjected to its self-weight.

It was assumed that masonry would behave elastically with a modulus of elasticity $E = 21 \text{ GPa}$, a density $\rho = 25.5 \text{ kN/m}^3$ and a Poisson modulus $\nu = 0.3$.

The Concrete Material Model implemented in the Workbench environment of Ansys® was used to model the nonlinear behavior of the masonry. This model, which is based on the well-known Willam and Warnke formulation [16-18], may efficiently simulate failure in cohesive-frictional materials taking both crushing and cracking failure modes into account.

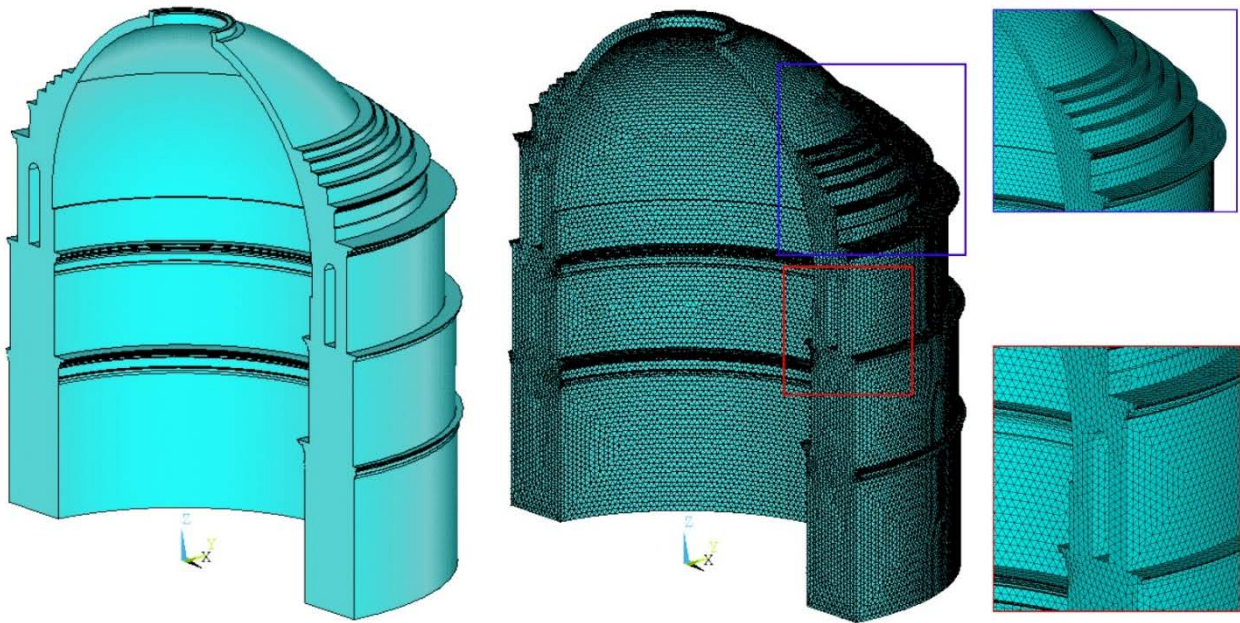


Fig. 6. Geometrical (left) and FE model of the Basilica of San Francesco di Paola dome (right).

4. Nonlinear FE analysis

Figs. 7-9 summarize the results of the performed numerical simulations in terms of Von mises and principal stresses and vertical and radial displacements. An onset of the cracking evolution is also shown in Fig. 8.

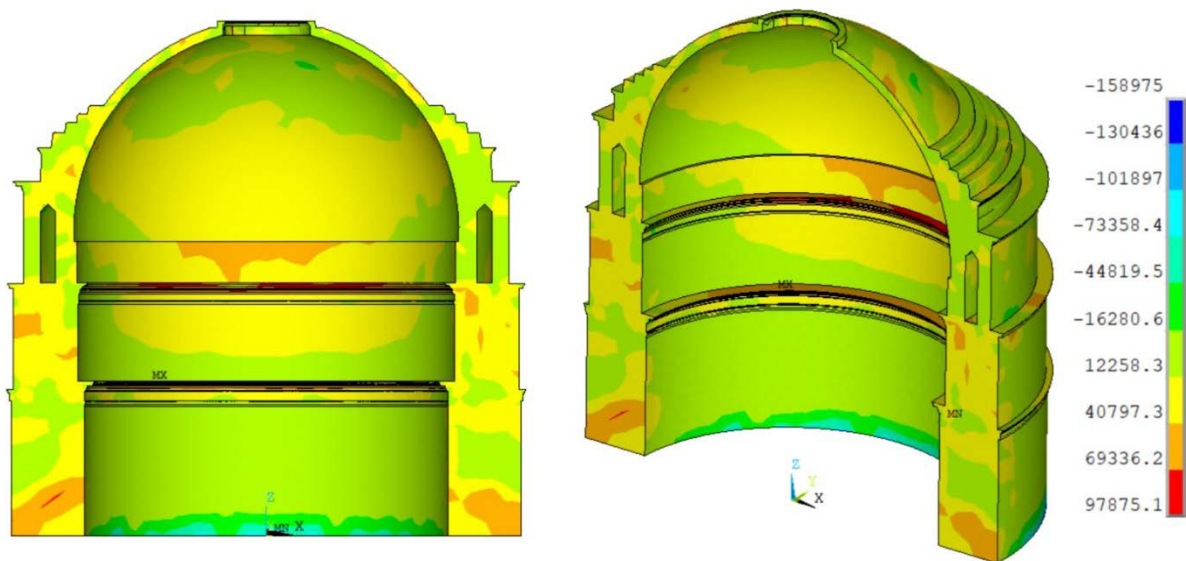


Fig. 7. 1st principal stress (Unit: Pa)

It is evident from the pictures that the behavior of the structure does not show any kind of regularity, as it could have been expected from a regular geometry, but rather a concentration of stress and fracture which coincide with the actual findings on the dome. In fact, apparently negligible, but diffuse imperfection from an ideal modelling lead to a substantially different state of stress from what could be expected.

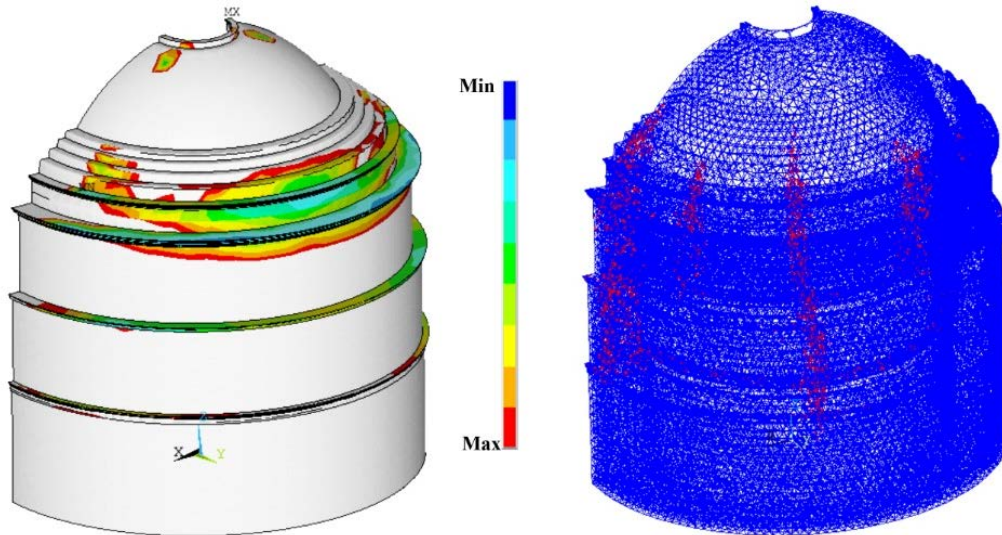


Fig. 8. Von Mises stress (left) and cracking onset (right).

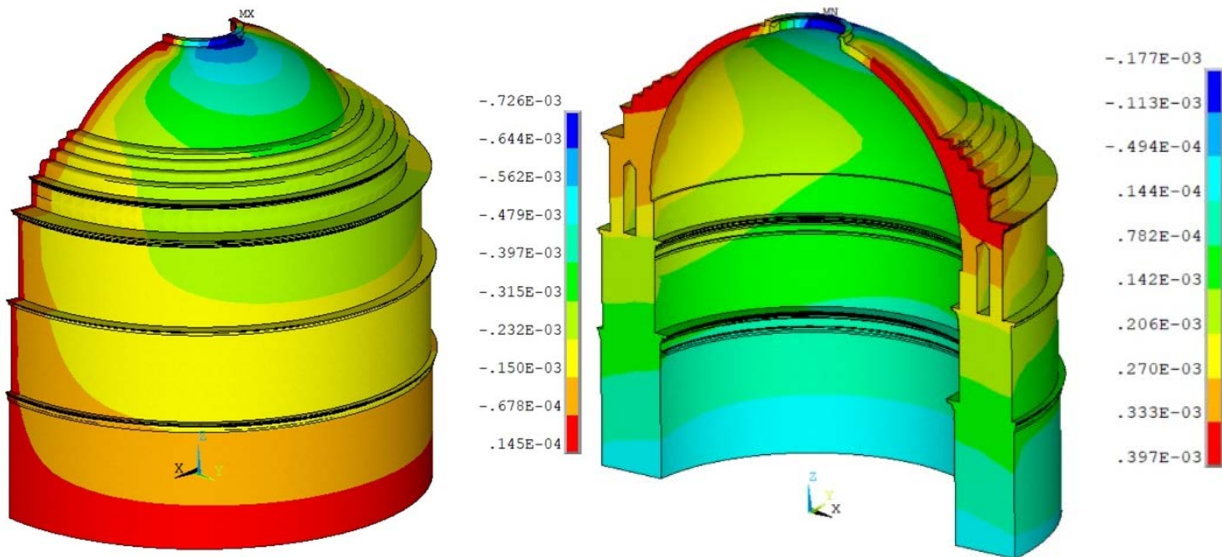


Fig. 9. Vertical (left) and radial (right) displacement (Unit: m), 1st principal stress (Unit: Pa)

5. Conclusions

The study has presented a numerical assessment of the state of stress of the dome of San Francesco di Paola Basil Naples without relying upon any regular geometrical model but, instead, on a scan of the actual structure which accounts for both the original plan and on the inevitable construction imperfections. Differently from previous works of some of the present authors [24-28], the modelling has been therefore particularly accurate.

It is found that diffuse imperfections from an ideal modelling leads to a substantially different state of stress from what could be expected from a regular geometry and give reason of the actual cracking patterns on the dome.

References

- [1] Real-time limit analysis of vaulted masonry buildings, Block P., Ciblac T., Ochsendorf J. Computers and Structures, 84(29), 1841–1852, 2006.
- [2] Seismic vulnerability of historical masonry buildings: A case study in Ferrara, Mallardo, V., Malvezzi, R., Milani, E., and Milani, G. Engineering Structures, 30, 2223–2241, 2008.
- [3] Resisting system and failure modes of masonry domes, Foraboschi, P. Engineering Failure Analysis, 44, 315–337, 2014. DOI: 10.1016/j.engfailanal.2014.05.005
- [4] Modeling strategies for the computational analysis of unreinforced masonry structures: review and classification. D’Altri, A.M., Sarhosis, V., Milani, G., Rots, J., Cattari, S., Lagomarsino, S., Sacco, E., Tralli, A., Castellazzi, G. and de Miranda, S., Archives of computational methods in engineering, 27, pp.1153-1185, 2020.
- [5] The majolica dome of Santa Maria della Sanità in Naples. Geometric configuration analysis and stability studies | La cúpula de mayólica de Santa Maria della Sanità en Nápoles. Configuración geométrica y estudios de estabilidad, Zerlenga, O., Cirillo, V., Cennamo, C., Cusano, C., Informes de la Construcción, 2021, 73(562), pp. 1–11.
- [6] Funicular analysis of masonry vaults. Computers and Structures, O’Dwyer, D.W. 73(1-5), 187–197, 1999.
- [7] Masonry constructions: mechanical models and numerical applications. Lucchesi, M., Padovani, C., Pasquinelli, G. and Zani, N. Lecture Notes in Applied and Computational Mechanics, 39, Springer, Berlin, 2008.
- [8] Failure pattern prediction in masonry. de Felice, G., and Malena M. J. Mech. Mater. Struct., 14, 663–682, 2019.
- [9] Statics of self-balancing masonry domes constructed with a cross-herringbone spiraling pattern. Paris, V., Pizzigoni, A. and Adriaenssens, S. Engineering Structures, 215, 110440, 2020. DOI: 10.1016/j.engstruct.2020.110440
- [10] Equilibrium of masonry vaults. Angelillo M., Fortunato A. in Lecture notes in applied and computational mechanics, novel approaches in civil engineering, Berlin: Springer, 14, (Fremond M, Maceri F Eds), 2004.
- [11] On rigid blocks detection in unilateral masonry-like structures: a continuous displacement fields approach. Mascolo I., Gesualdo, A., Olivieri C., Fortunato A. International Journal of Masonry Research and Innovation 7, no. 4 (2022): 395-405. DOI: [10.1504/IJMRI.2022.10046111](https://doi.org/10.1504/IJMRI.2022.10046111)
- [12] Arcos bóvedas y cúpulas. Geometría y equilibrio en el cálculo tradicional de estructuras de fábrica. Huerta, S. Madrid: Instituto Juan de Herrero, 2016.

- [13] Practical applications of unilateral models of masonry equilibrium. Angelillo, M. in Angelillo M. (ed.) *Mechanics of masonry structures*, Springer, Vienna, 109–210, 2014.
- [14] *Masonry Spiral Stairs: A Comparison between Analytical and Numerical Approaches*, Olivieri, C., Cennamo, C., Cusano, C., Cutolo, A., Fortunato, A., Mascolo, I., *Applied Sciences* (Switzerland), 2022, 12(9), 4274. DOI: 10.3390/app12094274
- [15] *Nonlinear FE Analysis of a Masonry Spiral Staircase in Nisida: a Refined Numerical Case Study*. Cutolo, A., Guarracino, F., Olivieri, C. and Mascolo, I., *International Journal for Multiscale Computational Engineering*, 2022, 20(5). DOI: 10.1615/IntJMultCompEng.2022042413
- [16] *Constitutive Model for the Triaxial Behavior of Concrete*, Willam, K.J. *Proc. of Int. Assoc. Bridge Struct. Engineers*, vol. 19, pp.1–30, 1975.
- [17] *The slicing technique for the evaluation of the formal efficiency: A comparative study*, Cusano, C., Cennamo, C., Guerriero, L., 16 - *Proceedings of the 23rd Conference of the Italian Association of Theoretical and Applied Mechanics*, 2017, 2, pp. 1543–1554.
- [18] *Stability of tunnels according to depth and variability of rock mass parameters*. Fraldi, M., Cavuoto, R., Cutolo, A. and Guarracino, F., 2019 *International Journal of Rock Mechanics and Mining Sciences*, 119, pp.222-229.
- [19] *The baroque skyline in naples. structural studies on 16TH and 17TH century domes in terms of form and stability*, Cennamo, C., Cusano, *Architecture and Engineering*, 2020, 5(2), pp. 8–16.
- [20] © Sir Bannister Fletcher, *Pantheon Section* is licensed under a Public Domain license. Accessed May 9, 2023, <https://ohiostate.pressbooks.pub/exploringarchitectureandlandscape/chapter/chapter-1/>
- [21] *La cupola di San Francesco di Paola. Geometria, costruzione e stabilità nel progetto di cupole nella prima metà del XIX secolo*. Phd Thesis, Cusano C. 2015, Università degli studi della Campania Luigi Vanvitelli.
- [22] *Considerations about the static response of masonry domes: A comparison between limit analysis and finite element method*, Cusano, C., Angjeliu, G., Montanino, A., Zuccaro, G., Cennamo, C., *International Journal of Masonry Research and Innovation*, 2021, 6(4), pp. 502–528.
- [23] *Automatic generation of statically admissible stress fields in masonry vaults*, De-Chiara, E., Cennamo, C., Gesualdo, A., Olivieri, C., Fortunato, A., *Journal of Mechanics of Materials and Structures*, 2019, 14(5), pp. 719–737.
- [24] *The neoclassical dome of san Francesco di Paola in Naples. A study on form and stability*, Cennamo, C., Cusano, C., Angelillo, M., 16 - *Proceedings of the 23rd Conference of the Italian Association of Theoretical and Applied Mechanics*, 2017, 1, pp. 1439–1448.

- [25] A study on form and seismic vulnerability of the dome of San Francesco di Paola in Naples, Cennamo, C., Cusano, C., Fortunato, A., Angelillo, M., *Ingegneria Sismica*, 2018, 35(1), pp. 88–108.
- [26] On the statics of large domes: A static and kinematic approach for San Francesco di Paola in Naples, Cennamo, C., Cusano, C., Angelillo, M., *Proceedings of the International Masonry Society Conferences*, 2018, 0(222279), pp. 504–517.
- [27] A limit analysis approach for masonry domes: The basilica of San Francesco di Paola in Naples, Cennamo, C., Cusano, C., Angelillo, M., *International Journal of Masonry Research and Innovation*, 2019, 4(3), pp. 227–242.
- [28] Graphical and analytical quantitative comparison in the domes assessment: The case of San Francesco di Paola, Cusano, C., Montanino, A., Olivieri, C., Paris, V., Cennamo, C., *Applied Sciences (Switzerland)*, 2021, 11(8), 3622.