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DEVELOPMENT OF FORMULAE FOR THE SECTION ROTATIONS DUE TO BENDING OF CURVED STEEL I-BEAMS THROUGH AI AND ML ALGORITHMS

Victor T. Chibaya[1], George Markou[1], and Nikolaos Bakas[2]

[1] University of Pretoria
cnr Lynwood Road and Roper Street, Hatfield, South Africa
u19144076@tuks.co.za; george.markou@up.ac.za

[2] National Infrastructures for Research and Technology – GRNET
7 Kifisias Avenue, 11523, Athens, Greece
nibas@grnet.gr

Abstract

Currently, there is no standardized method nor a reliable and easy to implement analytical relationship for obtaining the section rotations of horizontally curved steel I-beams that could be specified in design guides and used by practicing engineers. Analysis of horizontally curved I-beams is especially difficult to perform using traditional methods because, even under the influence of gravitational loads, the beams experience a combination of primary flexure and non-uniform torsion that result in vertical deflections that are coupled with twist rotations.

This research work proposes a systematic approach that was used to develop a simple analytical formula that could predict the sectional rotation of horizontally curved steel I-beams that were fully fixed at each end and subject to a vertical point load at their midspan. The development of such a formula had never been attempted before in international literature yet by combining data derived from 3D finite element analysis with a machine learning algorithm that makes use of higher-order nonlinear regression, a predictive formula was developed that could calculate the section's rotation of horizontally curved steel I-beams with a mean absolute error of just 1.44%*.*

Keywords: Finite Element Analysis, Machine Learning Algorithm, Steel Structures, Torsion, Horizontally Curved Beams, Closed Form Solutions.

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1 INTRODUCTION

The use of curved steel I-beams has been increasing due to, not only their aesthetic appearance in structures but also their effectiveness in industrial setups where straight beams may be impractical [1]. Horizontally curved steel I-girders are also commonly used in highway structures where they need to meet strict alignments and tight geometric restrictions, which are normally predetermined during the road design of these structures.

When a member is bent to a final radius of curvature, R, the material on the outside of the bend is stretched while the material on the inside of the bend is compressed and shortened. Figure 1 shows the curvature of a beam bent to its final radius, R, due to a bending moment, M, being applied on each end. These result in cross-sectional behaviour like that observed on a typical steel stress-strain curve as seen in Figure 2. Such behaviour of bent steel members has resulted in most guiding documents for curved member design using factors that reduce the yield strength while designing curved beams as straight members for simplicity.

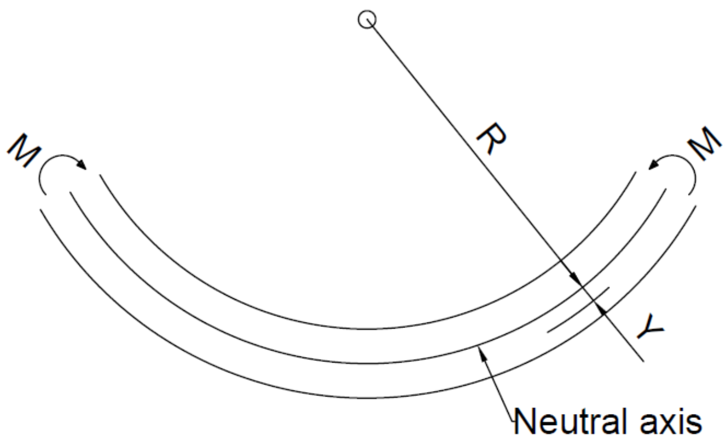

Figure 1: Curvature of a beam bent to a final radius, R, by the bending moment, M.

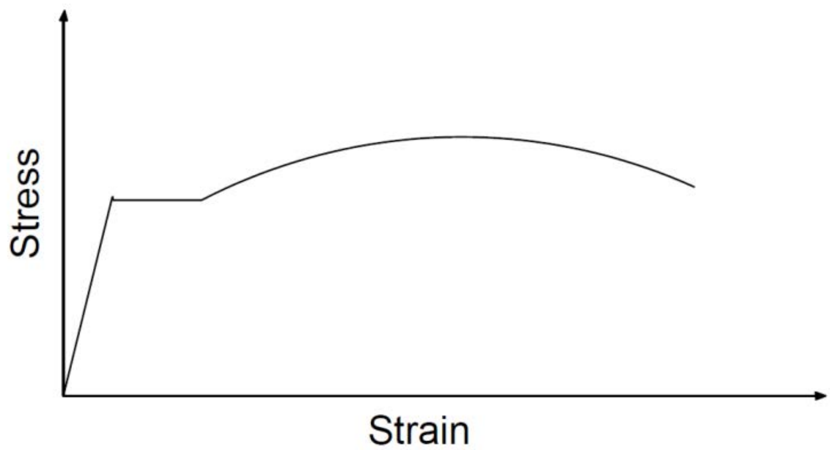

Figure 2: Typical steel stress-strain curve.

Another common method that has been used to analyze horizontally curved beams and cited in numerous literature is the use of Castagliano’s theorem [3,4,5]. Castigliano's theorem is a mathematical method used to determine the sensitivity of the total potential energy of a structural system to small displacements of its individual components. The theorem can be used to calculate the change in potential energy caused by changes in the load, shape, or material properties of the structure. The solution obtained from this theorem is typically in the form of a definite integral that is difficult to solve analytically and requires the use of numerical integration algorithms, such as those found in software like MATLAB, to obtain an accurate solution. Therefore, solving the final output obtained from Castigliano's theorem is not feasible nor is it practical for engineers.

Another method for the analysis of horizontally curved beams that is frequently cited in the literature [2,6,7] is the use of Finite Element Analysis (FEA) to model the structural members and obtain their mechanical responses when loaded. FEA modeling is currently the most well-known method used for developing the final design of curved I-beams [1] and has been proposed as the recommended method in design guides such as the American Institute of Steel Construction (AISC) design guide 33 [1]. Although accurate, this method does come with some challenges. Firstly, not all practicing engineers are equipped with either the software resources or the expertise to develop 3D detailed FEA models, to thus perform nonlinear analysis. This means that the design of horizontally curved structures becomes a very specialized task that must be reserved for larger firms with such resources and technical knowledge at their disposal. Another problem that FEA models come with is the required time that it takes to develop an accurate model and then the time it takes to run the model to obtain its structural response. This problem becomes especially evident when one must analyze a series of beams in a structure with slight variations in their geometry and loading conditions as they will be required to remodel the beam each time as well as reanalyze the models for each case.

This research work aimed to provide a simple closed-form formula that could be used by practicing engineers to obtain important serviceability values for curved steel I-beams, focusing on the prediction of the serviceability limit states rotations of horizontally curved steel I-beams with fixed supports and a vertical point load at the midspan. The study exploited the ability that ML has of identifying patterns that emerged when a large set of different horizontally curved steel I-beams were variably loaded and analyzed through FEA software. For this to be possible, a large enough dataset was developed through the numerical campaign discussed in section 2. Thereafter the data were used to train a machine learning (ML) algorithm as presented in section 3. Finally, the results, in the form of closed-form formulae, were obtained and validated as is going to be discussed in section 4 of this manuscript.

2 NUMERICAL CAMPAIGN

Currently, FEA modelling is the most common method used for analysing and designing curved members with the best level of accuracy thus far. The reliability and accuracy of FEA software is also reiterated in numerous literature and design guides including [1, 8-11]. For this reason, the use of 3D detailed modeling is adopted herein, to generate sufficient data for the development of a dataset that will be used for training a polynomial regression algorithm.

For this campaign, Reconan FEA (2020) [12] was used to generate the dataset that would train the ML algorithm. This software was selected for its ease of use, but more importantly, due to it having undergone a rigorous mesh sensitivity analysis process which validated its ability to predict the mechanical response of horizontally curved steel I-beams [11]. 300 unique data points were generated through the development of beams with different cross-sectional geometries, radius-to-span (R/L) ratios, and magnitudes of vertical point loads.

The models were developed using 8-nodal hexahedral finite elements with an element size of 50 mm, which was validated in[11]. All the models were assumed to be fixed at the supports and the vertical point load was distributed to the central cross-section of the beams to prevent local web buckling as shown in Figure 4 The models were then analyzed through the use of Reconan FEA nonlinear analysis program whereby the software was set to converge for a worktolerance of $1x10^{-4}$ thus ensuring accurate results during the vertical pushover analysis. The numerically obtained results were then stored in the form of a P-δ curve, where the deflections and sectional rotations that were assumed to form a part of the dataset derived for loads that did not exceed 50% of the ultimate failure load of the beams. This was done to restrain the rotations that derived from the analysis to a serviceability limit state loading condition. Table 1 shows a sample of the data that were used to feed the machine learning algorithm.

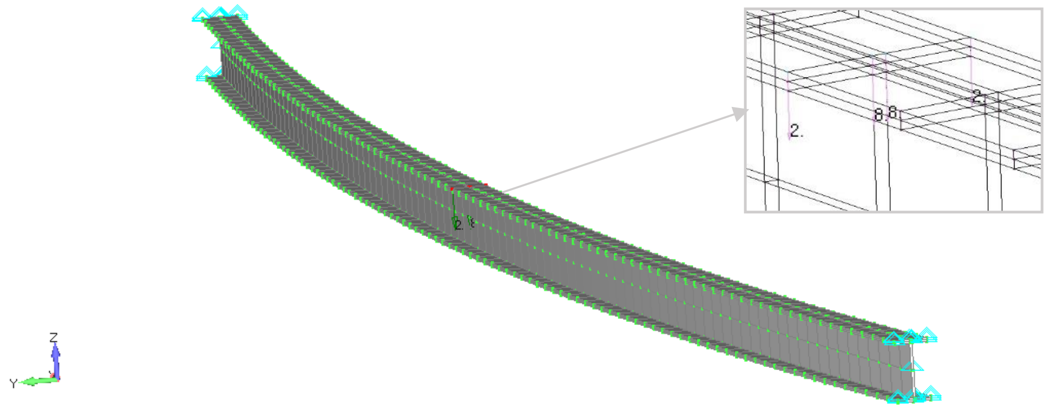

Figure 3: Example of a 3D FEA model showing the vertical point load implementation.

A	I_{xx}	I_{yy}	L	R	f_y	E	F	φ
1030	1710000	159000	5000	5000	350	200000	400	0.012828
1030	1710000	159000	5000	5000	350	200000	1000	0.032061
1030	1710000	159000	5000	5000	350	200000	2000	0.064052
1030	1710000	159000	5000	5000	350	200000	3000	0.09579
1030	1710000	159000	5000	5000	350	200000	4000	0.127568
1030	1710000	159000	5000	5000	350	200000	5000	0.160042
1030	1710000	159000	5000	5000	350	200000	7000	0.226467
1030	1710000	159000	5000	5000	350	200000	8000	0.26204
1030	1710000	159000	5000	5000	350	200000	9000	0.298897

Table 1: Sample of data that were used to train the ML algorithm.

where:

- φ Section rotation of horizontally curved I-section in radians (rad)
- A Cross-sectional Area of steel I-section in mm^2
- I_{xx} Moment of inertia of the steel I-section about the x-x axis in mm^4
- I_{yy} Moment of inertia of the steel I-section about the y-y axis in mm^4
- L Curved length of horizontally curved I-beam in mm
- R Radius-of-curvature of the horizontally curved I-beam in mm
- f_y Yielding stress of steel in MPa
- E Modulus of elasticity of steel in MPa
- F Applied vertical point load in N

Table 2 shows the range of values that were used for the generation of the data that would form the final dataset for training and testing. It is also important to state here that the proposed formulae that will be presented in section 4, are to be used for values found within the range of values presented in Table 2.

I-Beam Size	Max R/L Ratio	Min R/L Ratio	Max F (kN)	Min F (kN)
IPE100	36	1	18	0.4
305x165x46	28	1.2	250	15
406x178x54	25	3	270	21
533x210x109	35	2	500	40

Table 2: Range of values that were used for the training and the testing of the ML algorithm**.**

3 MACHINE LEARNING ALGORITHM

3.1 Julia ML Algorithm

The ML algorithm [14, 15] was developed using the Julia programming language and its basic functionality involves taking input in the form of a dataset made up of a single dependent variable and a series of independent variables for each dependent variable. The ML algorithm which is a polynomial regression method has the ability to pre-define the desired number of nonlinear features or terms the final predictive closed-form formula will have.

As previously mentioned, ML's power lies in its ability to generate predictive formulae when it identifies patterns that emerge in samples of data. This ability was exploited in this research when data was fed into the ML algorithm and a closed-form formula was expected as the output. A higher-order nonlinear regression modelling framework was used in this research mainly because of its ability to produce a closed-form formula which was also explicit [11, 14, 15]. The algorithm would then initialize the desired nonlinear features and then modify each feature to obtain a version of the feature that would result in the minimum error for the predictive formula. The program would end off by producing an analytical closed-form formula as an output in both latex and excel format with the former being for visualization and the latter being for analysis in excel spreadsheets. Bellow, the higher order regression algorithm is presented, as it was proposed in [15].

Algorithm: Higher Order Regression [15]

Input: *XX* (matrix of Independent Variables), *YY* (Vector of Dependent Variable),
*nlf (*number of nonlinear features to be kept in the model)
Output: Prediction Formulae
1. Create all nonlinear features* (*anlf*)
2. For *i* from 1 to *nlf* do
3. For *j* from 1 to *anlf* do
4. Add j^{th} feature to the model
5. Calculate Prediction Error, $MAPE_j$**
6. End
7. Keep in the model the j^{th} feature which yields the minimum prediction error
8. End

Return: Prediction Formula

*with all inter-item combinations up to the 3rd degree, **Mean Absolute Percentage Error (MAPE).

3.2 Predictive Model

Using the higher-order nonlinear regression algorithm, five different predictive models were developed for the calculation of the section rotation of curved steel I-beams by using 10, 15, 20, 50, and 100 formula features. The reason for creating five different equations was to observe the effects that varying the nonlinear features would have on the resulting predictive models' accuracy so that an optimum closed-form formula could be obtained. Fewer features would be ideal as they would result in a simpler equation however the accuracy must also be acceptable. Therefore, the objective herein was to develop a formula that had the highest accuracy possible with as few features as possible. It is important to state at this point that the proposed large in-terms of features number formulae are not to be used for calculating the sectional rotation by hand, but to be programmed in an Excel sheet or a windows-based application, where the user will be able to enter the input and in a matter of milliseconds obtain the required result.

4 RESULTS AND VALIDATION

4.1 Results

Five different predictive models were developed for the needs of this research work, each with a varying level of effectiveness when it came to predicting the section rotations of the beams that were investigated. The different models were evaluated against 300 additional results, known as out-of-sample data, obtained from additional numerical analysis performed to develop a new dataset for validation purposes. In order to describe how effective, the proposed models are, namely the R^2 value that was obtained from each predictive model and the standard error (SE) when predicting the section rotations of the horizontally curved steel I-beams were used. Equations 1, 2, 3, 4, and 5 show the latex versions of the 10, 15, 20, 50, and 100 feature formulae, respectfully.

$$\varphi = -8.03327E-16*E*E*A+9.41591E-21*E*E*Ixx+3.61267E-16*E*E*L-1.29349E-06*R+4.08848E-16*F*E*R+1.99174E-19*F*R*Ixx+4.48065E-16*fy*R*Iyy-7.32677E-19*R*L*Ixx-2.06655E-14*F*A*R+2.46929E-18*F*R*Iyy+1.13072E-01 \tag{1}$$

$$\varphi = -1.15005E-15*E*E*A+1.67432E-20*E*E*Ixx-4.00973E-17*E*E*L-1.68103E-06*R+3.52258E-16*F*E*R+2.13730E-19*F*R*Ixx-1.58515E-16*fy*R*Iyy-6.54406E-19*R*L*Ixx-1.87065E-14*F*A*R+1.58033E-18*F*R*Iyy-3.81676E-20*F*E*Ixx+2.50985E-10*F*L+2.30051E-15*E*R*A-3.04071E-19*F*L*Ixx+2.46966E-22*F*Iyy*Ixx+1.99542E-01 \tag{2}$$

$$\varphi = -1.52582E-16*E*E*A+1.01691E-21*E*E*Ixx+1.59637E-16*E*E*L-1.74218E-06*R-6.88025E-16*F*E*R-3.18988E-19*F*R*Ixx-8.91276E-18*fy*R*Iyy-2.21058E-19*R*L*Ixx+3.63844E-14*F*A*R-5.28108E-18*F*R*Iyy-3.35373E-20*F*E*Ixx+5.21729E-09*F*L+1.00235E-15*E*R*A+1.23758E-17*F*L*Ixx+2.28214E-22*F*Iyy*Ixx-1.31724E-12*L*A*F+1.64436E-16*F*Iyy*L+1.34740E-20*R*R*Ixx+9.73907E-12*R*R-1.33033E-15*R*R*A+1.82988E-02 \tag{3}$$

$$\varphi = 1.64715E-15*E*E*A-3.96688E-20*E*E*Ixx+6.04561E-10*E*L-1.09453E-10*E*R-3.85144E-16*F*E*R-3.23805E-19*F*R*Ixx-1.35781E-16*E*R*Iyy-1.87896E-17*R*L*Ixx+3.23452E-14*F*R*A-3.67593E-18*F*R*Iyy+4.47832E-18*F*fy*Ixx+9.80147E-15*F*E*L+5.20442E-08*R*A+5.29855E-18*F*L*Ixx+6.10568E-22*F*Iyy*Ixx-1.74520E-13*L*A*F-7.56775E-17*F*L*Iyy-6.59704E-19*R*R*Ixx+5.54764E-13*fy*R*R+5.70491E-14*R*R*A+4.00407E-15*F*L*R+1.18738E-18*F*A*Ixx-7.23681E-14*L*L*F+2.21952E-18*R*R*F-8.73430E-14*R*R*L+8.16511E-22*fy*Ixx*Ixx+2.17341E-15*R*L*Iyy-4.40380E-18*R*R*Iyy-4.68572E-12*R*L*A+2.32864E-12*R*L*L-1.01659E-13*E*L*L-1.84491E-21*R*Iyy*Ixx-1.08939E-19*F*R*F+8.02472E-24*F*Ixx*F+1.94167E-10*R*R-2.26679E-17*E*fy*Ixx-1.29234E-08*R*L+3.94738E-11*F*E-9.23041E-12*F*A*fy+8.02472E-24*Ixx*F*F+4.61683E-22*R*Ixx*Ixx+1.97059E-21*F*Iyy*Iyy+1.42889E-24*E*Ixx*Ixx+4.85550E-24*fy*fy*fy+4.47832E-18*F*Ixx*fy+1.71857E-12*Iyy*A+1.12782E-13*F*E*fy+1.35983E-17*Iyy*Ixx+3.45464E-07*fy*L-3.85144E-16*E*F*R-7.18972E-01 \tag{4}$$

$$\varphi = 3.43268E-16*E*E*A-4.49669E-15*E*Ixx+1.16876E-12*E*fy*L+5.33853E-16*E*E*R-1.28381E-16*F*E*R-3.23805E-19*F*R*Ixx-1.68196E-17*E*R*Iyy-4.78074E-17*R*L*Ixx+1.07817E-14*F*R*A-3.67593E-18*F*R*Iyy-1.32273E-21*F*E*Ixx+5.68313E-10*F*L+4.25262E-12*fy*R*A+2.33394E-18*F*L*Ixx+3.42581E-22*Iyy*Ixx*F-1.06528E-13*L*A*F-8.48082E-17*F*L*Iyy-3.29852E-19*R*R*Ixx+6.47225E-16*E*R*R+2.85246E-14*R*R*A+1.33469E-15*R*L*F-1.01755E-15*F*fy*Iyy-2.41227E-14*F*L*L+2.21952E-18*F*R*R-8.73430E-14*R*R*L+9.66870E-24*Iyy*Iyy*Iyy+1.75330E-15*R*L*Iyy-1.46793E-18*R*R*Iyy-7.78678E-13*R*L*A+1.16432E-12*R*L*L-5.44420E-12*fy*L*L+3.73363E-18*R*Ixx*A-1.08939E-19*F*F*R+8.02472E-24*F*Ixx*F+6.10118E-08*fy*R-1.28381E-16*E*F*R-7.04904E-23*R*Ixx*Ixx+8.02472E-24*Ixx*F*F-1.46793E-18*R*Iyy*R+5.68313E-10*L*F+1.33469E-15*F*L*R-3.52464E-11*fy*R*L-2.56762E-11*F*R-9.31124E-19*Iyy*Ixx-5.10857E-15*E*A*F+1.07817E-14*F*A*R-8.64388E-15*A*A*F+3.42581E-22*F*Iyy*Ixx-1.28381E-16*F*R*E-1.46793E-18*Iyy*R*R+1.62375E-12*F*fy*L+1.27644E-10*F*fy*fy+1.07817E-14*R*F*A-2.19478E-16*R*Iyy*A+1.85345E-18*E*Iyy*A-3.29852E-19*R*Ixx*R+2.85246E-14*R*A*R-1.32273E-21*E*F*Ixx+1.16432E-12*L*R*L+1.82151E-23*F*Ixx*Ixx+2.33394E-18*F*Ixx*L-7.33607E-14*F*fy*R-5.10857E-15*A*E*F-8.99338E-10*Ixx+3.42581E-22*Ixx*Iyy*F-4.71965E-14*R*A*A+3.90344E-11*fy*Iyy-7.33607E-14*F*R*fy+3.92307E-08*fy*A+3.55515E-13*R*Ixx+3.43268E-16*E*A*E-1.10139E-12*L*L*L+3.73363E-18*R*A*Ixx+3.55460E-18*F*A*Iyy+7.44208E-15*E*R*A+4.44375E-21*F*Iyy*Iyy-6.16812E-14*E*R*L-7.78678E-13*R*A*L+1.75959E-20*Iyy*Iyy*A-5.44420E-12*L*fy*L+3.73363E-18*Ixx*R*A-1.68196E-17*E*Iyy*R+2.12708E-13*fy*A*A+1.48842E-09*R*A+3.70690E-13*Iyy*A-2.41227E-14*L*L*F+1.29445E-10*R*R-3.56142E-13*F*Iyy-1.28477E-17*E*fy*Ixx+6.47225E-16*R*E*R-2.41227E-14*L*F*L+7.44208E-15*E*A*R-4.71965E-14*A*R*A+2.59436E-14*A*A*A+1.33469E-15*F*R*L-1.32273E-21*E*Ixx*F+1.06771E-10*E*R-3.36392E-12*R*Iyy+2.04532E-15*E*E*L+4.25262E-12*fy*A*R-5.80819E-01 \quad (5)$$

4.2 Coefficient of Determination R^2

To obtain the R^2 value of a regression model, five steps had to be taken. The first three steps involved the calculation of some values necessary to obtain the R^2 value namely the sum of squared errors (SSE), the sum of squares regression (SSR), and the sum of squares total (SST). The SSE was obtained by summing the square of the difference between each value predicted by the respective model and the mean of the data obtained from the numerical experiment. This relationship is shown in Equation 6 with φ_{mean} being the mean of the section rotations obtained from the numerical experiment, φ_i being the predicted section rotation in the *i* position of the dataset, and *n* being the total number of experimental results being used in the test.

$$SSE = \sum_{i=1}^{n} (\varphi_i - \varphi_{mean})^2 \quad (6)$$

The next value that was required to be computed, was the SSR, which is obtained by summing the squares of the difference between the predicted value and the respective value obtained from the numerical experiment as shown in Equation 7 where φ_{ei} is the section rotation obtained from the numerical experiment in the *i* position of the data set.

$$SSR = \sum_{i=1}^{n} (\varphi_i - \varphi_{ei})^2 \quad (7)$$

The final value that needed to be obtained before calculating the R^2 was the SST, which is the SSR, and the SSE added together as shown in Equation 8.

$$SST = SSR + SSE \tag{8}$$

The R^2 could then be calculated using Equation 9. This statistic always appears better when a dataset is large and may misrepresent the effectiveness of the model, which is why it was necessary to adjust it using Equation 10 where *k* was the number of terms present in the predictive model.

$$R^2 = 1 - \frac{SSR}{SST} \tag{9}$$

$$R^2{}_{adjusted} = 1 - (\frac{350 - 1}{350 - k - 1} \times (1 - R^2)) \tag{10}$$

Using this value as an initial check allowed to eliminate predictive models that were not statistically good enough by having an R^2 value lower than 0.9. It can be seen in Table 3 shows the R^2 values for the five predictive models, where it is easy to observe that the 10- and 15-feature formulae did not qualify for the 0.9 rule. It can also be seen that the 50-feature formula had the same R^2 value as the 100-feature formula even though there was an increase in the number of features being used in the 100-feature formula. This is attributed to the fact that the 50-feature formula manages to achieve maximum accuracy without overfitting the results.

K features	10	15	20	50	100
R^2 (adjusted)	0.68	0.73	0.94	0.95	0.95

Table 3: R^2 values for five predictive models containing k features.

4.3 Standard Error

The R^2 served as a starting point in validating the predictive model, whereas a more reliable validation that was used was the SE derived from the predictions of each regression model. What this value represents is how likely it is for the regression model to be inaccurate when predicting the section rotations of curved steel I-beams on the validation dataset. The SE was obtained using Equation 11 and the results for the five predictive models are shown in Table 4. As can be depicted, the formulae derived a relatively low error, with the 50-feature formula achieving a minimum error compared to the rest of the formulae.

$$SE = \sqrt{\frac{SSR}{n - k}} \tag{11}$$

K features	10	15	20	50	100
SE (%)	3.709	3.430	1.443	1.358	1.488

Table 4: Standard Error (SE) for the five predictive models containing k features.

To further observe the validation results, the data were plotted for visualization purposes. The first plot was generated to observe whether there did exist a R^2 between the predicted and the experimental data. Therefore, a plot was developed of the rotation data against one of the independent variables, the force in this case, thus observing whether the data set overlapped. The results of these plots can be seen in Figure 4 which shows the predicted and the numerical experiment data plotted against the applied force for the 10, 15, 20, 50, and 100 feature formulae, respectively.

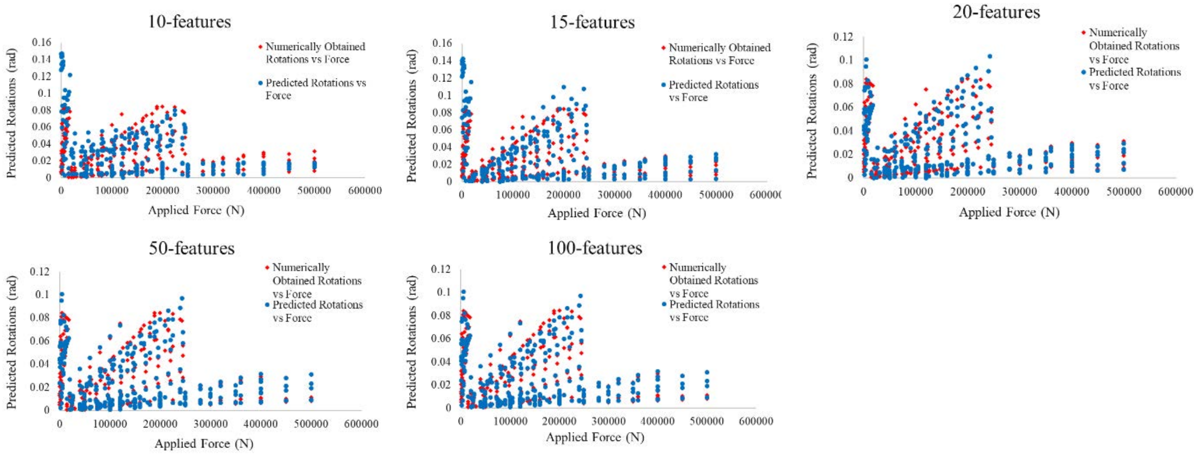

Figure 4: Predicted and numerical section rotations vs force as they resulted from each predictive formula.

It is easy to conclude that these graphs although helpful, fail to represent exactly how accurate the predictive models were, which is why the predicted rotations were then plotted against their respective values derived from the nonlinear analysis. Figure 5 shows the predicted section rotations plotted against the experimental values for all the developed predictive models.

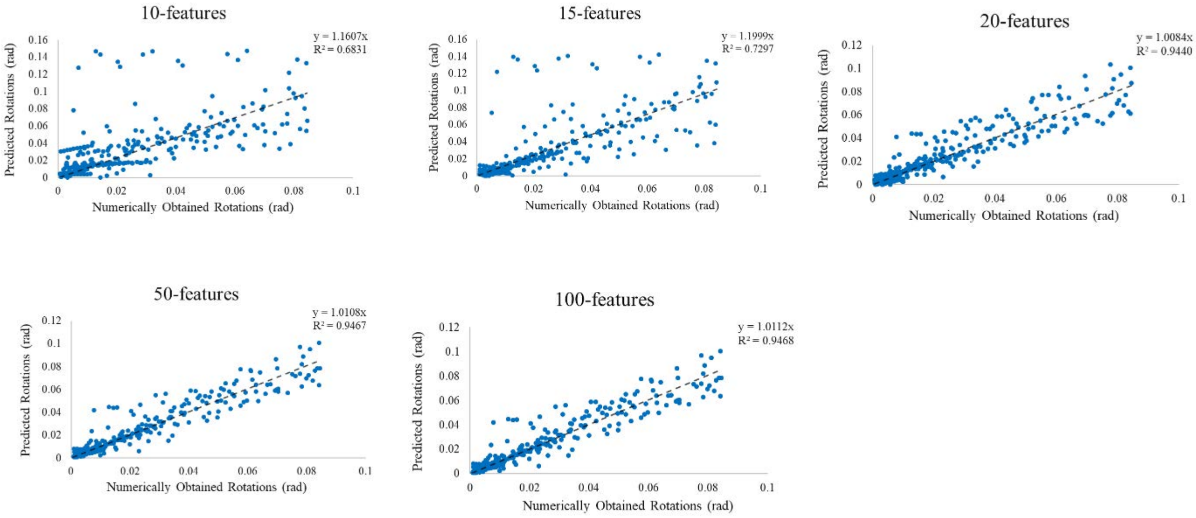

Figure 5: Predicted vs Numerical section rotations for predictive formulae.

Furthermore, the 20-, 50-, and 100-feature formulae were found to be the most accurate in producing results that correlate with the numerically obtained data producing slopes that range from 1.0084 to 1.0112, which are very close to 1 ($y = x$ curve). According to the form of these graphs, and the SE values derived from the proposed predictive models, it is safe to conclude that the 50- and 100-feature formulae derive the best fit to the under-study validation dataset.

5 CONCLUSION

The objective of this study was by using an ML algorithm to develop an analytical formula that could predict the section rotations of horizontally curved steel I-beams. This study foresaw the collection of data through the use of 3D detailed nonlinear FEA. According to the numerical results presented in this research work, the use of a higher-order nonlinear regression ML algorithm led to the development of a 50-feature formula that derived a standard error of 1.44%. This shows that the accuracy of the developed formula is extremely high, where the ability to predict the steel I-section rotations is very accurate. This formula is the first of its kind in the international literature, where additional research work is deemed necessary in expanding the current dataset.

In addition to developing a formula that could predict the rotation of curved steel I-beams, this study also highlighted the capabilities of ML and how it can be useful to civil engineering regulatory authorities by creating standard design formulae that can, soon, be featured in design codes as part of a reliable design approach adopted by practicing engineers in computing the expected rotations of curved steel I-beams.

6 ACKNOWLEDGMENTS

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