

TWO-BODIES VERTICAL SPANNING WALL RESTRAINED BY A FLEXIBLE TIE ROD

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Abstract

In unreinforced masonry structures, one of the most dangerous events that can occur during earthquakes is an out-of-plane mechanism. The use of tie rods can be a powerful tool in preventing these types of failure mechanisms, especially given the simplicity and the low cost of the strengthening intervention. When the wall is restrained to a horizontal element, such as a tie rod, its response significantly changes. The collapse, in this case, could take place for slipping / failure of the tie rod or for overturning of the wall, following the formation of a crack at an intermediate height between the base and the top.

In the literature are present models accounting for geometric non-linearities in the case of a wall rigidly restrained at the top. The same can be stated for the model free at the top, which considers four different patterns. No model accounting for all previously mentioned phenomena while also presenting a non-rigid restraint, is available. Therefore, a model formed by two bodies of finite thickness and a flexible restraint at the top capable to capture the dynamic response of the system is proposed here. The model is characterized by four motion patterns, and it can detect the transition among them, as well as the collapse mechanisms which can occur for overturning of the entire wall or of the upper body alone.

Explanatory time-histories describe the complex response of the system to recorded ground motion, pointing out its highly non-linear behavior. Finally, the investigation on the influence of the tie rods diameter confirms that the main advantages are obtained for tie rods characterized by a large cross-section.

Keywords: Multi-Rocking-Body Dynamics; Out-of-plane mechanism; Tie rods; Unreinforced masonry; Rocking motion.

1 INTRODUCTION

Out-of-plane (OOP) failure for masonry structures is one of the most complex and ill understood subject of seismic analysis according to Paulay and Priestley [1]. If the connections are inadequate, unreinforced masonry (URM) structures are particularly vulnerable to out-of-plane mechanisms during earthquakes [2–6], assuming no masonry disintegration occurs [4].

To prevent out of plane collapse several techniques were developed in the past [7,8], with tied rods being one of the most widely used intervention because of its simplicity and effectiveness [9]. However, if the walls are supported by horizontal elements, the response to seismic ground motion can be very different than when unrestrained. In these cases, collapse may occur as a result of failure of the tie rod (or its connection to the wall), or due to the overturning of the wall following the formation of a crack at an intermediate height between the base and the top [10–12]. In the case of small diameter and large length tie rods [13] assuming a rigid top support may be crude, and an elastic top restraint must be introduced [14]. This boundary condition delivers a system with two degrees of freedom (DOFs), similar to the one observed in a stack of two bodies that are free at the top, as studied by Psycharis [15] and Spanos et al. [16]. Therefore, the complexity of the problem increases as different patterns (or rocking modes) are possible [17].

Further, similar studies were conducted to assess the out of plane response of masonry walls connected to flexible diaphragms [18]. The response of single-story and two-story one-way spanning walls elastically restrained at the top was studied by Derakhshan et al. [19], who formulated a model disregarding the thickness of the façade. This assumption was also made by Landi et al. [20] in their study of a single-story URM wall. Penner and Elwood [21] conducted full-scale shaking table tests on five masonry wall specimens, which were connected to a steel frame by elastic springs. The inertia forces on the wall and spring reactions initiate the rocking motion as two semi-rigid bodies, causing a crack to form at an intermediate height. Derakhshan et al. [22] developed a three DOFs model to consider both the wall thickness and the deformation of the base diaphragm, emphasizing the strong influence of diaphragm stiffness on the response of the wall.

In the following, a the multi-rocking-body dynamic (MRBD) model to investigate the response of a two-bodies vertical spanning wall restrained by an elastic tie rod is briefly summarized, while a complete presentation can be found elsewhere [23]. Despite the simplicity of the model, the dynamic response is highly nonlinear. This behavior is due to different phenomena: when in motion the system may assume different configurations, with the transition between them due either to impacts or crack opening caused by ground acceleration. In this paper the response of a specific case study to a recorded accelerogram is shown, and the main features of the response are discussed. Finally, in addition to [23], the effect of the tie rod diameter is explored.

2 FORMULATION

The OOP response of a vertical spanning wall, restrained by an elastic tie rod, is modeled by means of two stacked rigid bodies connected to an elastic spring at the top (Figure 1). In the model, friction is assumed large enough to prevent any sliding. The behavior of the tie rod is infinitely elastic, meaning that the collapse can occur only for overturning of the wall (or part of it). The heights of the lower and upper bodies are $2h_1$ and $2h_2$, respectively, while $2h$ is the total height of the wall. The thickness of the lower body is $2b$, while the thicknesses of the interfaces at the bottom of the lower and upper bodies are $2b_1$ and $2b_2$, respectively. The masses of the lower and upper bodies are m_1 and m_2 , respectively, while m is the total mass of the wall. Further, the stiffness of the tie rod is k_{TR} . During the motion, the vertical spanning strip wall

attached to a flexible top restraint undergoes four different patterns, each having symmetric cases (Figure 1).

The formulation of the equation of motion of complex systems can rely on energy principles [24]. For each pattern, the analytical equation of motion is formulated within a Lagrangian approach to avoid the computation of internal forces. This approach allows for the use of generalized coordinates to describe the motion of the system and to define all kinematic quantities. In the case of rocking rigid bodies, it is convenient to describe the movement of the system (therefore of each point) by means of rotations, namely θ_1 for the lower body and that θ_2 for the upper body. The scalar parameters of kinetic energy T and potential energy V , as well as non-conservative generalized forces Q_i are computed, in order to assemble the Lagrangian equation of motion:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_i} \right) - \frac{\partial L}{\partial \theta_i} = Q_i \quad i = 1, 2 \quad (1)$$

where t is the time, $L = T - V$, $\dot{\theta}_i$ is the angular velocity, θ_i is the angular displacement, $i = 1$ or 2 makes reference to the lower or the upper body, respectively.

During the motion of the system a pattern change can occur for two reasons: a) sudden accelerations; b) impacts. To detect the first type of pattern change, it is necessary to determine the threshold acceleration for which this event is triggered. To this purpose, it is required to compare the internal moment M_I , which typically stabilizes the bodies, with the external moment M_E , which tends to overturn the bodies. Regarding the pattern change due to impacts, it is necessary to calculate the angular velocities of the blocks after the impact. To this end, the coefficients of restitution are calculated for every possible pattern transition using the conservation of angular momentum principle.

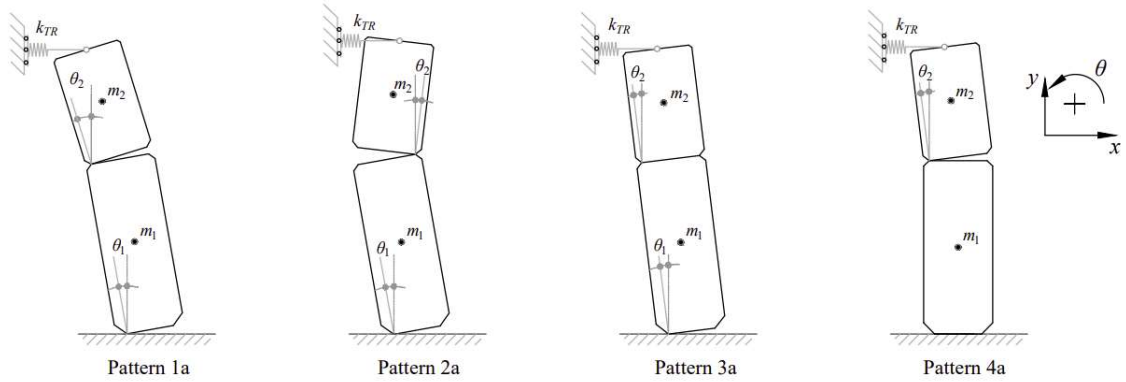


Figure 1 Elastically-restrained two-bodies system and possible (positive) motion patterns.

3 TIME HISTORY EXEMPLIFICATION

In order to exemplify the potential of the model and the event procedure, the response of a specific wall to a given earthquake record (Figure 2) is shown in Figure 3. It is assumed: $2b_1 = 2b_2 = 0.29$ m, $2h_1 = 2.20$ m and $2h_2 = 1.80$ m, consequently the ratio between the height of the lower body and the total height is $\xi = 0.55$ [21]. The masonry density is assumed as $\rho = 2000$ kg/m³, and the stiffness of the top spring is $k_{TR} = 1833$ kN/m. The response is analyzed for an

accelerogram (Figure 2) recorded during the Friuli, Italy earthquake (date: 1976-03-15, station: Tolmezzo, component: EW).

The wall is at rest until the rocking motion is initiated in pattern 3a ($\ddot{x}_g = 0.16 \text{ g}$ at $t = 3.60 \text{ s}$) (Figure 3). Then, the system continues to rock in pattern 3 impacting the foundation until the acceleration is strong enough ($\ddot{x}_g = 0.27 \text{ g}$) so that the two portions of the wall separate (Figure 3a) at instant $t = 3.92 \text{ s}$, and the system starts to move in pattern 2b (Figure 3b). Then, the rotations grow considerably while the two block are impacting one on each other, in this phase the system moves mostly in pattern 2a and 2b (Figure 3b). Subsequently, when the acceleration has decreased (approximately at $t = 7.50 \text{ s}$), the upper body tends to impact several times on the lower one (Figure 3a) dissipating energy until the system stops at $t = 8.80 \text{ s}$. In this phase the motion is characterized by fast transitions between pattern 1 and pattern 2.

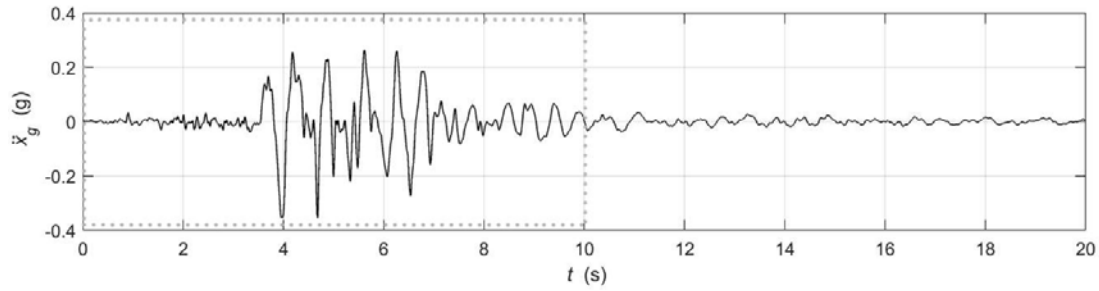


Figure 2 Accelerogram used in the time history analyses. Boxed time window related to Figure 3 and Figure 4

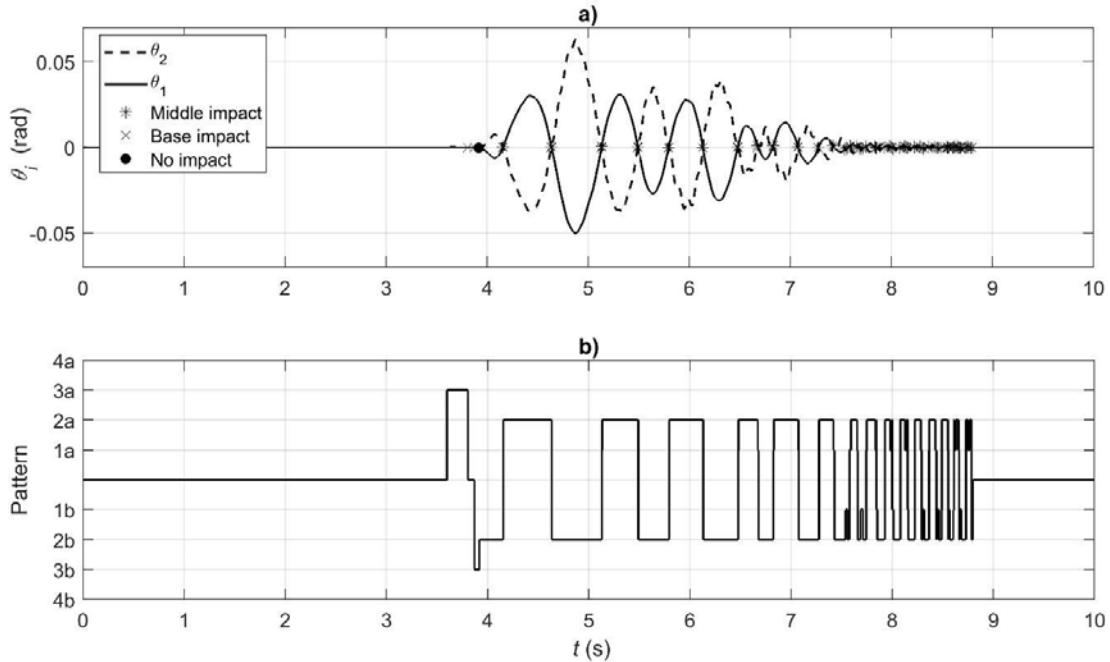


Figure 3 Rotations (a) and pattern (b) vs time

4 TIE RODS EFFECT

In this section the wall used for the previous numerical exemplification is restrained by different diameter tie rods (Figure 4). The stiffness related to each diameter is calculated under the assumption that a single tie rod, with a length $l_{TR} = 6$ m, has a tributary wall length $l_w = 6$ m (Table 1).

In this case (Figure 4), the rotations (of the lower and upper body) obtained using the model are normalized with respect to the corresponding slenderness parameter $\alpha_i = \arctan(b_i/h_i)$. Therefore, if the system is rotating in pattern 3, the rotations are normalized with respect to the slenderness α of the wall considered monolithic. When the system is moving in pattern 1, 2 and 4 the rotations of each block are normalized with respect to the corresponding slenderness: α_1 for the lower block and α_2 for the upper block.

ID	l_w m	Φ mm	l_{TR} m	k_{TR} kN/m
1	6	30	6	660
2	6	20	6	1833
3	6	12	6	4123

Table 1 Length of the wall l_w , Diameter Φ , length L_{TR} and stiffness k_{TR} of the considered tie rods

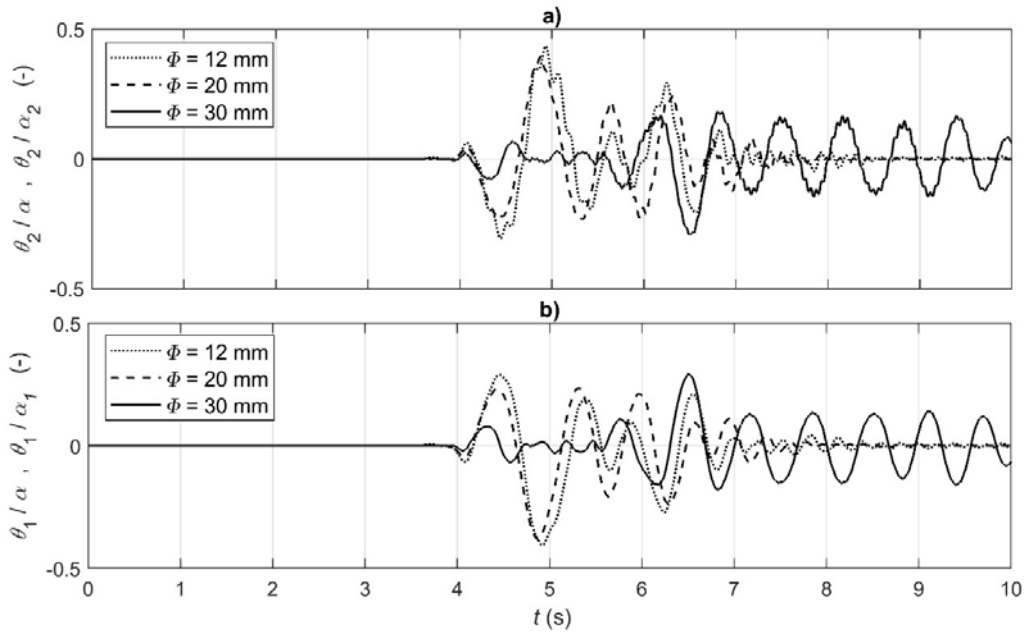


Figure 4 Rotations vs time for different size of the tied rod: a) upper body; b) lower body

The results (Figure 4) pointed out that, for large cross-section tie rods (case 1 in Table 1), the maximum normalized absolute rotations for both upper and lower block are reduced if compared with the cases where a tie rod with a smaller diameter is used. In this case, even though rotations are reduced, the wall continues to vibrate for a longer period. For medium diameter tie rod (case 2) the absolute normalized maximum rotations, for both, the upper (Figure 4a) and the lower (Figure 4b) block, are slightly smaller than those of the case in which

a smaller diameter is used (case 3). Moreover, in case 2 the system tends to dissipate energy faster especially if compared with case 1.

Further, for small cross-section tie rods (case 3) the absolute normalized maximum rotations are larger than those of the other cases confirming that, in general, the absolute normalized maximum rotations decrease as the diameter of the tie rod increases.

5 CONCLUSIONS

In the case of a wall rigidly restrained at the top, models that account for geometric non-linearities are present in the literature. The same can be stated for the model free at the top, which considers four different patterns. No model accounting for all the previously-mentioned phenomena (models accounting for all the geometric non-linearities and with a flexible restraint at the top) is available. Therefore, in this paper it is shown a multi-rocking-body dynamic (MRBD) model formed by two bodies of finite thickness and with a flexible restraint.

After a summary of the formulation, the MRBD model is used to investigate the response to a recorded accelerogram of a specific vertical spanning wall restrained by an elastic tie rod. The analysis confirms the strong non-linearity of the response, mainly due to the several patterns transition that the system undergoes during the motion. Finally, the effect of the tie rod diameter is investigated. The study concluded that, at least for the case at the hand, the normalized maximum absolute rotations decrease as the diameter of the tie rod increases. This behavior is particularly evident for large cross-sections ($\Phi = 30$ mm), while this beneficial effect is negligible for smaller size tie rods, meaning that the normalized maximum rotations are comparable when medium ($\Phi = 20$ mm) or small ($\Phi = 12$ mm) diameter tie rods are used.

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