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FINITE ELEMENT TECHNOLOGY-BASED SELECTIVE MASS SCALING FOR SHEAR DEFORMABLE STRUCTURAL ELEMENT FORMULATIONS

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Abstract. *We present a new concept for selective mass scaling (SMS) in the context of shear deformable structural element formulations. The novel SMS scheme is based on a method from the field of finite element technology (FET), in particular the discrete strain gap (DSG) method [1], and is inspired by the recently introduced concept of intrinsically selective mass scaling (ISMS) [2]. So far, theoretical connections of efficient methods from the fields of FET and SMS have not been systematically investigated in literature. But, in this contribution, we show that this particular theoretical connection leads to novel SMS schemes of high accuracy. Our theoretical hypothesis is tested via a numerical example, which underlines the high potential of the proposed concept.*

Keywords: selective mass scaling, finite element technology, Timoshenko, Reissner-Mindlin.

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1 INTRODUCTION

Explicit time integration algorithms are well suited for highly non-linear and non-smooth problems because they do not require iterative solution of the global equations of motion. However, the conditional stability of explicit algorithms limits the allowed, so-called critical time step size Δt_{crit}. It depends on the highest natural frequency $\omega_{\max}$ of the discrete system in the form of

$$\Delta t_{\mathrm{crit}} = \frac{2}{\omega_{\max}}. \tag{1}$$

There are several approaches to reduce the numerical costs (and thus increase efficiency) of explicit simulations, as can be seen for instance in [3]. Different approaches are usually used in combination, whereas the simultaneous use of locking-free finite elements and mass scaling concepts is state-of-the-art in practically all applications of explicit methods. So far, however, these two strategies for increasing the efficiency have only been applied independently and theoretical connections of efficient methods from the fields of FET and SMS have not been investigated. It is precisely this particular theoretical connection that forms the basis of the novel unified approach for SMS presented herein. That is, we present recent investigations on SMS techniques, which are based on concepts from FET, in particular the DSG method [1]. Some similarities and opposites to the recently introduced concept of ISMS [2] are discussed.

The present contribution focuses on the development of novel SMS schemes for structural element formulations, since for shear deformable structural element formulations existing SMS strategies either lack efficiency or accuracy for general applications.

The present contribution is organized as follows. Section 2 provides a brief overview on the state-of-the-art in mass scaling methods. Section 3 introduces the novel FET-based SMS conpept, which is tested in Section 4 via a numerical example. Section 5 concludes the achieved results and gives an outlook on open issues and further necessary developments.

2 MASS SCALING

With the goal of increasing the critical time step size in explicit dynamics, mass scaling methods artificially add inertia to the system. It needs to be distinguished between conventional mass scaling (CMS) and SMS. CMS increases inertia simply by artificially adding mass to the system. In the context of diagonal, lumped mass matrices (LMM), the diagonal entries are increased and the diagonal structure desired for reasons of efficiency is retained.

When CMS is applied to translational degrees of freedom, the translational inertia is increased. Thus, the use of CMS for all elements of a finite element mesh leads to a severe increase in linear momentum of the overall structure. This influences all natural frequencies and eigen modes, including those related to the rigid body translation and rigid body rotation, as shown schematically in Figure 1. In this diagram, the ratio of scaled natural frequencies ω° to unscaled natural frequencies ω is plotted over the number of modes. Due to its low accuracy, CMS usually cannot be applied in the overall structure. The use of CMS is then limited to small parts of the mesh that limit the critical time step size Δt_{crit}.

SMS concepts aim at modifying the mass matrix such that total mass and thus the translational inertia of the structure are preserved. In the context of solid or solid-shell elements that possess solely translational degrees of freedom, this is only possible if artificial inertia is added to diagonal and off-diagonal terms of the mass matrix. In this way, the highest natural frequencies, which are usually irrelevant for the actual structural behavior, can be reduced. The more

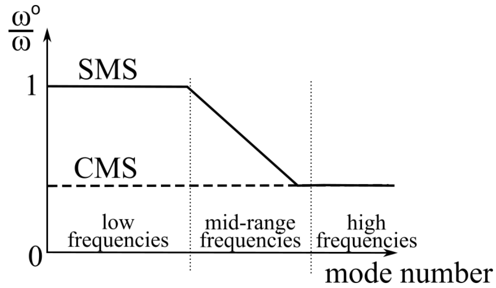

Figure 1: Schematic sketch of the ratio of scaled to unscaled natural frequencies of a system using CMS and SMS.

important low frequencies should be modified as little as possible, as shown in the schematic representation in Figure 1. In some applications, a good compromise between accuracy and critical time step size can be achieved, but a linear system of equations has to be solved in each time step due to non-diagonal mass matrices.

The general form of SMS methods can be represented as

$$\mathbf{M}^{\circ} = \mathbf{M} + \boldsymbol{\lambda}^{\circ}, \tag{2}$$

where $\mathbf{M}^{\circ}$ is the scaled mass matrix, $\mathbf{M}$ is the initial mass matrix and $\boldsymbol{\lambda}^{\circ}$ describes the artificial mass matrix.

Different SMS methods essentially differ in the construction and structure of the artificial mass matrix $\boldsymbol{\lambda}^{\circ}$. There are numerous SMS methods that are based on algebraic constructions. Stiffness-proportional SMS, i.e. $\boldsymbol{\lambda}^{\circ} = \alpha\mathbf{K}$ is described, for example, in [4, 5, 6] and is characterized by a high level of accuracy. However, the practical application is limited to linear dynamic problems, since the stiffness matrix changes in nonlinear problems and the scaled mass matrix has to be reassembled accordingly. The additional numerical costs caused by the reassembly of mass matrix and by assembly of the stiffness matrix, which is actually not required in explicit algorithms, represent the main disadvantages and are the main reasons why stiffness-proportional SMS methods are not implemented in commercial FE codes. Further algebraic SMS methods have been developed in [7, 8, 9, 10], among others. In addition to the algebraic methods there is also a class of variational methods for selective mass scaling (VSMS), see for instance [11, 12, 13].

As an alternative to discretizing thin-walled structures by solid or solid shell elements, modeling with (four-node) shear deformable Reissner-Mindlin shell elements represents the state-of-the-art for many applications. In these cases, the highest natural frequencies are associated with transverse shear modes. In dynamic problems, shear deformable structural elements provide physically more sound results than structural elements neglecting transverse shear deformation. But, also when using shear deformable elements, the highest shear frequencies are practically insignificant for the structural behavior and solely limit the allowed critical time step size Δt_{crit}. Thus, the use of mass scaling suggests itself. However, in the case of structural elements with rotational degrees of freedom, the aforementioned SMS methods cannot be easily extended to the scaling of the rotational inertia, which will be shown later, in Section 4. Nevertheless, since for shear deformable formulations the translational and rotational parts of the mass matrix can be constructed independently, the CMS of the rotational inertia already represents a kind of semi-selective mass scaling method. The translational inertia is not affected by this mass scaling. But, the method is not preserving angular momentum and is not very accurate, since bending-dominated frequencies are strongly affected, as well. The origins of rotational mass scaling go back to [14, 15, 16]. Due to the preservation of the diagonal structure of the LMM and the simplicity of the method, rotational mass scaling, which is denoted as CMS in this paper, represents the state-of-the-art in commercial codes for numerical simulations with shear deformable shell elements, see for instance [17].

3 NOVEL SELECTIVE MASS SCALING CONCEPT

In Oesterle et al. [2], we presented the concept of intrinsically selective mass scaling (ISMS), which is essentially based on some favorable properties of hierarchic, intrinsically locking-free shear deformable beam, plate and shell formulations from [18, 19, 20]. These structural element formulations are intrinsically free from transverse shear locking based on clever reparameterizations of the kinematic equations. That is, shear locking is already avoided at the level of theory, independent of the choice of the discretization scheme. This reparameterization of the kinematics leads to the fact that distinct shear degrees of freedom, such as shear rotations or shear displacements, are parameterized as primary variables. In the case of beam formulations, bending and shear deformations can be represented in a fully decoupled fashion. On the one hand, this leads to intrinsically locking-free structural formulations, since pure bending deformations can be displayed without any further strategies from the field of FET like reduced integration, etc. On the other hand, these specific reparameterizations directly allow ISMS of the high-frequency shear modes without significantly affecting the bending-dominated modes that are relevant for the dynamic structural response. ISMS enables the high accuracy of an SMS, but retains the simplicity of a CMS and the diagonal structure of the LMM. In this simple and effective way, however, ISMS is only applicable to hierarchical structural formulations that require C^1-continuous shape functions. That is, ISMS is promising for any smooth discretization scheme, but not directly applicable to standard C^0-continuous finite elements. Exactly for this class of C^0-continuous discretization methods, which are still state-of-the-art in most areas of numerical simulations of dynamic processes, innovative, accurate and efficient mass scaling concepts are required.

However, looking at the ISMS concept, it has to be remarked that methods from FET, initially designed for avoiding specific locking effects, can help to find simple, accurate and efficient novel SMS schemes. Additionally, one can see that the parameterization of transverse shear degrees of freedom leads to a particularly effective SMS method and represents are highly important property. How this property is related to the DSG method, will be shown in the subsequent derivations.

As starting point, we consider the removal of transverse shear locking for two-node, shear deformable Timoshenko beam elements. The kinematic equation used to identify transverse shear locking is the discrete transverse shear strain

$$\gamma \approx \gamma_{\mathrm{h}} = v_{\mathrm{h}}' + \varphi_{\mathrm{h}}, \tag{3}$$

where v_{h} and φ_{h} describe the ansatz functions for the displacement of the midline and for the total rotation of the cross-section of the beam. Furthermore, $(\bullet)' = \frac{(\bullet)}{\mathrm{d}x}$ holds. The imbalance in the ansatz spaces or derivatives, that can be seen in Eq. (3), leads to overstiff element behavior due to well-known transverse shear locking. Element-wise linear, parasitic transverse shear strains and stresses are present in slender elements, since the condition $\gamma_{\mathrm{h}} = 0$ cannot be satisfied pointwise. The DSG method from [1] was initially designed to remove transverse shear locking from shear deformable beam, plate and shell elements and can be summarized in four basic steps. For the simple case of Timoshenko beam elements these steps are:

1. integration: $v_{\mathrm{s}}(x) = \int_0^x \gamma_{\mathrm{h}} \mathrm{d}x$

2. collocation: $v_{\mathrm{s}}^{i} = v_{\mathrm{s}}(x)|_{x=x_i}$

3. interpolation: $v_{\mathrm{s}}^{\mathrm{mod}}(x) = \sum_{i=1}^{n} N_i(x) v_{\mathrm{s}}^{i}$

4. differentiation: $\gamma^{\mathrm{mod}}(x) = (v_{\mathrm{s}}^{\mathrm{mod}}(x))'$

This procedure leads to locking-free Timoshenko beam elements in four steps. For further details about this method from FET, we refer to [1] and subsequent publications. The most important observation is that in step 3, the interpolation, a modified shear displacement $v_{\mathrm{s}}^{\mathrm{mod}}$ appears, in which the previously integrated and collocated discrete shear gaps are interpolated. This modified shear displacement $v_{\mathrm{s}}^{\mathrm{mod}}$ is closely related to the reparameterized shear displacement v_{s} of hierarchic Timoshenko beam formulations, as described in [18, 19, 20]. But inhere, based on the DSG method, $v_{\mathrm{s}}^{\mathrm{mod}}$ is expressed as a function of the usual degrees of freedom of a Timoshenko beam element $\mathbf{d}^{\mathrm{T}} = (v_1, \varphi_1, v_2, \varphi_2)$ and leads to the requirement of at least C^0-continuous shape functions, which includes standard finite element discretizations.

The main idea of the novel SMS scheme, denoted as DSGSMS, is the extension of the virtual internal work for the Timoshenko beam by an additional virtual kinetic work expression stemming from the DSG method, more precisely from the modified shear displacement $v_{\mathrm{s}}^{\mathrm{mod}}$. The modified virtual internal work reads

$$\begin{aligned} \delta W^{\mathrm{int}} = & \underbrace{\int_0^L (\delta\gamma GA\gamma + \delta\kappa EI\kappa)\,\mathrm{d}x}_{\delta W^{\mathrm{int}}} + \underbrace{\int_0^L (\delta v \rho A \ddot{v} + \delta\varphi \rho I \ddot{\varphi})\,\mathrm{d}x}_{\delta W^{\mathrm{kin}}} \\ & \underbrace{+\, \alpha_{\mathrm{DSG}} \int_0^L \left(\delta v_{\mathrm{s}}^{\mathrm{mod}} \rho A \ddot{v}_{\mathrm{s}}^{\mathrm{mod}}\right)\,\mathrm{d}x}_{\delta W^{\mathrm{kin}}_{\mathrm{DSGSMS}}}, \end{aligned} \tag{4}$$

where E and G denote Young's modulus and shear modulus. The cross-sectional area, moment of inertia and density are denoted by A, I and ρ. The parameter α_{DSG} describes the scalar mass scaling parameter of the DSGSMS method.

The consistent mass matrix (CMM) of a two-node Timoshenko beam element is constructed by

$$\mathbf{m}_{\mathrm{C}} = \int_0^{l_{\mathrm{e}}} \mathbf{N}^{\mathrm{T}} \begin{bmatrix} \rho A & 0 \\ 0 & \rho I \end{bmatrix} \mathbf{N}\,\mathrm{d}x, \tag{5}$$

where $\mathbf{N}$ is the matrix of shape functions in terms of midline displacement v and total cross-sectional rotation φ. The CMM ($\mathbf{m}_{\mathrm{C}}$) and the LMM ($\mathbf{m}_{\mathrm{L}}$, via row sum lumping) result in

$$\mathbf{m}_{\mathrm{C}} = \frac{\rho l_{\mathrm{e}}}{3} \begin{bmatrix} A & 0 & \frac{A}{2} & 0 \\ 0 & I_{yy} & 0 & \frac{I_{yy}}{2} \\ \frac{A}{2} & 0 & A & 0 \\ 0 & \frac{I_{yy}}{2} & 0 & I_{yy} \end{bmatrix} \quad \text{and} \quad \mathbf{m}_{\mathrm{L}} = \frac{\rho l_{\mathrm{e}}}{2} \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & I_{yy} & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & I_{yy} \end{bmatrix}, \tag{6}$$

where l_{e} denotes the element length. In this case, a CMS is achieved by multiplying the rotational inertias of $\mathbf{m}_{\mathrm{L}}$ (entries [2,2] and [4,4]) by a factor α_{CMS} . The naive extension of the SMS concept from Olovsson et al. [5] (which is only described for solid elements) to scale the rotational inertia, and the newly proposed DSGSMS according to Eq. (4) lead to the artificial element mass matrices of the form

$$\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}} = \beta \frac{\rho l_{\mathrm{e}} I_{yy}}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS}} = \alpha_{\mathrm{DSG}} \frac{\rho l_{\mathrm{e}} A}{3} \begin{bmatrix} 1 & -\frac{l_{\mathrm{e}}}{2} & -1 & -\frac{l_{\mathrm{e}}}{2} \\ -\frac{l_{\mathrm{e}}}{2} & \frac{l_{\mathrm{e}}^2}{4} & \frac{l_{\mathrm{e}}}{2} & \frac{l_{\mathrm{e}}^2}{4} \\ -1 & \frac{l_{\mathrm{e}}}{2} & 1 & \frac{l_{\mathrm{e}}}{2} \\ -\frac{l_{\mathrm{e}}}{2} & \frac{l_{\mathrm{e}}^2}{4} & \frac{l_{\mathrm{e}}}{2} & \frac{l_{\mathrm{e}}^2}{4} \end{bmatrix}, \tag{7}$$

where β and α_{DSG} represent corresponding scaling factors. SMS of the LMM is performed on element level in the form

$$\mathbf{m}^{\circ}_{\mathrm{Olov}} = \mathbf{m}_{\mathrm{L}} + \boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}} \qquad \text{and} \qquad \mathbf{m}^{\circ}_{\mathrm{DSGSMS}} = \mathbf{m}_{\mathrm{L}} + \boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS}}. \tag{8}$$

For a stiffness proportional mass scaling we get $\mathbf{m}^{\circ}_{\text{k-prop}} = \mathbf{m}_{\mathrm{L}} + \boldsymbol{\lambda}^{\circ}_{\text{k-prop}} = \mathbf{m}_{\mathrm{L}} + \alpha_{\text{k-prop}}\mathbf{k}$, where $\mathbf{k}$ is the element stiffness matrix.

Multiplication of the artificial mass matrices $\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}}$, $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS}}$ and $\boldsymbol{\lambda}^{\circ}_{\mathrm{k-prop}}$ with a vector of unit angular acceleration $\mathbf{a} = \begin{bmatrix} \frac{l_{\mathrm{e}}}{2} & 1 & -\frac{l_{\mathrm{e}}}{2} & 1 \end{bmatrix}^{\mathrm{T}}$ results for all cases in zero inertia forces. Thus, the balance of angular momentum is preserved for all cases on element level. Only the LMM modified by CMS is not preserving angular momentum. The accuracy and effectiveness of the four mass scaling concepts are examined using a simple numerical example, which is shown next.

4 NUMERICAL EXAMPLE

Let us consider a beam of length $L = 1$, thickness $t = 0.01$ and width $b = 1$. Young's modulus, Poisson's ratio and density are set to $E = 1000$, $\nu = 0.0$ and $\rho = 1$. The beam is discretized with 20 linear (two-node) locking-free (via DSG method) Timoshenko beam elements. The results for the generalized eigenvalue problem of the form

$$(\mathbf{K} - \omega^2 \mathbf{M})\boldsymbol{\phi} = \mathbf{0} \tag{9}$$

are shown in Figure 2 for all the four mass scaling concepts introduced.

The scaling parameters α_{CMS}, α_{DSG} and $\alpha_{\mathrm{k-prop}}$ are selected each in such a way that the maximum natural frequency is scaled down to 20% of the original natural frequency (obtained by the use of LMM), that is $\frac{\omega^{\circ}}{\omega_{\mathrm{LMM}}} = \frac{1}{5}$. The SMS inspired by Olovsson et al. [5], see Eq. $(7)_1$ and Eq. $(8)_1$ is, independent of the scaling parameter β, not able to scale the maximum natural frequency and turns out to be unusable in the context of structural elements with rotational degrees of freedom. Both stiffness-proportional mass scaling and DSGSMS are clearly superior to CMS in terms of accuracy. While the highest frequency is effectively scaled to 20% of the original frequency by all these three methods, CMS shows very poor accuracy in the bending-dominated, first part of the frequency spectrum. This indicates that CMS may significantly change also the relevant natural frequencies. Due to its simplicity and its isotropic structure, CMS still represents the state-of-the-art in commercial FE programs in the area of explicit dynamic simulations with structural elements, although CMS shows poor accuracy and does not preserve angular momentum. In contrast, the presented DSGSMS is highly accurate. It seems to be even slightly more accurate than stiffness-proportional mass scaling. As can be seen in Figure 2, the bending-dominated branch of the spectrum is nearly unaffected by DSGSMS.

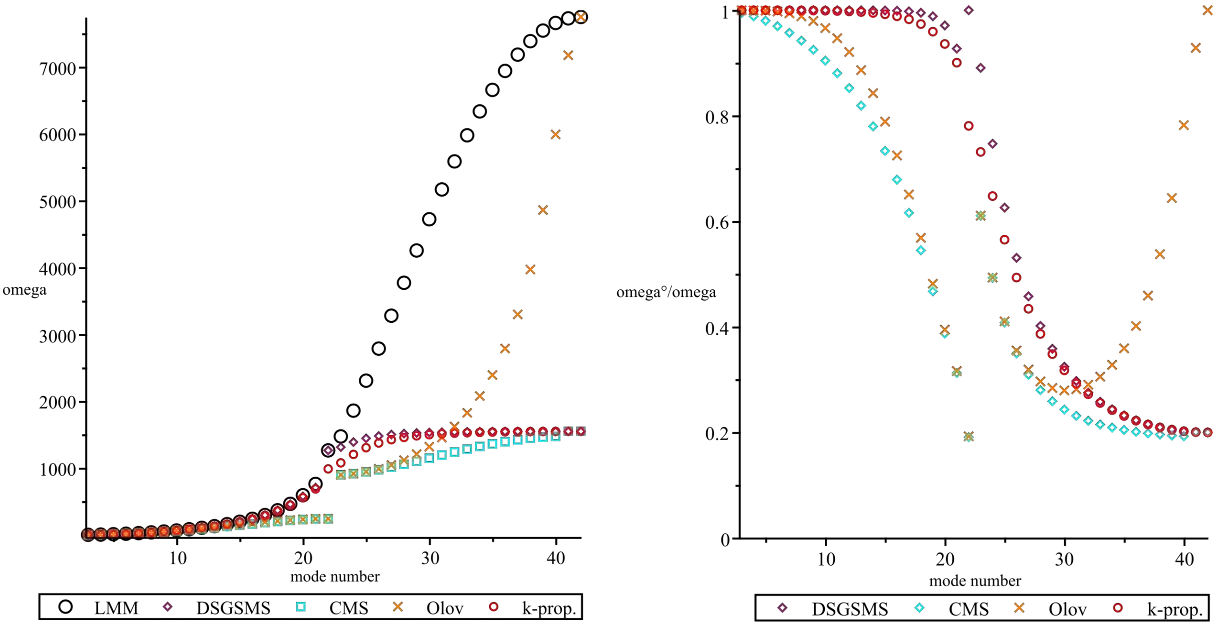

Figure 2: Timoshenko beam problem, frequency spectra (without rigid body modes) for different mass scaling methods. Left: (scaled) natural frequencies, right: ratio of scaled to unscaled natural frequencies.

5 CONCLUSIONS AND OUTLOOK

Due to its simplicity and its isotropic structure, CMS still represents the state-of-the-art in commercial FE programs in the field of explicit dynamic simulations with structural elements, although CMS provides poor accuracy and does not preserve angular momentum. But, both stiffness-proportional mass scaling and the newly presented DSGSMS are anisotropic. That is, corresponding mass matrices change with rotation of the elements. This results in the need for reassembly of scaled mass matrices in the case of non-linear problems with large rotations. The development of an isotropic version of the DSGSMS for beam and shell elements with rotational degrees of freedom is therefore a central goal of future work. Once this is done, the development of efficient and accurate time step estimates is required. DSGSMS represents a novel class of unified SMS concepts, applicable to various element types and applications. For instance, DSGSMS can be extended to efficient and accurate explicit dynamics of thin-walled structures using solid elements in straightforward way. The extension to solid elements and related first attempts for isotropic versions of DSGSMS, including results on transient problems, are described in [21].

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REFERENCES

[1] Bletzinger, K.-U., Bischoff, M. and Ramm, E., A unified approach for shear-locking-free triangular and rectangular shell finite elements, *Computers & Structures*, **75**, 32–334, 2000.

[2] Oesterle, B., Trippmacher, J., Tkachuk, A. and M. Bischoff, M., Intrinsically Selective Mass Scaling With Hierarchic Structural Element Formulations, *Book of Extended Abstracts of the 6th ECCOMAS Young Investigators Conference, 7th-9th July 2021, Valencia, Spain*, 99–108, 2021.

[3] Benson, D. J.: Explicit Finite Element Methods for Large Deformation Problems in Solid Mechanics. In: *Encyclopedia of Computational Mechanics*. John Wiley & Sons, Ltd, 2007.

[4] Macek R. W. and und Aubert, B. H., A mass penalty technique to control the critical time increment in explicit dynamic finite element analyses. *Earthquake Engineering & Structural Dynamics*, **24**, 1315–1331, 1995.

[5] Olovsson, L., Simonsson, K. and Unosson, M., Selective mass scaling for explicit finite element analyses. *International Journal for Numerical Methods in Engineering*, **63** , 1436–1445, 2005.

[6] Askes, H., Nguyen, D. C. D. and Tyas, A. Increasing the critical time step: micro-inertia, inertia penalties and mass scaling. *Computational Mechanics*, **47** , 657–667, 2011.

[7] Olovsson, L., Unosson, M. and Simonsson, K., Selective mass scaling for thin walled structures modeled with tri-linear solid elements. *Computational Mechanics*, **34**, 134–136, 2004.

[8] Cocchetti, G. , Pagani, M. and Perego, U. Selective mass scaling and critical time-step estimate for explicit dynamics analyses with solid-shell elements.*Computers & Structures*, **127**, 39–52, 2013.

[9] Mattern, S., Schmied, C. and Schweizerhof, K., Highly efficient solid and solid-shell finite elements with mixed strain–displacement assumptions specifically set up for explicit dynamic simulations using symbolic programming, *Computers & Structures*, **154**, 210–225, 2015.

[10] González, J. A. and Park, K. C., Large-step explicit time integration via mass matrix tailoring, *International Journal for Numerical Methods in Engineering*, **121**, 1647–1664, 2020.

[11] Tkachuk, A. and Bischoff, M., Variational methods for selective mass scaling, *Computational Mechanics*, **52**, 563–570, 2013.

[12] Tkachuk, A. and Bischoff, M., Local and global strategies for optimal selective mass scaling, *Computational Mechanics*, **53**, 1197–1207, 2014.

[13] Stoter, S. K. F., Nguyen,T.-H., Hiemstra, R. R. and Schillinger, D. Variationally consistent mass scaling for explicit time-integration schemes of lower- and higher-order finite element methods.*Computer Methods in Applied Mechanics and Engineering*, **399**, 115310, 2022.

[14] Key, S. W. and Beisinger, Z. E., The transient dynamic analysis of thin shells by the finite element method, *Proceedings of the Third Conference on Matrix Methods in Structural Mechanics*, 1978.

[15] Hughes, T. J. R., Cohen, M. and Haroun, M., Reduced and selective integration techniques in the finite element analysis of plates. *Nuclear Engineering and Design*, **46**, 203–222, 1978.

[16] T. Belytschko und W. L. Mindle. Flexural wave propagation behavior of lumped mass approximations, *Computers & Structures*, **12**, 805–812, 1980.

[17] Hartmann, S. and Benson, D., Mass scaling and critical time step estimates for isogeometric analysis, *International Journal for Numerical Methods in Engineering*, **102**, 671–687, 2015.

[18] Oesterle, B., Ramm, E. and Bischoff, M.: A shear deformable, rotation-free isogeometric shell formulation. *Computer Methods in Applied Mechanics and Engineering*, **307**, 235–255, 2016.

[19] Oesterle, B., Sachse, R., Ramm, E. and Bischoff, M.: Hierarchic isogeometric large rotation shell elements including linearized transverse shear parametrization. *Computer Methods in Applied Mechanics and Engineering*, **321**, 383–405, 2017.

[20] Oesterle, B., Bieber, S., Sachse, R., Ramm, E. and Bischoff, M.: Intrinsically locking-free formulations for isogeometric beam, plate and shell analysis. *PAMM*, **18**, e201800399, 2018.

[21] Hoffmann, M., Tkachuk, A., Bischoff, M., Oesterle, B., Finite element technology-based selective mass scaling for explicit dynamic analyses of thin-walled structures using solid elements. *COMPDYN Proceedings*, 2023.