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STOCHASTIC MODEL UPDATING OF RC TIED-ARCH BRIDGE

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Abstract

The finite element (FE) method is a powerful tool in the civil engineering field for the design, analysis, and assessment of structures. Although FE models attempt to replicate structures' behaviour, discrepancies always exist between experimental and numerical model responses. These are related to aleatoric uncertainties, such as measurement noise and epistemic uncertainties, like modelling assumptions. Epistemic uncertainties may negatively affect the FE model to the point that it may not correctly represent the structure behaviour, becoming unreliable for structural assessment. The FE model updating method has been successfully used to reduce the error between experimental tests and numerical output by updating uncertain FE model parameters, usually adopting data from dynamic tests. In this framework, experimentally derived dynamic responses in terms of natural frequency and mode shape can be used to reduce uncertainties. Although the general methodology for updating the FE model is well known, specific implementation techniques should be investigated and adapted to the analysed structure to fulfil more accurate results. In this paper, a model updating procedure through experimental data is applied to an existing Reinforced Concrete (RC) tied-arch bridge. The structural characteristics were assessed by an onsite test campaign, including and ambient vibration test (AVT). The updating procedure involves applying a stochastic method for quantifying the variability of parameters.

Keywords: Model updating; RC tied-arch bridge; Dynamic identification; Experimental test.

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1 INTRODUCTION

The finite element (FE) method is widely used in the engineering field to simulate real structures, making model-based activities easier, including but not limited to damage identification, structural health monitoring (SHM), structural safety, and risk assessment. Therefore, a "high-fidelity" FEM that accurately characterises structural behaviour is of fundamental importance. However, it is known that the results obtained from an FE model built only on the original designs, may differ from the actual measurements made on the real structure. Therefore, experimental measurements should be used to validate the numerical results. The structural response can be thoroughly described in terms of natural frequencies and mode shapes through dynamic identification test, particularly by ambient vibration test (AVT).

To reduce the discrepancies between experimental and numerical dynamic characteristics, specific methodologies have been implemented [1–6]. These methods use uncertain parameters including material quality, geometric characteristics, boundary conditions, etc., to calibrate the numerical model.

In general, the model updating problem can be divided into two groups: deterministic and stochastic [7, 8].

In the deterministic model updating method, a single FE model is optimised by minimising the error between the numerical results and the test data of the physical structure. However, predictions based on a single calibration of model parameters cannot give a measure of confidence in the numerical models' capability in representing the physical structure [1, 2, 9]. Instead, stochastic model updating refers to the numerical model with measurement variability, and the updating parameters are estimated from the interval or probability density function (PDF).

This paper presents a case study of probabilistic model updating of a historic RC tied-arch bridge (1931) located in Padua, northern Italy. The paper is organized as follows: in chapter 2, the description of the case study is presented, including the modal parameters for the model updating, and the preliminary FE model. In chapter 3, the results of AVT and the deterministic and stochastic model updating are discussed.

2 MATERIAL AND METHODS

2.1 Description of the case study

The case-study selected is a bridge situated in Padua, northeast Italy, built in the 1930s. The bridge is a reinforced concrete tied-arch bridge with 13 vertical hangers per side and 3 horizontal braces between the arches. The deck consists of longitudinal and transverse girders and a flat reinforced concrete slab. One abutment of the bridge has sliding supports, called pendulums, while the other end has fixed supports. During the restoration works carried out in 1994, the flat slab was repaired, and two cantilevered cycle-pedestrian walkways were added to either side of the bridge. The general configuration of the bridge is reported in Figure 1, with the dimensions of its components being consistent with those reported in the original project documents.

2.2 Dynamic identification

An Ambient Vibration Test (AVT) was performed to extract the modal characteristics of the bridge. The test was conducted by means of 14 uniaxial accelerometers, performing the recordings in two separate setups, namely North and South (as illustrated in Figure 2). Seven recordings were made with the North setup and four with the South setup.

The experimental parameters were extracted using MACEC 3.3 toolbox [10] applying the poly-reference Least Square Complex Frequency domain (p-LSCF) technique [11–13]. Modal parameters were estimated for each recording and the resulting data from the two setups were combined, obtaining 28 configurations. The correlation between the modal shapes of these configurations was calculated using the MAC index [14].

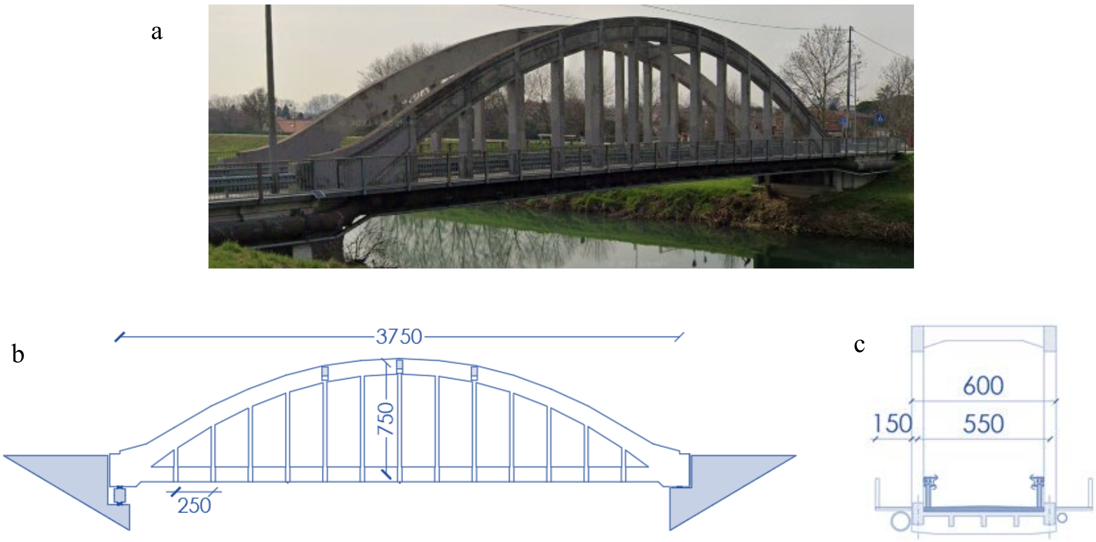


Figure 1: a) Case study view b) longitudinal view; c) transverse view of the bridge.

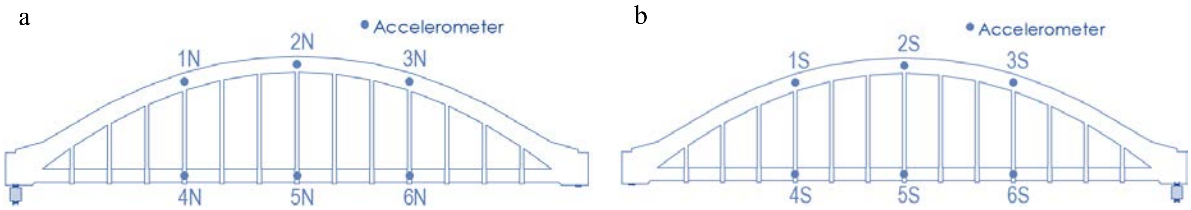
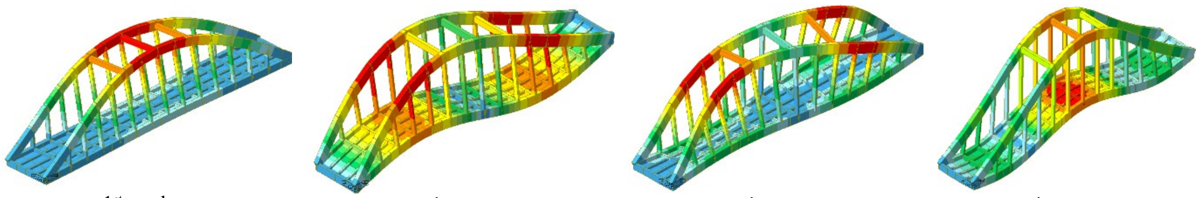


Figure 2: Ambient vibration test: a) North setup; b) South setup.

2.3 Numerical model and model updating

A 3D FE model was realised through the utilization of Midas Civil [15]. In this FE model, all bridge members were modelled via beam elements, except for the slab which was modelled as a plate. The loads of the road pavement, barriers, and cycle-pedestrian walkways were applied as non-structural masses. The arch-hanger, tie-hanger and arch-brace connections are simulated by inserting rotational springs, while the boundary conditions were simulated by translational springs, except for the vertical direction, assumed fixed. An eigenvalue analysis was conducted, and the first four mode shapes are presented in Figure 3: Frist four modal shapes of the case study bridge.



1st mode 2nd mode 3rd mode 4th mode

Figure 3: Frist four modal shapes of the case study bridge.

Finally, the FE model updating was performed using the software FEMtools 3.6 [16]. The updating process consists into two parts: deterministic updating and stochastic updating. First, deterministic model updating was performed. The procedure is based on a sensitivity formulation that can be written as [17]:

$$\{R_e\} = \{R_a\} + [S](\{P_u\} - \{P_0\}) \quad \text{or} \quad \{\Delta R\} = [S]\{\Delta P\} \tag{1}$$

where $\{R_e\}$ is the vector containing the reference system responses (experimental data); $\{R_u\}$ is the vector containing the predicted system responses for a given state $\{P_0\}$ of the parameters values; $\{P_u\}$ is the vector containing the updated parameter values, and $[S]$ is the sensitivity matrix. Eq. (1) is usually underdetermined, so the Bayes Parameter Estimation (BPE) [18] technique was performed to solve it.

Parameter values updated with the deterministic process are assumed as the mean value for the probabilistic model updating. The other stochastic characteristics of the parameters are estimated with the Mean Value First Order Reliability Method (MVFOSM) [19]. The MVFOSM method assumes a normal distribution for the model parameters, while the standard deviation of the parameter distributions is estimated using the following relationship [20]:

$$\sigma_{R_i}^2 \approx \sum_{j=1}^{n} s_{ij}^2 \sigma_{P_j}^2 \tag{2}$$

where s_{ij} is the sensitivity coefficient of response R_i with respect to the parameter P_j, σ_{R_i} is the standard deviation of response R_i, and σ_{P_j} is the standard deviation of the parameter P_j. The sensitivity coefficient $s_{ij}$is computed using the FE model, while the standard deviation σ_{R_i} is estimated from the available test data. In this way, Eq. (2) allows the estimation of σ_{P_j}.

The statistical properties of the parameters are used to perform a Monte Carlo Simulation [21] and re-analyse the FE model with the obtained sample values to investigate the scatter of the numerical frequencies in relation to the experimental ones.

3 Results

3.1 Summary of AVT results

Table 1presents statistical information of the 28 experimental configurations concerning the first four identified frequencies and the error in relation to numerical model. Correlations between experimental modal shapes are calculated with the MAC index, and the results are reported in Figure 4. Figure 5 shows the correlation between the average of experimental and the numerical modal shapes. The maximum error in both frequency and MAC index occurs in the third modal shape.

n° mode	Mode type	Mean [Hz]	Std dev	Std rsd	Initial FEM [Hz]	Error
1	I° Trans	3.015	0.009	0.30%	2.768	8.19%
2	I° Vert	4.057	0.079	1.94%	3.094	23.74%
3	I° Tors	4.796	0.009	0.18%	3.633	24.25%
4	II° Vert	7.216	0.063	0.88%	5.986	17.16%

Table 1: Statistics of the first four identified frequencies.

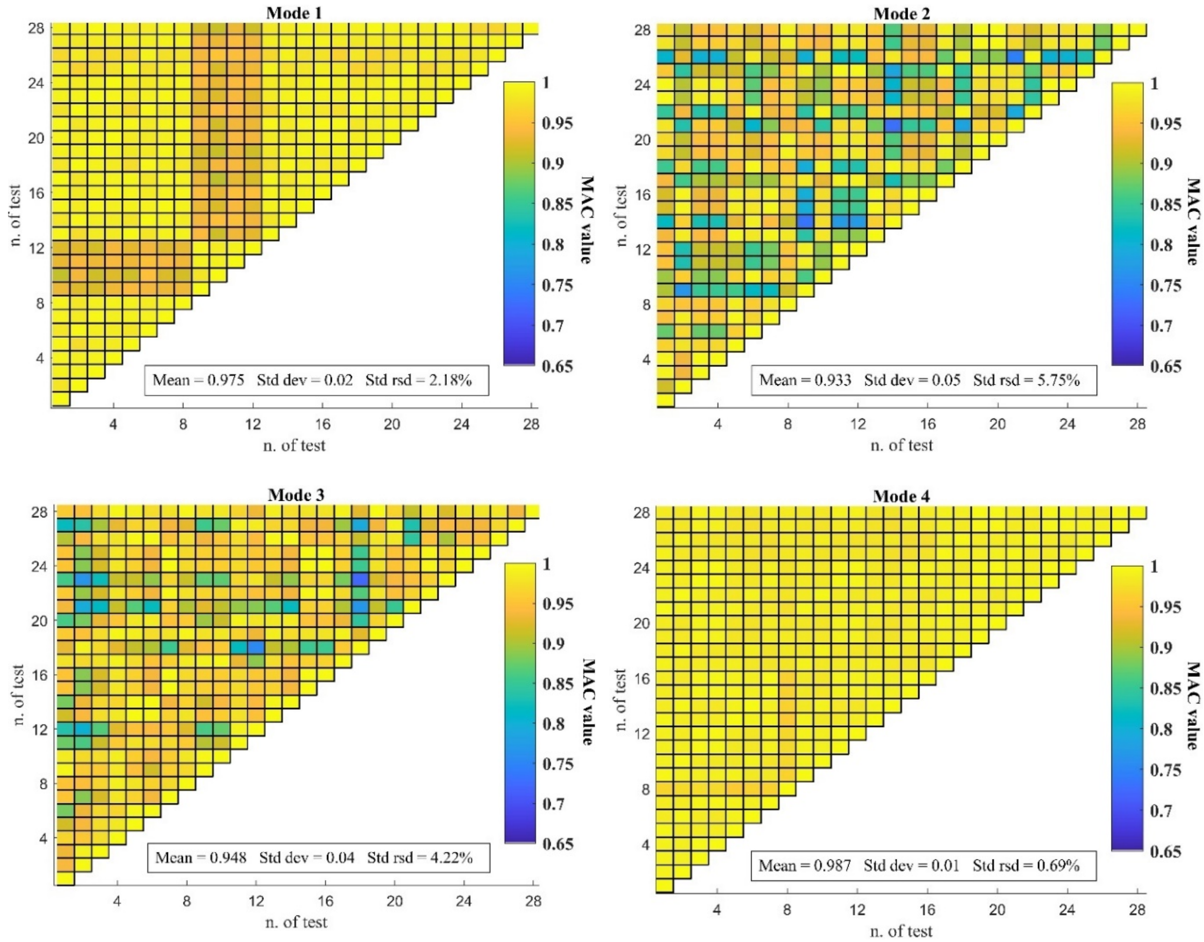
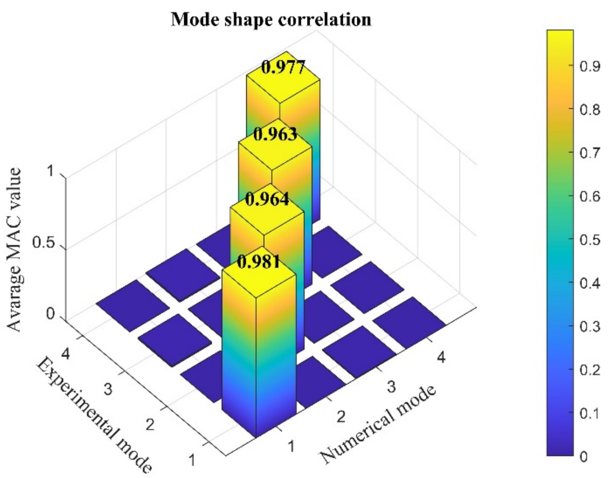


Figure 4: Statistics of the first four identified modal shape.



n° mode	Mean	Std dev	Std rsd
1	0.981	0.022	2.26%
2	0.964	0.022	2.33%
3	0.963	0.025	2.58%
4	0.977	0.009	0.94%

Figure 5: Mode shape correlation between test and numerical model.

3.2 Deterministic model updating

First, a sensitivity analysis was employed to investigate the impact of various parameters on the first four frequencies of the numerical model. Initially, 9 variables were selected for the updating procedure, including the elastic modulus (E), material density (ρ), non-structural mass (NSM), rotational stiffness of the arch-hanger and tie-hanger connections (k_1), rotational stiffness of arch-braces connections (k_2), and horizontal boundary conditions (k_3). The 9 variables were presented in Table 2 with the respective reference values. The results of the sensitivity analysis reported in Figure 6 indicated that the elastic modulus and mass density exhibited the highest influence on the frequencies response. As a result, only these two parameters were taken into consideration for the model updating procedure.

n°	Description	Symbol	Ref. value	Units
1	Elastic modulus	E	34.3	GPa
2	Mass density	ρ	2.40	t/m^3
3	Non-structural mass	NSM	0.31	t/m^2
4	Rotational spring stiffness hangers (x-axis)	k_{1x}	10^5	kNm/rad
5	Rotational spring stiffness hangers (y-axis)	k_{1y}	10^4	kNm/rad
6	Rotational spring stiffness braces (x-axis)	k_{2x}	10^5	kNm/rad
7	Rotational spring stiffness braces (z-axis)	k_{2y}	10^5	kNm/rad
8	Translational spring stiffness (x-axis)	k_{3x}	10^{10}	kN/m
9	Translational spring stiffness (y-axis)	k_{3y}	10^{10}	kN/m

Table 2: Initial Input parameters of the FEM base model.

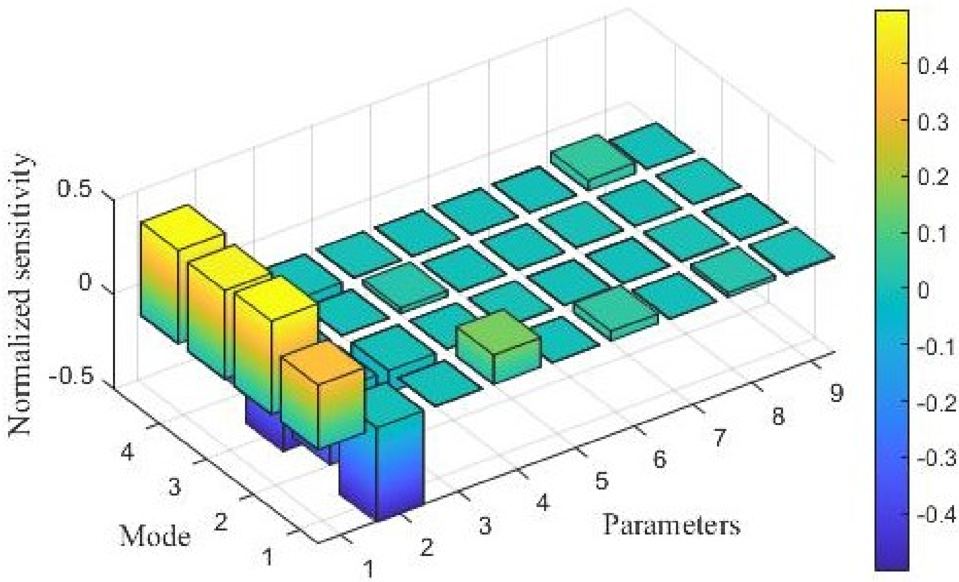


Figure 6: Sensitivity of numerical model frequencies to initial input parameters.

Initially, a first-step calibration strategy was performed on a FE model, whose elements have the same values of elastic modulus and mass density that have been varied in a range between -30% and +30% of the reference values. The average values of the experimental frequencies were assumed as targets.

The final results of the deterministic calibration process are presented in Table 3 while the updated parameter values are summarized in Table 4. This calibration strategy reduced the maximum error between the experimental and model frequencies from 24.25% to 15.20%. The updated model presents an increment of +30% of the elastic modulus and +10.5% of the mass density. Despite only the experimental frequencies were used as target, the average MAC index value indicates a solid correlation between the experimental and numerical modal shapes.

After the first calibration process, a second-step calibration was performed varying the elastic modulus and mass density parameters for specific elements, such as ties, arches, hangers, braces, and deck. The decision to model separately ties and deck is based on the assumption that ties were constructed with a higher performance concrete mix-design [22]. The initial values and range of variation of the parameters are the same as those used in the first-step updating process, and the sensitivity analysis is reported in Figure 7. Through the second-step deterministic model updating, the maximum error between experimental and numerical frequencies was reduced from 24.25% to 8.23%, as reported in Table 5. As reported in Table 6, the updating process caused an increase of +30% of the elastic modulus of ties and arches and a reduction of about -30% in all other elements. Mass density decreased in all elements except in the arches, where it remained almost unchanged, and in the braces (+30% increment). Also, in this case no significant changes in the MAC index are reported.

n° mode	Mode type	Exp freq. [Hz]	FEM freq, [Hz]	Δf_{resp} [%]	Mean MAC
1	I° Trans	3.015	3.057	-1.39	0.951
2	I° Vert	4.057	3.486	14.07	0.972
3	I° Tors	4.796	4.067	15.20	0.923
4	II° Vert	7.226	6.777	6.21	0.878

Table 3: Comparison between experimental and numerical modal characteristics of the updated FE model with first-step strategy.

n°	Description	Symbol	Ref. value	Update value	Δ_{par} [%]	Units
1	Elastic modulus	E	34.3	44.5	+30	GPa
2	Mass density	ρ	2.40	2.65	+10.5	t/m^3

Table 4: Comparison between the structural properties of the FEM model before updating (Ref. value) and after updating (Updated value) with first-step strategy.

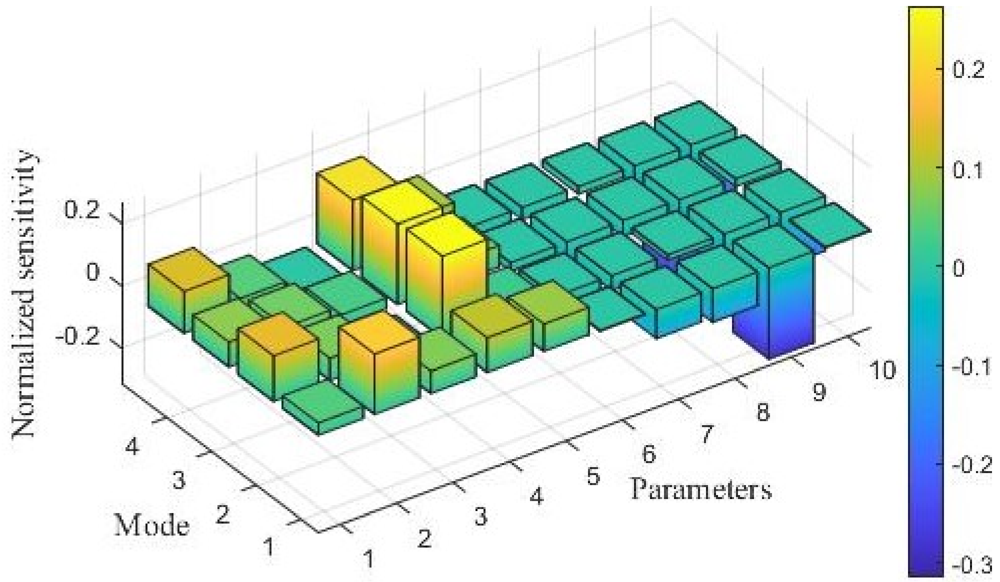


Figure 7: Sensitivity of numerical model frequencies to second-step calibration input parameters.

n° mode	Mode type	Exp freq. [Hz]	FEM freq, [Hz]	Δf_{resp} [%]	Mean MAC
1	I° Trans	3.015	2.973	1.39	0.939
2	I° Vert	4.057	3.723	8.23	0.991
3	I° Tors	4.796	4.465	6.90	0.938
4	II° Vert	7.226	6.977	3.44	0.930

Table 5: Comparison between experimental and numerical modal characteristics of the updated FE with second-step strategy.

n°	Description	Symbol	Ref. value	Update value	Δ_{par} [%]	Units
1	Tie elastic modulus	E_t	34.3	44.5	+30	GPa
2	Hanger elastic modulus	E_h	34.3	24.8	-27.5	GPa
3	Brace elastic modulus	E_b	34.3	24.0	-30	GPa
4	Arch elastic modulus	E_a	34.3	44.5	+30	GPa
5	Deck elastic modulus	E_d	34.3	24.0	-30	GPa
6	Tie mass density	ρ_t	2.40	1.68	-30	t/m^3
7	Hanger mass density	ρ_h	2.40	1.68	-30	t/m^3
8	Brace mass density	ρ_b	2.40	3.12	+30	t/m^3
9	Arch mass density	ρ_a	2.40	1.68	-30	t/m^3
10	Deck mass density	ρ_d	2.40	2.38	-1.0	t/m^3

Table 6: Comparison between the structural properties of the FEM model before updating (Ref. value) and after updating (Updated value) with second-step strategy.

3.3 Stochastic model updating

For all parameters, a normal distribution was assumed, with the mean value assumed from the deterministic model updating results [20]. The standard deviation was estimated using Eq. (2), which uses the dispersion of the experimental tests and the sensitivity values calculated on the calibrated FE models. The resulting probabilistic parameters proprieties of the first-step and second-step calibration strategies are shown respectively in Table 7 and Table 8.

n°	Description	Symbol	Mean value	Std dev	Std rsd	Units

1	Elastic modulus	E	44.5	0.046	0.10%	GPa
2	Mass density	ρ	2.65	0.045	0.02%	t/m^3

Table 7: Probabilistic parameters proprieties of the first-step strategy.

n°	Description	Symbol	Mean value	Std dev	Std rsd	Units
1	Tie elastic modulus	E_t	44.5	0.069	0.16%	GPa
2	Hanger elastic modulus	E_h	24.8	0.114	0.46%	GPa
3	Brace elastic modulus	E_b	24.0	0.030	0.13%	GPa
4	Arch elastic modulus	E_a	44.5	0.094	0.21%	GPa
5	Deck elastic modulus	E_d	24.0	0.103	0.43%	GPa
6	Tie mass density	ρ_t	1.68	0.039	2.32%	t/m^3
7	Hanger mass density	ρ_h	1.68	0.015	0.89%	t/m^3
8	Brace mass density	ρ_b	3.12	0.043	1.38%	t/m^3
9	Arch mass density	ρ_a	1.68	0.079	4.70%	t/m^3
10	Deck mass density	ρ_d	2.38	0.100	4.20%	t/m^3

Table 8: Probabilistic parameters proprieties of the second-step strategy.

The probabilistic properties of parameters were used to generate 200 random Monte Carlo samples and re-analyse the FE model with the sample values. Natural frequencies for all the samples are reported in Table 9 considering only those values that fall within the 95th percentile ellipse. The results indicate that the second-step strategy shows better convergence with the experimental data cloud, with the centre of the 95th percentile ellipses positioned closer. Specifically, the Euclidean distance calculated between the experimental and numerical data obtained with the first-step strategy is 0.589 [Hz] for the scatter plot between frequency 1 and frequency 2 (Figure 8a), and 0.887 [Hz] for that between frequency 3 and frequency 4 (Figure 8b). Whereas, for the second-step strategy, these values drop to 0.186 [Hz] (Figure 9a) and 0.290 [Hz] (Figure 9b), respectively, providing further evidence of its better convergence with the experimental data.

n° mode	Mode type	Mean freq. [Hz]			Std dev			Std rsd		
		Exp	Prim.	Sec.	Exp	Prim.	Sec.	Exp	Prim.	Sec.
1	I° Trans	3.015	3.038	2.968	0.009	0.091	0.074	0.30%	3.00%	2.49%
2	I° Vert	4.057	3.466	3.877	0.079	0.097	0.101	1.95%	2.80%	2.61%
3	I° Tors	4.796	4.041	4.581	0.009	0.121	0.123	0.19%	2.99%	2.69%
4	II° Vert	7.226	6.739	7.025	0.011	0.186	0.171	0.15%	2.76%	2.43%

Table 9: Comparison between experimental and numerical frequencies after Monte Carlo Simulation.

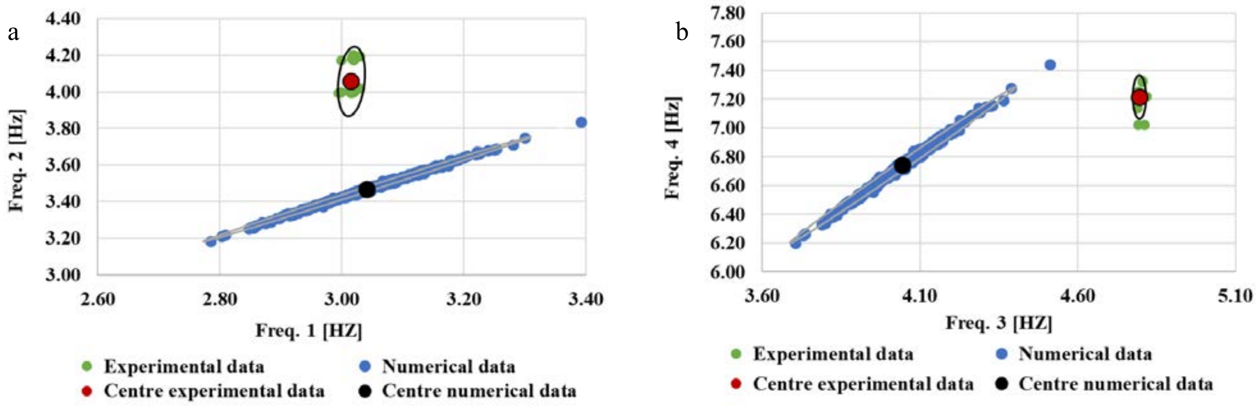


Figure8: Overlaid scatter plots of experimental and numerical frequencies for first-step strategy: a) freq.1-freq.2, b) freq.3-freq.4.

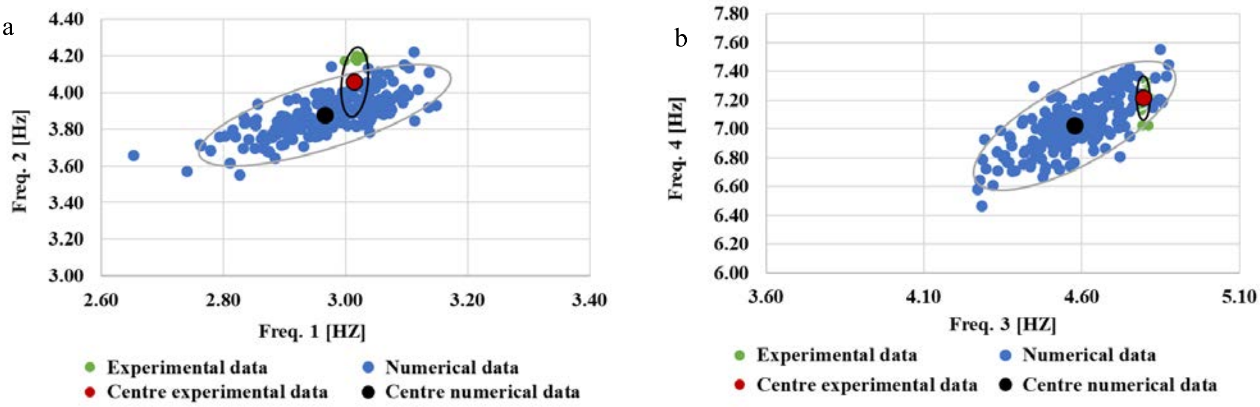


Figure 9: Overlaid scatter plots of experimental and numerical frequencies for second-step strategy: a) freq.1-freq.2, b) freq.3-freq.4.

4 CONCLUSION

This paper presents a stochastic model updating of an RC tied-arch bridge. The initial model was calibrated using dynamic identification test results. The recordings were made on two set-ups, and the extracted modal parameters were appropriately combined create 28 combination tests. The model updating procedure assumed the frequencies of the first four modes as targets, and a sensitivity analysis was conducted to determine the most sensitive parameters. The analysis revealed that the elastic modulus and mass density were the most sensitive parameters. Next, deterministic model updating was performed using an iterative procedure based on the Bayes Parameter Estimation (BPE) technique. Two strategies were adopted, first-step and second-step, where the first considered a FE model with identical material characteristics for all elements, while the second considered different material characteristics for specific elements. The results showed that the second-step strategy provided better results, reducing the frequencies error from 24.25 % to 8.23 %, while the first-step strategy stopped at 15.20 %. The FE model that best represented the real behaviour of the bridge present an increase in elastic modulus for the arch and tie elements and a reduction for all other elements. As for mass density, a reduction was observed for all elements except for the arch, which remained constant, and for the braces, where it increased. Next, stochastic model updating was performed, where the statistical properties of the parameters were estimated using the Mean Value First Order Reliability

Method (MVFOSM) assuming as mean values the results value of the deterministic model updating.

Finally, a Monte Carlo simulation was performed using the statistical properties of the model parameters to generate 200 random samples and re-analyse the FE model with these values. The frequencies of these models were statistically processed to obtain a scatter plots. It has been demonstrated that in probabilistic model updating, more reliable results can be obtained by using a larger number of calibration parameters. In particular, it was shown how the scatter of the numerical data generated by the Monte Carlo Simulation of the second-step strategy is closer to the distribution of the experimental data, reducing the Euclidean distance from 0.589 [Hz] to 0.186 [Hz], for the scatter plot between frequency 1 and frequency 2, and from 0.887 [Hz] to 0.290 [Hz] for frequency 3 and frequency 4, compared with the first-step strategy.

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