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Eccomas Proceedia COMPDYN (2023) 4402-4415

ECCOMAS
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COMPDYN 2023
9th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Athens, Greece, 12-14 June 2023

SEMI-ANALYTICAL CHARACTERIZATION OF A SLOSHING FLUID MASS USING A SMOOTHED PARTICLE HYDRODYNAMIC BASED METHOD

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Abstract. *Sloshing phenomena regard the dynamic excitation of a partially filled tank which leads to the motion of the fluid within. The dynamic effects, especially if the excitation frequency approaches the natural frequency of the tank (near resonance), may compromise the structure stability. Amplification of motion may be either pursued or avoided, depending on the design purposes; regardless, it is important to predict the coupled effects generated by the tank interacting with the contained liquid. The present study aims to offer a simplified approach along with valid parameters that link the filling level of the tank to the sloshing mode of vibration. The parameterization procedure is numerically supported by a Computational Fluid Dynamics (CFD) solver based upon the Smoothed Particles Hydrodynamics (SPH) meshless method. Within the proposed framework, a parametric sloshing tank is simulated and validated using experimental data as a test-rig, therefore used to perform a wide sensitivity study by changing the filling ratio. The numerical results interpretation leads to a method which can synthetically predict the characterizing frequency of the phenomenon using a semi-analytical fitting, with improved accuracy over previous literature methods. This procedure is established within the open-source SPH-based DualSPHysics, and provides a reliable alternative to traditional approaches, proving that validated numerical methods can directly support the design of sloshing tanks, with all the advantages related: quickness, affordability and reproducibility.*

Keywords: Sloshing, free-surface, Fluid-Structure Interaction, CFD, Smoothed Particle Hydrodynamics, DSPH

doi: 10.7712/120123.10728.20753

1 INTRODUCTION

The motion of the liquid in partially filled tanks subject to dynamic excitation, or to any sort of disturbance, is called sloshing [1]. The free-surface displacement, depending on the container shape and on the external forcing condition, may range from planar to chaotic. The following dynamic effects are of great interest in various fields of engineering: likewise physics is recognized, among others, in soil-fixed tanks subject to extreme events (i.e. earthquakes), fuel or propellant tanks of crafts and vehicles in motion, transportation of liquid fuels (i.e. liquid natural gas – LNG), and offshore petroleum factories (Floating Production Storage and Offloading – FPSO) - an extensive consultation is referred in [2]. The present craft industry trend, in particular, advances with increasingly bigger models. Global demand of trade of commercial products and fuel requests greater cargo ships. Parallel to that, increasing demand for commercial trips is leading air-craft industry to enlarge its capacities producing bigger models - an article entitled 'Big Jets Fly Again As Long-Haul Travel Picks Up' was issued on the February 6^{th}, 2023 edition of the Wall Street Journal ([3]).

Hence, the comprehension of the dynamics involved and its effects are of current practical interest. The sloshing topic is therefore widely investigated, to the point of recognizing the need to establish reference conditions for experimental tests: in 2012, a benchmark on sloshing model tests was held with the participation of world-wide universities and research institutes [4] and other experimental rigs were conducted in order to provide useful information for further sloshing tests in various conditions and still capture experimental uncertainties [5]. Even though experimental tests remain crucial for the comprehension of the sloshing mechanics, analytical and numerical approaches developed upon the consistent experimental background can provide useful alternatives or completions of the field campaigns. In order to obtain the response spectrum of the liquid motion, analytical equations with rigorous solution may be used [6]. Nonetheless, when describing non-linear phenomena, their handling may thoroughly outcome too complex, as closed explicit solutions are feasible only for a few special cases such as upright cylindrical and rectangular containers [1].

In the latter case, an alternative to the costly physical tests to investigate non-linear dynamics is offered by numerical methods, within which it is possible to vary dimensions and conditions of the fluid-tank system. Generally, Computational Fluid Dynamics (CFD) methods are preferred when studying extreme phenomena with significant nonlinearities, as it may be in sloshing near resonance, and mesh-less approaches are likely more feasible than mesh-based ones when dealing with free-surface flows and considerable displacements. In mesh-based procedures, in fact, the relative position of the lattice nodes is predetermined and difficulties arise when simulating large deformations, discontinuities or even phase transformations [7]. Mesh-less methods, on the other hand, do not require any fixed grid to connect the computational nodes, which, hence, develop their own physical evolution, including changes of position, according to governing equations.

Nevertheless, mesh-based methods have been extensively used to investigate weakly non-linear phenomena and with very accurate results [8]. The re-meshing computational cost, however, may turn them into less efficient solution when compared to mesh-less methods for FSI studies, having to couple different meshing procedures between fluid and structure phases, especially for deformable structures [9]. Further methods, such as Arbitrary Lagrangian-Eulerian (ALE), were proposed in attempt to capture deformations, but their efficiency is limited for small deformations [10], and still require another Lagrangian mesh to discretize the structure [8].

In particle methods [11] the domain can be discretized in nodes (particles) carrying their own assigned information about the physical properties. Among the particle methods, the Smoothed Particles Hydrodynamic (SPH) method is used for this investigation.

The SPH is a meshless discretization technique that has been widely used to CFD applications [12]. The inherent suitability of this approach for reproducing the physics featured in sloshing problems greatly helped in divulging the SPH method as a real alternative to traditional grid-based solvers [13]. Even the aforementioned issue of sloshing in LNG tankers was investigated through SPH simulations [14]. It was recognized to be a promising tool, so further optimized application matured. Specifically regarding sloshing, benchmark experiments aimed to inquire the SPH simulation suitability: Vorobyev et al. [15] validated a 3-D Weakly Compressible formulation for central sloshing, whereas Zhang et al. [16] implemented a Riemann-based SPH solver with dissipation limiter (in order to restrain dissipation for violent free-surface phenomena) and applied it to violent sloshing phenomena. Generally, free-surface motion and impact pressure showed great accordance with experimental data.

The SPH implementation in DualSPHysics [17] provides several tools and features that make the code ready for engineering applications, and very efficient when fluid phases are relevant [more in 18]. During the last few years, the code has seen an increase in availability of functionalities that are very apt for the simulation of Fluid-Structure Interaction in different engineering fields, ranging from renewable energy device [19, 20, 21] to hydroelasticity problems [22, 23, 24, 25]. The code, moreover, proved its reliability for nearly any kind of free-surface flow interacting with rigid structures: dambreak-induced violent flows on downhill structures [26, 27], coastal breakwaters [28, 29] or floating objects [30, 31, 32]. Sloshing simulations as well have been performed using DualSPHysics. Green et al., 2021 [33] recently tackled complex 3-D cases of long duration sloshing in tanks, and Kotsarinis et al. 2023 [34] modeled sloshing damping for spacecrafts; a traditional sloshing problem, indeed, is utilized in English et al. 2022 [35] to assess the performance of the newly developed modified Dynamic Boundary Conditions (mDBC).

Undertaken this premise, we propose an analytical simplified approach, whose parameters are retrieved from numerical simulations. Our research consists in numerically simulating the fluid sloshing response induced by tank motion considering different filling ratios, i.e. the ratio between water level d and the tank's semi-base l - (d/l). The tank is initially displaced using a smooth sinusoidal function that lasts one cycle and then the tank is kept still and the fluid motion is observed. The sloshing natural frequency is computed using an analytical relationship. The SPH solutions are validated against the reference solution for sloshing parametrization, and the results are compared with the outcomes obtained from other CFD approaches.

2 SPH DISCRETIZATION

In the Smoothed Particle Hydrodynamics (SPH) method, the value of the particle's properties derives from the interpolation, through a weighting function and its derivatives, of the neighboring particles' values. Hereupon, the hydrodynamic of fluid particles is calculated with the locally averaged Navier-Stokes equations, with conservation of mass and linear momentum [11].

The interpolation takes place within an established domain of influence (of the single particle) defined upon the so-called smoothing length (h). This interpolation is weighted by the smoothing function or kernel function (Figure 1), which corresponds to the discrete Dirac delta function at a continuous level. The continuous SPH interpolation of a generic function reads:

$$< f(\mathbf{r}) >= \int_{\Omega} f(\mathbf{r'}) W(\mathbf{r'} - \mathbf{r}, h)\, d\Omega \tag{1}$$

where f is the interpolation function (expressed in its integral average), Ω is the compact support, $\mathbf{r}$ and $\mathbf{r'}$ the position vectors of two different particles in space.

In discrete form, the function is approximated by summing up the values of the nearest neighbor particles that possess fixed mass m_b, density ρ_b, and volume V_b. The discrete form of Equation (2) for a particle a, being $b \in [1, N_p]$ part of its support domain is:

$$< f(\mathbf{r}_a) >= \sum_{b=1}^{N_p} \frac{m_b}{\rho_b} f(\mathbf{r}_b) W_{ab} \tag{2}$$

The smoothing length must be well-determined in order to maintain a compact support of the kernel - that is, to guarantee convergence. The latter kernel must satisfy several properties, like positivity on the compact support, normalization, and monotonically decreasing with distance. Kernel functions may assume different forms, which derive from the original Gaussian function; Gaussian kernels, however, are deprecated because of the heavy computational cost they require. The weighting function used in this work, hence, is defined by a piecewise polynomial expression, known as the Quintic Wendland kernel [36].

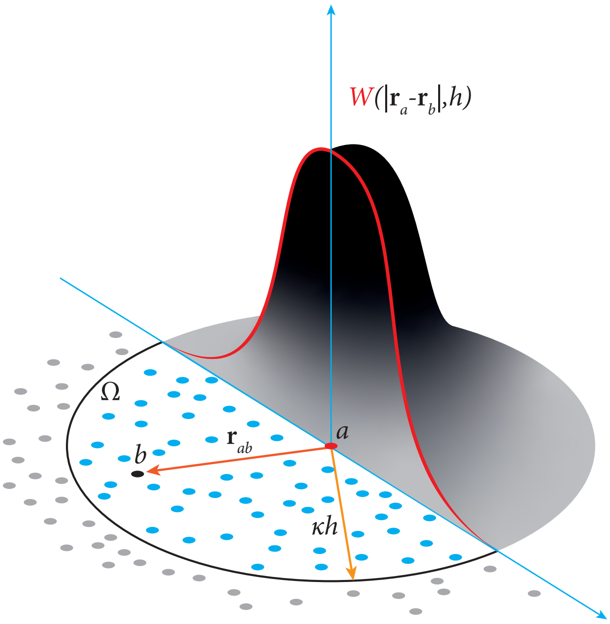


Figure 1: Example of a kernel function (reproduced from [26]).

DualSPHysics uses the so-called modified Dynamic Boundary Condition (or mDBC) to implement solid geometry discretizations within the SPH domain [35]. They overcome some issues of the former implementation, Dynamic Boundary Condition (DBC) – Crespo et al. [37]. The novel procedure is able to avoid the nonphysical gap between boundary and fluid particles that occur in dry boundary-wet boundary transition phases even using coarse resolution; this makes the mDBC perfectly suitable for the purpose of this work especially for the modeling of sloshing fluid masses. Details about their implementation are in [35] in which a sloshing tank with very energetic motion is validated; other application can be found in Capasso et al. [38] and Ruffini et al. [39].

3 SLOSHING FLUID EQUIVALENT MODEL

The present investigation was done on a prismatic tank, and the motion imposed to it is purely horizontal: likewise, the sloshing motion is assumed to be horizontal. This premise allows us to study through a linearized two-dimensional model, as the flow motion does not present relevant three-dimensional features. By means of linearization, the motion is studied via the combination of the m harmonic motions that compose it. Moreover, the fluid is considered to be irrotational and inviscid. Boundary conditions are withal inferred.

3.1 Fundamental assumptions

The liquid contained in the tank is considered to be incompressible, giving rise to the continuity equation:

$$\nabla \cdot \mathbf{u} = 0 \tag{3}$$

with $\mathbf{u} = \{u, w\}$ being the velocity field. The fluid is also inviscid, and its motion can be assumed irrotational, so that the velocity may be deduced from a velocity potential $\phi = \phi(x, z)$. In this case, the velocity is given by the spatial derivative of the defined potential, known as Laplace equation:

$$\nabla^2 \phi = 0 \tag{4}$$

Previous studies demonstrated that the moving fluid may be partitioned in an impulsive and a convective portion [40]. The former moves along with the tank whilst the latter oscillates relatively to it. The velocity potential, likewise, may be split into impulsive and convective:

$$\phi = \phi_i + \phi_c \tag{5}$$

The boundary conditions for the velocity potentials with respect to tank walls are:

$$\frac{\delta \phi_i}{\delta n} = V; \frac{\delta \phi_c}{\delta n} = 0 \tag{6}$$

where V is the velocity of the tank in the direction of the imposed motion, and n the direction orthogonal to the tank walls, retained undeformable [6]. This condition indicates that the motion of the particles near the tank wall is reconducted to the sole motion of the tank wall, that is, near the boundary, the motion is exclusively given by the impulsive portion of fluid.

As mentioned before, the sloshing phenomenon can be represented by m modes of vibration. Of particular interest are the anti-symmetric modes, which horizontally shift the liquid's center of gravity position (Figure 2) producing lateral motion in order to re-establish equilibrium [41]. The convective oscillation also discloses free-surface phenomena which can often be nonlinear, except for small amplitude waves under weak oscillation.

3.2 Linearized flow equations

Our study proposes the numerical simulation of a sloshing problem included in the group of cases that have explicit solution by pure analytical methods, as the sloshing is purely horizontal and the tank has rigid upright walls with rectangular geometry. The latter cases are also feasible

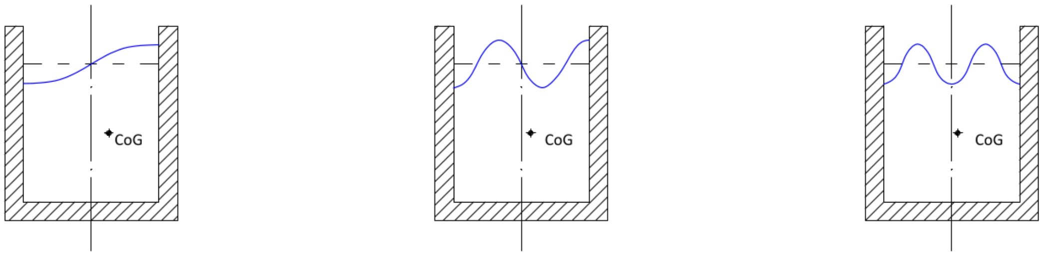


Figure 2: center of gravity shift in the first three anti-symmetric modes of vibration.

for mesh-based simulation. We are thus able to compare results from all three methods. The setup was arranged as illustrated in Figure 3. In order to analytically compare the results, the case must be linearized following the hypothesis hereunder exposed.

Inflicting a horizontal impulse to the tank, the displacement in time $X(t)$ may be written as:

$$X(t) = -iX_0 e^{i\omega t} \tag{7}$$

where X_0 is the initial position of the tank and ω is the angular frequency. Deriving $X(t)$ in time, we obtain the horizontal velocity of the tank $u(t)$:

$$u(t) = iX_0 \omega e^{i\omega t} \tag{8}$$

Given the system's geometrical and forcing conditions, waves generated with a single impulse tend to be regular. The free-surface elevation description is, hence, explicited by a sinusoidal function η:

$$\eta(x,t) = Asin(\omega t - kx) \tag{9}$$

being A the amplitude of the wave, $k = 2\pi/\lambda$ the wave number (where λ is the wave length). Moreover, the motion is considered to be smooth, so particles outlining the free-surface are always the same. At the same time, given the reference system as per Figure 3, with x_1 and x_2 being two consecutive positions of the same particle and t_1 and t_2 the respective instants observed ($\Delta t = t_2 - t_1$), we may linearly describe the free-surface as:

$$\eta|_{(x_2,t_2)} = \eta|_{(x_1,t_1)} + w\Delta t \tag{10}$$

At the same time, assuming that the elevation may be described as a harmonic function (sinusoidal), it can be explicited as a Taylor series. In our case, the series may be trimmed at its first order differential without considerable error; rewriting (10) as so, bringing $v\Delta t$ to the left term and dividing by Δt gives:

$$w = \frac{\delta\eta}{\delta t} \tag{11}$$

namely, the kinematic free-surface boundary condition assuming small amplitudes ($\delta\phi/\delta x$ is negligible). The second fundamental assumption, as mentioned before, is that fluid is inviscid, that is, with no internal friction, so no dissipation is computed inside the system. The Bernoulli equation, hence describes the system's equilibrium. The kinematic term u^2+w^2 can be considered negligible. Knowing that the fluid pressure at the surface ($z = 0$) is equal to the atmospheric pressure, we can set the dynamic free-surface boundary condition:

$$\frac{\delta\phi}{\delta t} + g\eta = 0 \tag{12}$$

where g is the gravitational acceleration.

The velocity potential ϕ, must then obey the two aforementioned boundary conditions, and may be written in the form:

$$\phi(x, z, t) = [C_1 \cosh k(z + d)] \sin(\omega t - kx + \phi_i) \tag{13}$$

in which C_1 is a case-dependent constant and d is the water depth.

This potential description is suitable for our purpose of study, given that we aim to characterize the motion with a dispersion relation. In fact, fitting curves were proposed in literature with the angular frequency depending on the sole filling ratio (d/l). This means that for a tank of determined dimensions (semi-base l), d becomes the only variable of the curve's equation. The curves previously proposed were obtained by the parametrization of the wave number, which is substituted by a fitting scalar. The fitting outcomes from data obtained with experimental tests, analytical solution or mesh-based simulation. We propose a further parametrization through data obtained in meshless simulation. Hereafter, comparison of our solution with the ones proposed in literature is done, aiming to optimize the fitting parameters.

In order to describe the free-surface, we recall that for our reference system the latter is located at $z = 0$ (Figure 3); hence, Equations (9) and (13) are substituted in Equation (12). We note that $w = \delta\eta/\delta t$, and at the same time, $w = \delta\phi/\delta z$. By these means, we conclude that:

$$\omega^2 = gk \tanh kd \tag{14}$$

which is an example of dispersion relation.

4 VALIDATION

The numerical setup is implemented in DualSPHysics according to Figure 3. The geometry of the tank is used throughout the testing phase, and only the water level (d) changed according to the specific test. The four walls are discretized using the mDBC technique and as such, they comprise seven layers of particles. As in Figure 3, two numerical gauges were positioned at a small distance δ from the two lateral solid boundaries to capture the dynamic evolution of the free-surface (η) during the simulation. The water is set into motion by an initial translation along the x-axis.

Aiming at the characterization of the fluid motion through its free-surface profile, which in pure horizontal motion may be approximated with the sole first mode shape, we simulated a decay test in DualSPHysics. The test consists in imposing to the tank a single horizontal sinusoidal impulse with determined amplitude and period. Next, the external impulse is ceased and the liquid is left to slosh without any external addition of energy. Using data sampled by the two wave gauges, the peak heights were identified, and eventually used to identify the period T of the waves, as explained in the following. Specifically, the time series of the wave peaks was processed using a properly developed software routine, that purged the initial noise by backtracking data with circular shift. Using the up-zero-crossing method, we obtained the wave height referred to the undisturbed free-surface level.

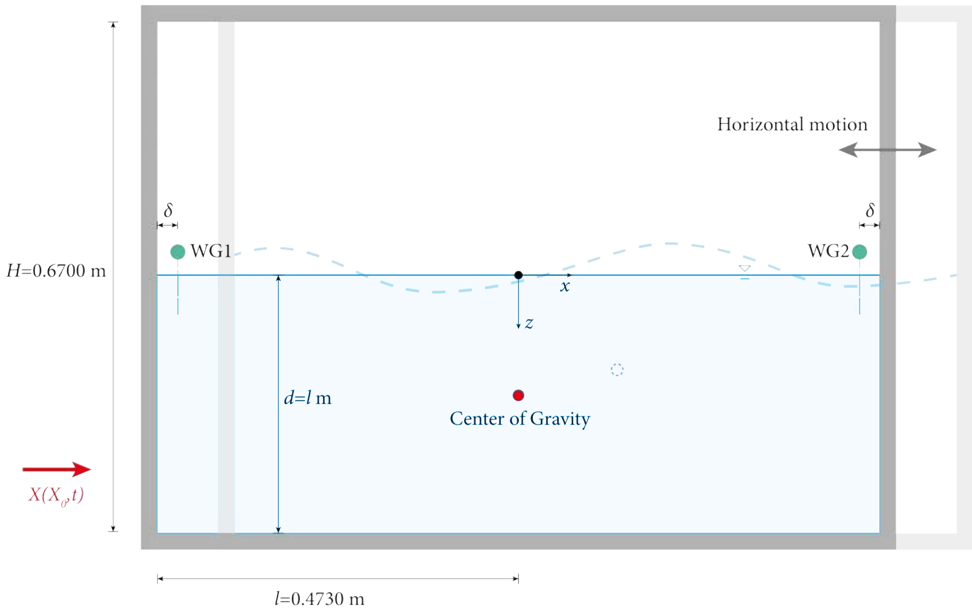
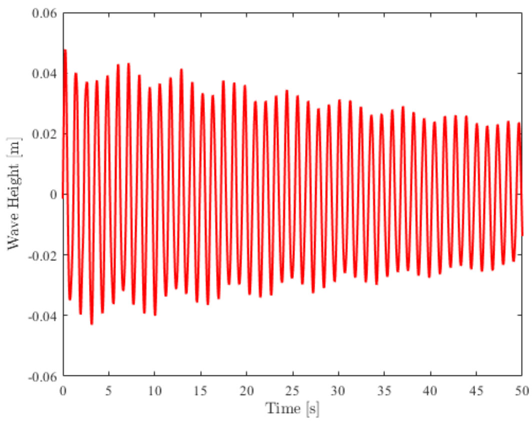
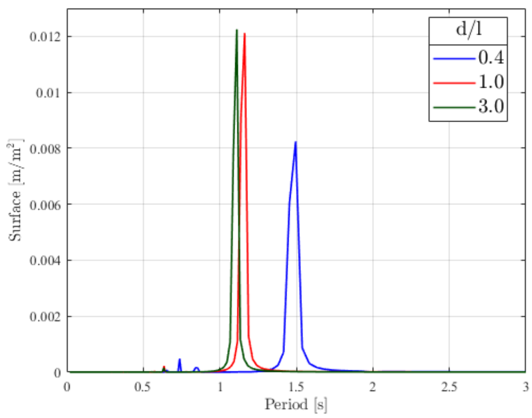


Figure 3: Sketch of the numerical setup implemented in DualSPHysics.



(a) Wave height measurements at WG1 for d/l = 1.0.



(b) Period spectrum for different filling ratios.

Figure 4: Transformation of the wave height signal from decay test simulations in DSPH.

The graphic Figure 4a shows the free surface time history gauged nearby the fixed wall of the tank. It can be observed that no substantial noise affects the decaying behavior of the fluid mass. Therefore, it is valid to determine the wave period as the interval between two subsequent peaks. We thus obviate the direct computation of λ.

The convolution of the period signal towards the the liquid was transferred through the Fast Fourier Transform (FFT) analysis tool, which operates with sinusoidal functions as basis. The period spectrum is respectively restored through sine functions [42].

Reading the period spectrum in Figure 4, whither the higher peak corresponds to the natural period (T_1) of the sloshing (higher density value of the spectrum). At this point, the natural

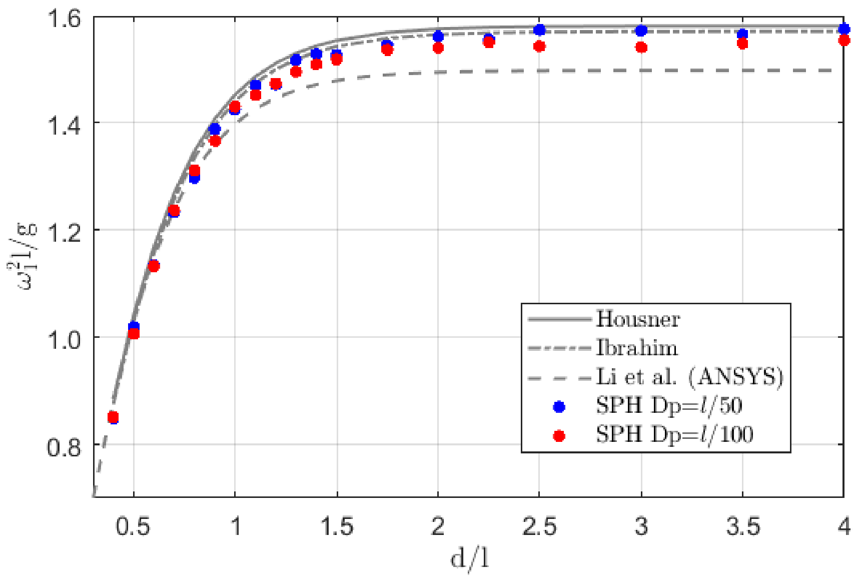


Figure 5: Comparison between results from simulations in DSPH ($Dp = l/50$ and $Dp = l/100$) and frequency curves previously proposed in literature ($Housner$ and $Ibrahim$).

frequency (ω_1) is quantified using the following relationship:

$$\omega_1 = \frac{2\pi}{T_1} \tag{15}$$

The natural frequencies obtained, each regarding a specific filling ratio, were likewise charted. As previously mentioned, fitting curves linking ω to the filling ratio d/l appear in literature. Specifically for the first mode of vibration, Housner (1963) [40], with an analytical-experimental method, proposed:

$$\omega_1^2 = \sqrt{2.5}.\frac{g}{l}.\tanh\sqrt{2.5}\frac{d}{l} \tag{16}$$

Further, finer research, headed by Ibrahim [1], explicited the equation with a purely analytical approach on different tank geometries (as a combination of primary geometry solutions). For anti-symmetric modes in a rectangular tank, it follows:

$$\omega_1^2 = \frac{g\pi}{2l}\tanh\frac{d\pi}{2l} \tag{17}$$

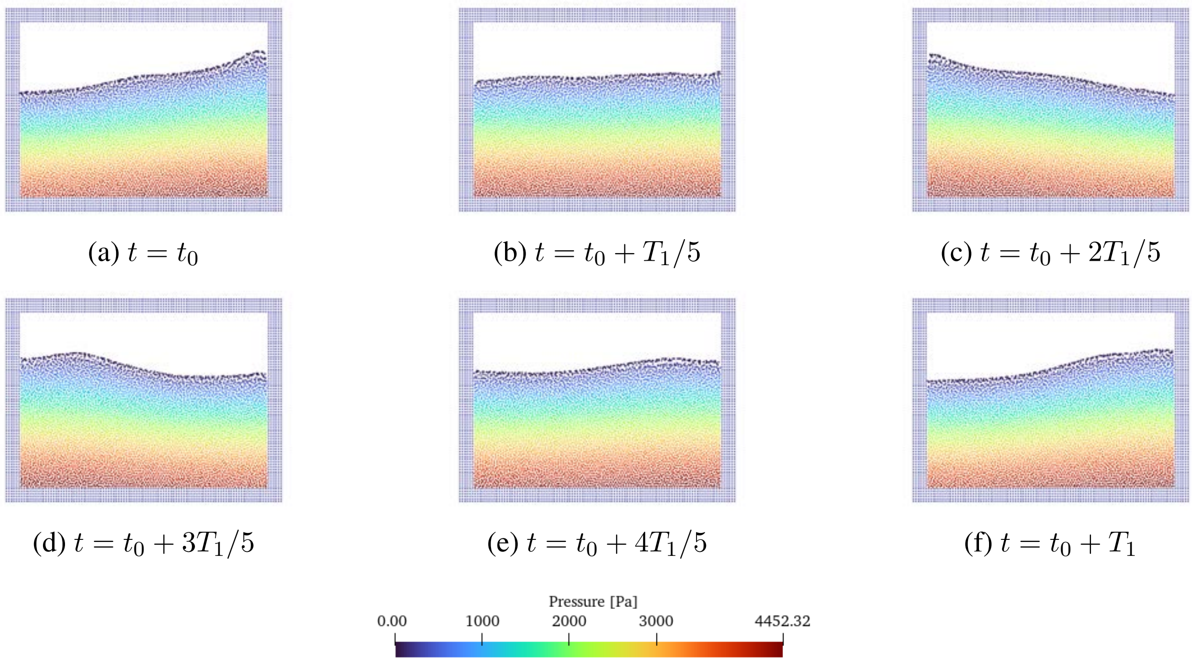
The approach was later optimized by Li et al. [43] with a simpler method using the CFD-based solver available in ANSYS, whose data was analytically synthesized by means of a least square fitting curve, giving the following equation:

$$\omega_1^2 = 1.498\frac{g}{l}\tanh 1.681\frac{d}{l} \tag{18}$$

We notice that all the proposed equations are parameter calibrations of Equation (14). Our numerical simulation was carried out in two different resolutions of discretization for the initial inter-particle distance (Dp). In particular, since $Dp = l/100$ showed good accordance with the fitting curves mentioned above, an ulterior simulation was processed with $Dp = l/50$, and good accordance persisted (see Figure 5).

As a matter of fact, comparison between our results and the equations proposed in literature, even when simulation was carried out in low resolution ($Dp = l/50$ and $Dp = l/100$), shows that the natural frequency obtained was comprised between the curves delineated by equations by Li [43], Housner [40], and Ibrahim [1]. We may then conclude that great accordance was observed even for low resolution simulation (as for $Dp = l/50$ in Figure 5). This is an important result in an SPH simulation, given that high resolutions demand greater computational effort - thus, we were able to characterize a free-surface phenomenon with minor computational demand and no economical cost. Charting the solution equations proposed by Housner and Ibrahim, we observe that they are almost superimposed, whereas the simplification proposed by Li et al. addresses an underestimated value of ω_1. In order to illustrate our considerations, we took the example of the simulation conducted for a 1.0 filling ratio. Transforming the data from gauges through the FFT, we had a natural period T_1 of 1.163 s, consequently, a natural frequency ω_1 of 5.403 rad/s. For the same filling ratio, results from literature were: 5.489 rad/s (Equation (16)); 5.466 rad/s (Equation (17)); 5.384 rad/s (Equation (18)).

Another available option for analyzing the simulation is an optical approach. By identifying frames with the same free-surface profile, the time interval between two consecutive matching profiles corresponds to the wave period. As illustrated above, wave periods may be computed based on the free-surface profile. A decay test was launched imposing a 0.040 m amplitude and 1.133 s period impulse to the partially filled tank with d/l = 1.0.



(a) $t = t_0$ (b) $t = t_0 + T_1/5$ (c) $t = t_0 + 2T_1/5$

(d) $t = t_0 + 3T_1/5$ (e) $t = t_0 + 4T_1/5$ (f) $t = t_0 + T_1$

Figure 6: Post-process visualization of the decay test simulation launched in DSPH during a time-lapse of $1.16s$.

In Figure 6, frames were captured in a time-lapse corresponding to the natural period T_1 obtained through the FFT of the wave signal captured. Even though profiles at the first (Figure 6a) and last (Figure 6f) frame are similar, they do not exactly correspond to each other, as we would expect for a regular wave at the completion of its period. A part from the fact that post-process filter for the free-surface was not applied, other explanations may be seized: the waves are not purely regular; in the FFT the energy dissipation by collision with the tank wall is not computed; moreover, in the spectrum itself we notice that other than the higher density value there is a second minor peak at the left indicating another mode that still interferes (Figure 4).

5 CONCLUSIONS

The present study aimed to characterize the sloshing motion in a partially filled tank within a simplified method, that could nonetheless well capture free-surface outline and relative non-linearities. A numerical meshless approach was undertaken to avoid drawbacks observed in other methods – despite the accuracy, purely analytical solutions are not explicit if not in simple cases, such as rectangular rigid tank. Experimental tests offer indisputable reference data, though less affordable insight of economical costs and facilities needed. We thus proceeded with CFD simulations using the open-source code DualSPHysics. Depicting a case that could be described by explicit analytical formulae, numerical results were compared to theoretical solutions in sight of validating the use of SPH method.

Simulations consisted in imposing a force of determined amplitude (A_t) and period (T_t) to the tank through a horizontal thrust kept for a single cycle. Hence, ceased the motion, liquid was left to slosh and settled gauges measured the wave height at fixed points. The signal read was transformed into a spectrum, giving back the natural period of the fluid's motion. Once the natural period is known, we immediately calculate the natural frequency ω_1. Our results, at that point, were compared to the ones proposed in literature so to assure that the method was valid. Accordingly, the numerical simulation results were actually close to that of other methods, and differently from the mesh-based model proposed by Li et al., in our case w_1 was not much underrated if compared to experimental results. The underestimation of the motion, indeed, could be a dangerous imprecision when foreseeing its effects on structures.

Once the motion is thus properly characterized, apropos of the specific filling ratio, sloshing may be tuned towards or against the natural frequency of the structure depending on the purpose of interest. By that premise, given the validation of the method and its potential to solve non-linear phenomena, it could represent a solution tool for further cases of study in which explicit analytical solving is not possible.

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