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FINITE ELEMENT TECHNOLOGY-BASED SELECTIVE MASS SCALING FOR EXPLICIT DYNAMIC ANALYSES OF THIN-WALLED STRUCTURES USING SOLID ELEMENTS

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Abstract. *We present a novel class of selective mass scaling (SMS) concepts in the context of explicit dynamic analyses of thin-walled structures using solid elements. The novel SMS schemes are based on the discrete strain gap (DSG) method [1], a method from the field of finite element technology (FET). Thus, they are denoted as DSGSMS schemes and they extend the initial work of [2] for shear deformable structural element formulations to the application in thin solid elements. We show that these novel DSGSMS concepts naturally preserve both translational and rotational inertia and possess high accuracy. Additionally, having non-linear problems including large rotations in mind, we show how efficient isotropic DSGSMS concepts can be developed such that the need for reassembly of scaled mass matrices is avoided.*

Keywords: selective mass scaling, finite element technology, thin-walled structures, solid elements, solid shell elements.

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1 INTRODUCTION

The critical time step in explicit transient analyses depends on the highest frequency of the discretized system, that is

$$\Delta t_{\mathrm{crit}} = \frac{2}{\omega_{\max}}. \tag{1}$$

In case of thin-walled structures discretized by solid or solid shell elements, the critical time step and thus the efficiency is limited by the highest frequencies related to thickness stretch of the elements [3].

Mass scaling methods aim at increasing the critical time step size Δt_{crit} and thus increasing efficiency by artificially adding inertia to the system. For discretizations being composed of solid or solid shell elements the simplest version of mass scaling, that is conventional mass scaling (CMS), is very limited in its use. CMS influences the entire frequency spectrum and increases linear momentum of the structure in an unphysical way. The use of CMS for solid elements is limited to small parts of the mesh that limit Δt_{crit} .

SMS concepts aim at scaling down the highest frequencies while keeping the low frequencies as unaffected as possible. This typically comes at the cost of non-diagonal mass matrices. The general form of SMS methods can be represented as

$$\mathbf{M}^{\circ} = \mathbf{M} + \boldsymbol{\lambda}^{\circ}, \tag{2}$$

where $\mathbf{M}^{\circ}$ is the scaled mass matrix, $\mathbf{M}$ is the initial mass matrix and $\boldsymbol{\lambda}^{\circ}$ describes the artificial mass matrix. Existing SMS concepts mainly differ in the construction and the structure of the artificial mass matrix $\boldsymbol{\lambda}^{\circ}$.

Most SMS concepts from literature are designed for discretizations composed of solid or solid shell elements, as can be seen in [4, 5, 3, 6, 7, 8, 9, 10], among others. To fulfil a basic requirement, they are designed such that translational inertia is preserved. To achieve higher accuracy, some SMS concepts can be extended in such a way that, additionally, rotational inertia is preserved. But this typically comes along with additional computational costs, which do not pay off in most applications.

In this contribution, we present a novel class of selective mass scaling (SMS) concepts in the context of explicit dynamic analyses of thin-walled structures using solid elements. The novel SMS schemes are based the DSG method, known from FET. In our initial work [2], we show that the use of methods from FET for the design of novel SMS schemes is of high potential. This promising concept is further extended to so-called DSGSMS schemes being applicable to discretizations of thin solid or solid shell elements. The resulting DSGSMS concepts naturally preserve both translational and rotational inertia and possess high accuracy. Additionally, we show how efficient isotropic DSGSMS concepts can be developed such that reassembly of scaled mass matrices is not needed. This is particularly important for explicit simulations of non-linear problems including large rotations.

The present contribution is organized as follows. Section 2 introduces the novel DSGSMS concept for thin solid elements. We develop both isotropic and anisotropic DSGSMS concepts and discuss their properties. In Section 3, the novel DSGSMS concepts are tested and compared to existing SMS via numerical examples. Section 4 concludes the achieved results and gives an outlook on open issues and necessary future developments.

2 THE DSGSMS CONCEPT FOR THIN SOLID ELEMENTS

As described in [2], the inspiration for the DSGSMS concept stems from the recently presented ISMS concept [11]. The DSGSMS concept is based on the theoretical connection of shear deformable, hierarchic structural element formulations and DSG formulations to avoid transverse shear locking. As shown in [2], the four main steps of the DSG method can be summarized as 1. integration, 2. collocation, 3. interpolation and 4. differentiation. For the sake of simplicity, in [2], the DSG method is explained for the simple case of a two-node, two-dimensional and straight Timoshenko beam element. For further details on the DSG method, we refer to [1].

As can be seen from [1, 12, 13], the DSG method is a general method for eliminating any type of geometrical locking effects in beam, plate, shell, solid, or solid shell elements. The four steps of the DSG method described above can be applied analogously to every single strain component of the corresponding mechanical model. In three-dimensional discretizations of thin-walled structures, the highest natural frequencies are usually associated with thickness stretch modes, as shown in [3]. In order to scale the highest natural frequencies by the DSGSMS method, the discrete strain gaps of the normal strain component in thickness direction, ε_{33}, are integrated (1.), collocated (2.) and interpolated (3.). From the resulting strain gap function u_3^{mod} an additional contribution to the virtual kinetic energy is constructed by

$$\delta W^{\mathrm{kin}}_{\mathrm{DSGSMS-33}} = \alpha_{\text{DSG-33}} \int_{\Omega} \left(\delta u_3^{\mathrm{mod}} \rho \ddot{u}_3^{\mathrm{mod}} \right) \,\mathrm{d}\Omega, \tag{3}$$

where ρ and $\alpha_{\text{DSG-33}}$ denote the density and the artificial, scalar mass scaling parameter of the DSGSMS-33 method, in which the artificial mass matrix is solely constructed by a strain gap function stemming from integration of ε_{33}. In general, dependent on the particular application, further strain components may be taken into account. The artificial mass matrix $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ calculated from the interpolated strain-gap function is aligned with the 3 axis or thickness direction. $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ is anisotropic, that is, only artificial inertia is added in thickness direction. In subsequent derivations, the algebraic structure of the artificial mass matrix $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ is compared to the structure of $\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}}$, according to Olovsson et al. [5]. This is of particular interest, since the concept of Olovsson et al. [5] represents the state-of-the-art SMS in commercial codes like LS-DYNA.

In the case of an eight-node hexahedron element, $\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}}$ takes the form

$$\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}} = \begin{bmatrix} \boldsymbol{\lambda}^{\circ,8\times8}_{\mathrm{Olov}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\lambda}^{\circ,8\times8}_{\mathrm{Olov}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\lambda}^{\circ,8\times8}_{\mathrm{Olov}} \end{bmatrix}, \quad \text{with} \quad \boldsymbol{\lambda}^{\circ,8\times8}_{\mathrm{Olov}} = \frac{\beta m^{\mathrm{e}}}{56} \begin{bmatrix} 7 & -1 & \dots & -1 \\ -1 & 7 & & \\ \vdots & & \ddots & \\ -1 & & & 7 \end{bmatrix}, \tag{4}$$

where m^{e} represents the total mass of the element and a blockwise ordering of the element degrees of freedom of the form $\mathbf{d} = \left[d_x^1 \, d_x^2 \, \dots \, d_x^8 \, d_y^1 \, d_y^2 \, \dots \, d_y^8 \, d_z^1 \, d_z^2 \, \dots \, d_z^8 \right]^{\mathrm{T}}$ is assumed. β represents a scalar mass scaling parameter. From the algebraic structure of $\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}}$ in Eq. (4) we can see:

- SMS from Olovsson et al. [5] is *isotropic*, i.e. each node has the same inertia in x, y and z direction.

- The inertia artificially added to the main diagonal entries is subtracted row by row and column by column. Thus, the translational inertia of $\mathbf{m}^{\circ}_{\mathrm{Olov}}$ is preserved.

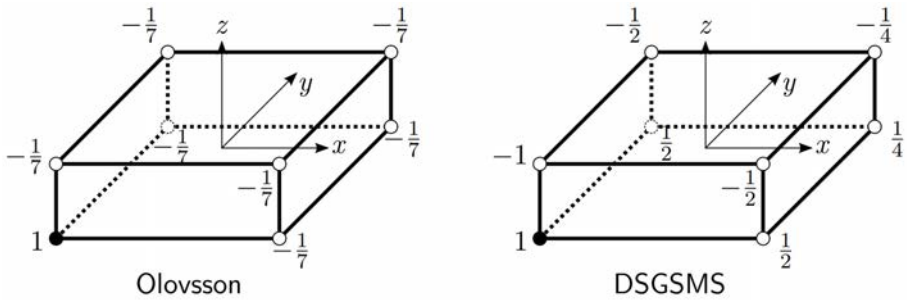


Figure 1: Exemplary representation of the artificial mass distribution for a thin hexahedron. The value "1" represents unit value of the main diagonal entry of the corresponding artificial mass matrix $\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}}$ (left) or $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ (right). Further values represent off-diagonal entries.

- The rotational inertia of $\mathbf{m}^{\circ}_{\mathrm{Olov}}$ is *not* preserved.

Sorting the element degrees of freedom in blocks, see above, results in the algebraic structure of the artificial element mass matrix $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$, derived from Eq. (3) as

$$\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\lambda}^{\circ,8\times 8}_{\mathrm{DSGSMS-33}} \end{bmatrix}, \tag{5}$$

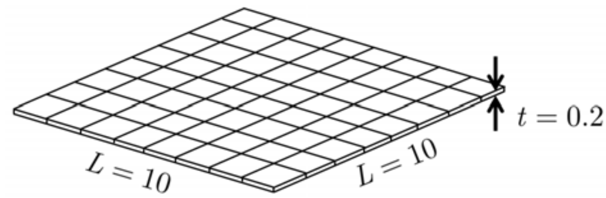
with

$$\boldsymbol{\lambda}^{\circ,8\times 8}_{\mathrm{DSGSMS-33}} = \alpha_{\mathrm{DSG-33}} \cdot f \begin{bmatrix} 1 & 1/2 & 1/4 & 1/2 & -1 & -1/2 & -1/4 & -1/2 \\ 1/2 & 1 & 1/2 & 1/4 & -1/2 & -1 & -1/2 & -1/4 \\ 1/4 & 1/2 & 1 & 1/2 & -1/4 & -1/2 & -1 & -1/2 \\ 1/2 & 1/4 & 1/2 & 1 & -1/2 & -1/4 & -1/2 & -1 \\ -1 & -1/2 & -1/4 & -1/2 & 1 & 1/2 & 1/4 & 1/2 \\ -1/2 & -1 & -1/2 & -1/4 & 1/2 & 1 & 1/2 & 1/4 \\ -1/4 & -1/2 & -1 & -1/2 & 1/4 & 1/2 & 1 & 1/2 \\ -1/2 & -1/4 & -1/2 & -1 & 1/2 & 1/4 & 1/2 & 1 \end{bmatrix}, \tag{6}$$

where $\alpha_{\mathrm{DSG-33}}$ represents the scaling factor, and f represents an additional factor, which is dependent on element geometry and material data. As stated above, $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ is anisotropic. In order to analyze other properties, the algebraic structure of $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ is compared with the structure of $\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}}$. For this purpose, Figure 1 shows, for a representative node (node 1) and for the inertias in the z axis or 33 direction, how the artificial mass in $\boldsymbol{\lambda}^{\circ}_{\mathrm{Olov}}$ or $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ are distributed to the corresponding nodal degrees of freedom. The shown distribution factors correspond to the first row of the mass matrices.

Since an isotropic artificial mass matrix is desirable for efficiency reasons, a first attempt is made in this regard, as subsequently outlined. The anisotropic artificial mass matrix $\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33}}$ only includes artificial inertias, which are related to the degrees of freedom of displacement in the z direction, i. e. d^i_z, see Eq. (6). In the following, these artificial inertias are transferred to the x and y direction in a similar fashion, leading to an artificial mass matrix of the form

$$\boldsymbol{\lambda}^{\circ}_{\mathrm{DSGSMS-33-iso}} = \begin{bmatrix} \boldsymbol{\lambda}^{\circ,8\times 8}_{\mathrm{DSGSMS-33}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\lambda}^{\circ,8\times 8}_{\mathrm{DSGSMS-33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\lambda}^{\circ,8\times 8}_{\mathrm{DSGSMS-33}} \end{bmatrix}, \tag{7}$$



$E = 200\,\text{GPa}$
$\nu = 0.3$
$\rho = 8000\,\text{kg/m}^3$

Figure 2: Simply supported quadratic plate, problem setup.

This results in an isotropic artificial mass matrix $\boldsymbol{\lambda}^{\circ}_{\text{DSGSMS}-33-\text{iso}}$ that remains constant during rotation. This property is particularly desirable for non-linear problems and leads to less computational effort compared to the anisotropic case. The non-zero entries of $\boldsymbol{\lambda}^{\circ}_{\text{DSGSMS}-33-\text{iso}}$ and of $\boldsymbol{\lambda}^{\circ}_{\text{Olov}}$ are located at exactly the same positions, but their values differ, as can be seen in Fig. 1. Furthermore, it is visible that in the construction of $\boldsymbol{\lambda}^{\circ}_{\text{DSGSMS}-33}$ the smallest and decisive edge length, i. e. the thickness direction (here the z direction), was considered in special way. The inertias of nodes lying on top of each other have identical absolute values, but different signs. With the Eqs. (6) and (7) and with Fig. 1, we remark that:

- DSGSMS-33-iso is *isotropic*, i.e. the inertias in the x, y and z direction are identical.
- Artificial inertia added to the main diagonal entries is subtracted row by row and column by column and, thus, $\boldsymbol{\lambda}^{\circ}_{\text{DSGSMS}-33-\text{iso}}$ is preserving linear momentum.
- $\boldsymbol{\lambda}^{\circ}_{\text{DSGSMS}-33-\text{iso}}$ is naturally preserving angular momentum (not shown), while $\boldsymbol{\lambda}^{\circ}_{\text{Olov}}$ is not.
- The non-zero entries of $\boldsymbol{\lambda}^{\circ}_{\text{DSGSMS}-33-\text{iso}}$ and of $\boldsymbol{\lambda}^{\circ}_{\text{Olov}}$ are located at exactly the same positions, solely their values differ. This results identical memory requirements.
- The conditioning of the mass matrices (not shown) is also almost the same, which is important for the efficiency of iterative solvers.

3 NUMERICAL EXAMPLES

Next, the performance of novel DSGSMS concepts presented herein is tested via two numerical examples. First, in Section 3.1, frequency spectra obtained with different SMS concepts are compared for a plate problem, discretized with solid elements. In Section 3.2 the performance in a transient linear problem is studied.

3.1 Simply supported plate

The effectiveness of the newly presented DSGSMS-33 and DSGSMS-33-iso is demonstrated by frequency spectra of a simply supported plate, discretized with eight-node enhanced assumed strain (EAS) solid elements, see Fig. 2. The results for the generalized eigenvalue problem of the form

$$(\mathbf{K} - \omega^2 \mathbf{M})\boldsymbol{\phi} = \mathbf{0} \tag{8}$$

are shown in Fig. 3. All scaling parameters are selected such that the maximum natural frequency is scaled down to approximately 60% of the original natural frequency, which was obtained via a row-sum lumped mass matrix (LMM). That is, $\frac{\omega^{\circ}}{\omega_{\text{LMM}}} = \frac{3}{5}$. CMS scales the entire frequency spectrum to $\frac{\omega^{\circ}}{\omega_{\text{LMM}}} = \frac{3}{5}$ and is therefore unsuitable in most situations. When SMS

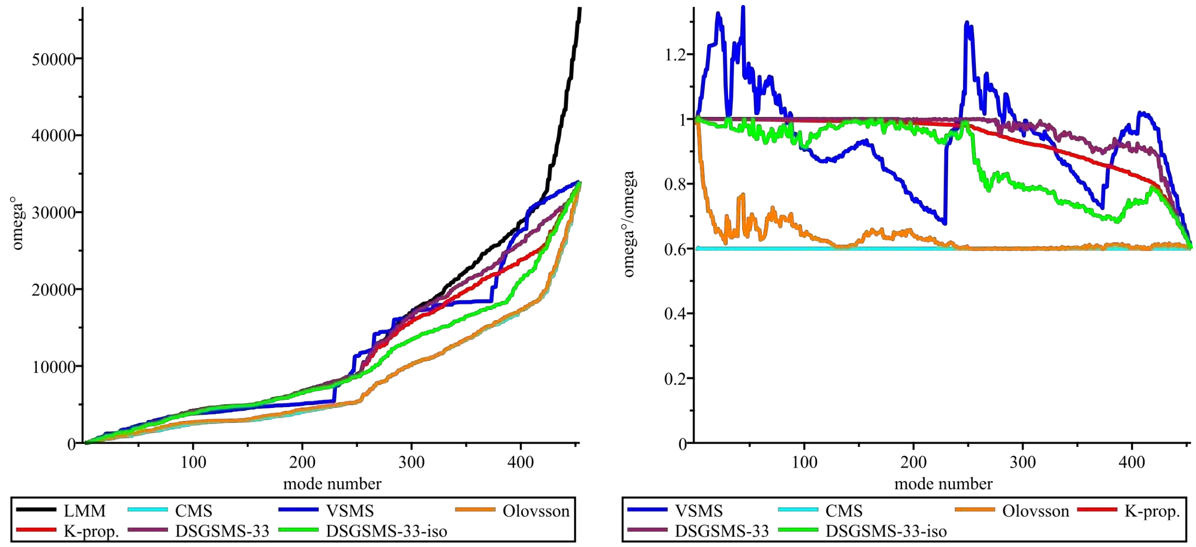
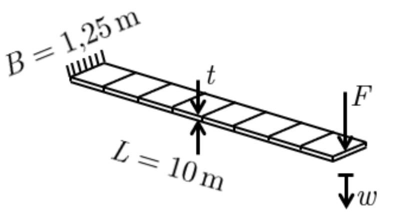


Figure 3: Simply supported plate, frequency spectra (without the six rigid body modes) of different mass scaling methods in comparison. Left: scaled frequency spectra, right: ratio of scaled to unscaled frequencies.



$E = 200\,\mathrm{GPa}$

$\nu = 0.3$

$\rho = 8000\,\mathrm{kg/m^3}$

Figure 4: Tip-loaded cantilever, problem setup.

according Olovsson et al. [5], see Eq. (4), is used, only a few low frequencies are preserved with high accuracy, while large deviations occur over large parts of the eigenvalue spectrum, see Fig. 3. Variational selective mass scaling (VSMS) according to [6] is also not very accurate for such slender element geometries. The most accurate SMS method for the example presented herein is DSGSMS-33, which is even more accurate than stiffness proportional mass scaling (k-prop), according to [4]. However, these two mass scaling methods are anisotropic and, most likely, numerically too inefficient for non-linear simulations including large rotations.

For this problem, previously introduced DSGSMS-33-iso seems to provide a good compromise between efficiency and accuracy. While DSGSMS-33-iso shows only minor loss in accuracy in the relevant first half of the spectrum, it is isotropic and will lead to comparable numerical costs as the SMS method by Olovsson et al. [5], also for non-linear problems.

3.2 Tip-loaded cantilever

Next, the presented *isotropic* mass scaling concepts are compared using a transient, geometrically linear numerical simulation. The cantilever shown in Fig. 4 is abruptly loaded by a concentrated force F (distributed to the four edge nodes), while initial displacements and initial velocities are assumed to be zero. The material parameters and the element formulation are identical to the simply supported plate problem from Figure 2.

In Fig. 5, results for two different thicknesses, that is $t = 0.2\,\mathrm{m}$ (left) and $t = 0.05\,\mathrm{m}$ (right), are shown. In both cases, the cantilever is discretized with 8×1 elements and the load F is chosen such that the maximum static displacement according to Bernoulli beam theory is $w = 0.04\,\mathrm{m}$. The structural response is studied for different isotropic mass scaling methods by

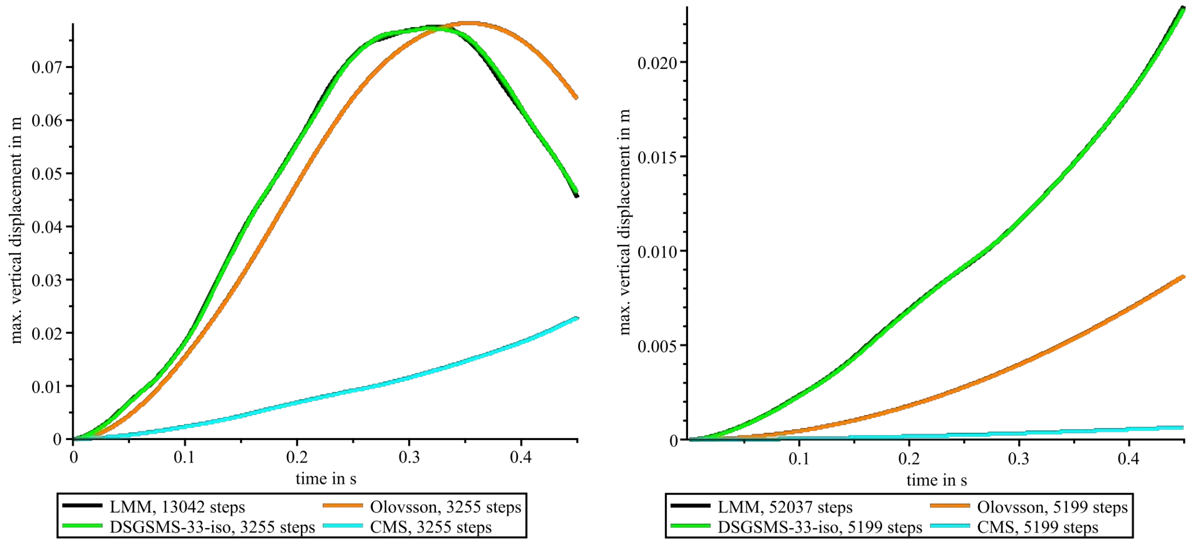


Figure 5: Tip-loaded cantilever, left: transient solution for thickness $t = 0.2\,\mathrm{m}$, reduction of max. frequency by approx. 75% with respect to LMM, right: transient solution for thickness $t = 0.05\,\mathrm{m}$, reduction of max. frequency by approx. 90% with respect to LMM.

comparing the time history of the maximum vertical displacement w up to the the time of $0.45\,\mathrm{s}$. We consider a geometrically linear dynamic simulation with explicit time integration by central difference method. In all cases, the critical time step size Δt_{crit} is determined exactly by solving the generalized eigenvalue problem according to the Eq. (8). The obtained solutions for LMM (without mass scaling) with 13042 required time steps (left, thickness $t = 0.2\,\mathrm{m}$) and 52037 required time steps (right, thickness $t = 0.05\,\mathrm{m}$) serve as reference. For the thickness $t = 0.2\,\mathrm{m}$ the maximum natural frequency $\omega_{\max}$ is reduced by approximately 75% by all mass scaling methods, so that only 3255 time steps are required. In the thin case, for thickness $t = 0.05\,\mathrm{m}$, the maximum natural frequency $\omega_{\max}$ is reduced by about 90%, such that 5199 instead of 52037 time steps (for LMM, without mass scaling) are sufficient.

The results shown in Fig. 5 underline the potential of DSGSMS-33-iso presented herein. DSGSMS-33-iso provides practically identical results as LMM at only 25% or 10% of the required number of time steps. For both thicknesses CMS proves to be absolutely inaccurate. But, also the SMS method from Olovsson et al. [5] shows relatively large deviations compared to the reference solution (LMM), already for thickness $t = 0.2\,\mathrm{m}$. For more slender elements, that is the thin case with $t = 0.05\,\mathrm{m}$, even larger differences between the SMS concepts can be observed. The high accuracy of DSGSMS-33-iso underlines the high potential of the novel concepts presented in this contribution.

4 CONCLUSIONS AND OUTLOOK

We presented a novel class of SMS concepts for efficient and accurate explicit dynamic analyses of thin-walled structures using solid elements. The novel DSGSMS schemes extend the initial work of [2] for shear deformable structural element formulations to the application in thin solid elements. Resulting scaled mass matrices naturally preserve translational and rotational inertia, coming along with high accuracy of large parts of the frequency spectra. Both anisotropic and isotropic DSGSMS methods have been developed, that is DSGSMS-33 and DSGSMS-33-iso. The latter provides a good compromise between accuracy and efficiency and is, thus, a highly promising SMS concept for non-linear transient analyses to be studied in future. Additionally, practical use of the presented concepts requires the development of efficient and accurate time step estimates.

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