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NUMERICAL MODEL CALIBRATION OF A LOW IMPACT STRENGTHENING TECHNIQUE BASED ON BED-JOINTS SLIDING IN SEISMIC RETROFITTING OF MASONRY INFILLS

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Abstract

The challenge against the waste of resources in the world of construction industry, starts from the seismic and thermal retrofitting of existing buildings with low environmental impact innovative technologies. When the buildings are especially vulnerable, for instance in the case of double-layer infills - often without thermal insulation - it's really hard to achieve satisfactory performance levels without resorting to radical interventions.
In these cases the interaction with the building internal activities is unavoidable and forms a further design constraints.
This work shows an innovative technology based on recycled plastic parts, optimized for the double-layer infills of existing buildings in which the maintaining of the internal activity is fundamental for the successful of the intervention. The system with recycled plastic joints is aiming at improving thermal and seismic performances, avoiding the residents relocation, consistently reducing the demolitions of existing infills and using plastic from recycled waste.
Such system was preliminarily tested on a real-scale double-layer infill equipped with two joint lines under horizontal force at the testing laboratory of materials and structures (LPMS) of the University of L'Aquila.
On the basis of this previous experimental tests, the present study proposes the calibration of a macroelement and a later numerical modelling of a multi-storey frame representative of r.c. buildings of the 80's.
The results of the retrofitted configuration with plastic joints were compared with bare frame and infilled frame with different macroelement available for traditional infills.

Keywords: in-plane retrofitting of masonry buildings, bed-joints sliding, macroelements calibration, recycled plastic joints, double-layer infills, sustainable materials.

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1 INTRODUCTION

In the past 12 years different effective and innovative techniques were developed to improve the seismic performances of single layer infills [1], [4].

However, for the double-layer configuration, no specific solution was developed and the predominant approach is still referred to reinforced coatings using either composite materials as Fiber Reinforced Polymers (FRP) in strips or sheets [5], [6], or fiber Reinforced Mortar (FRCM or FRLM) as external coating [7][8].

This type of infill can show high fragility in case of dynamic excitation; in fact, under out-of-plane force, the two layers generally have different motion since unlinked. This effect reduces the whole strength because the instability generally comes before ultimate strength.

Since 2012, one of the authors has started developing innovative devices for new building aiming at reducing in-plane fragility and preventing out-of-plane instability of the double-layer infills, through the combined effect of horizontal flexible joints and vertical bands made of recycled plastic [10], [11].

In further studies the previous joint waw upgraded to be use in the retrofitting of double-layer infills in which the mitigation of the pervasiveness is a priority requirement [12], also providing satisfactory results from a thermo-mechanical standpoint [13].

To date, the proposed technique stands out from the previous solutions by providing to the out-of-plane resistant mechanism two different mechanical contributions: a) the shear resisting mechanism due to plastic plate that links the two layers, b) the flexural resisting mechanism provided by the cantilevering elements placed behind the bricks where joint opens.

Figure 1 shows the mechanisms involved in the equilibrium of the infill undergoing out-of-plane inertia force induced by earthquake.

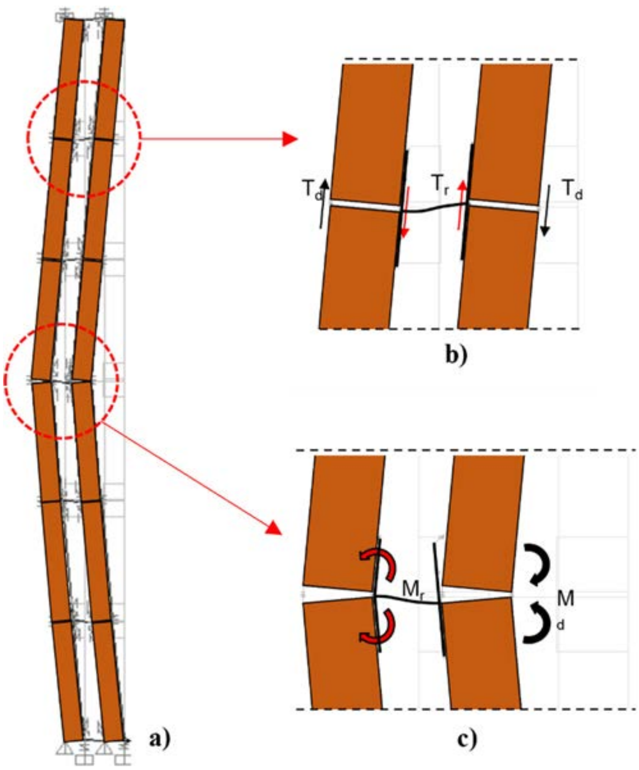


Figure 1. Resistant mechanisms involved in the equilibrium of the double-layer infill undergoing OoP loads: a) deformed configuration, b) shear resisting mechanism, c) flexural resisting mechanism.

2 NUMERICAL MODELLING OF BED-JOINT SLIDING

Bare concrete frames are structures with significantly more flexibility respect those infilled with common bricks and mortar, therefore when they work in parallel the stiffer element takes a large part of the inertia forces.

As a consequence, the elements are strengthened until to satisfy the performance demands provided by the current standard: this is what we can call “strength-based approach”.

Following this approach, the stiffness inevitably grows and therefore the demand on both structural and non-structural element, as a consequence of a stiffer structure.

The proposed technique based on the bed-joint sliding mechanism follows the opposite philosophy. All the deformations are channeled toward predefined lines in which special deformable joints are placed. In this way, blocks can slide without any cracks and the interaction between frame and infill can be neglected: this is what we have called “flexibility-based approach”. Following this second approach, we have low increasing in stiffness as well as having more dissipation – due to plastic deformation of the joints – and low interaction with the frame.

2.1 Calibration of a simplified macroelement

In this work the authors propose a modelling strategy via simplified macro-model developed for infill masonry panels undergoing in-plane forces [14] and implemented in Seismostruct [15], a FEM program of the software house SeismoSoft.

The calibration of the macroelement was carried out on a previous experimental campaign at the University of L'Aquila, in which a double-layer infilled frame of 12 and 8 cm thickness with 12 cm intermediate hollow space was tested [12].

The aim of the experimental test was to verify the triggering of the sliding mechanism due to special deformable joints made of recycled plastic, placed within an infill built in a wood frame with hinged ends. Figure 2 shows some assembly phases of the tested infill, while figure 3 shows the testing setup with the corresponding mechanical scheme with nonlinear central element developed by [14], respectively.

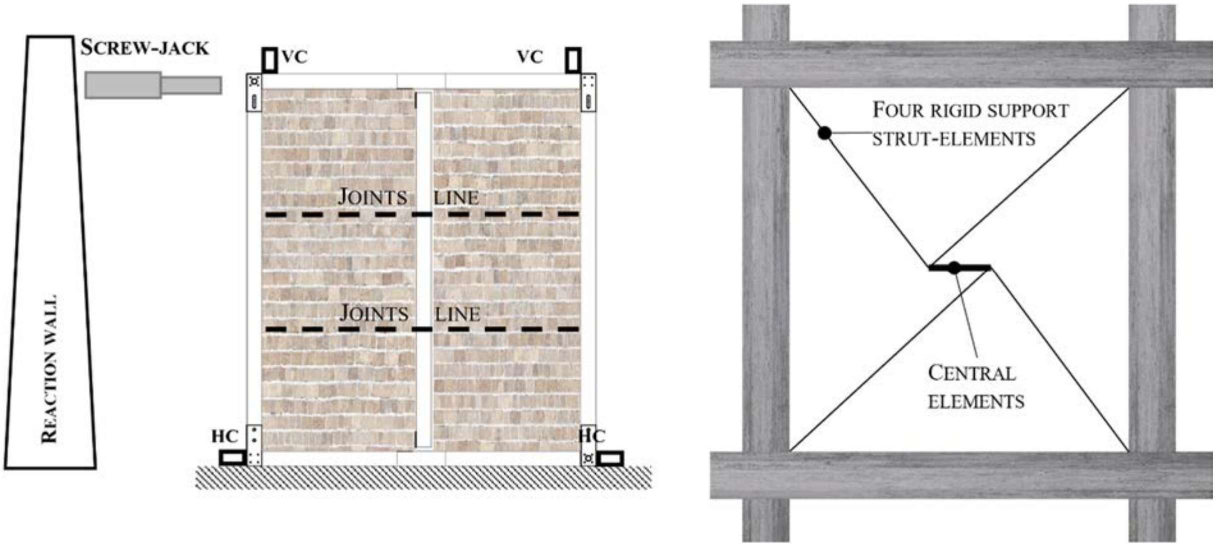
The non-linear behavior resulting from the arrangement of the plastic joints such as shown in figure 3, is concentrated in the central element, since its in-plane behavior is a function of the interstorey drift.

The advantage of the adopted macroelement is in the independence from the number of joint lines, so they can be always represented by an equivalent system in which the nonlinear properties are focused in the central element.

a) **b)** **c)**

Figure 2. Some assembly phases: a) first blocks rows placed into the frame, b) completed infill; c) removing of blocks to simulate the seismic retrofitting of masonry infills.

In figure 4 the experimental curve, the backbone curve - defined as a trilinear relationship - and the calibration curve are overlapped, while the bottom right graph represents the pinched asymmetric curve model, in which $da= pinchdisp\cdot(dA'-dA'')+ dA''$ and $fa= pinchforce\cdot fkp$.



a) **b)**

Figure 3. a) Test setup: in evidence the placement of the plastic joints. Vertical Constrains (VC) and Horizontal constrains (HC) fix the frame to the ground, allowing the horizontal deformation; b) the mechanical scheme of the adopted macroelement with non linear central element.

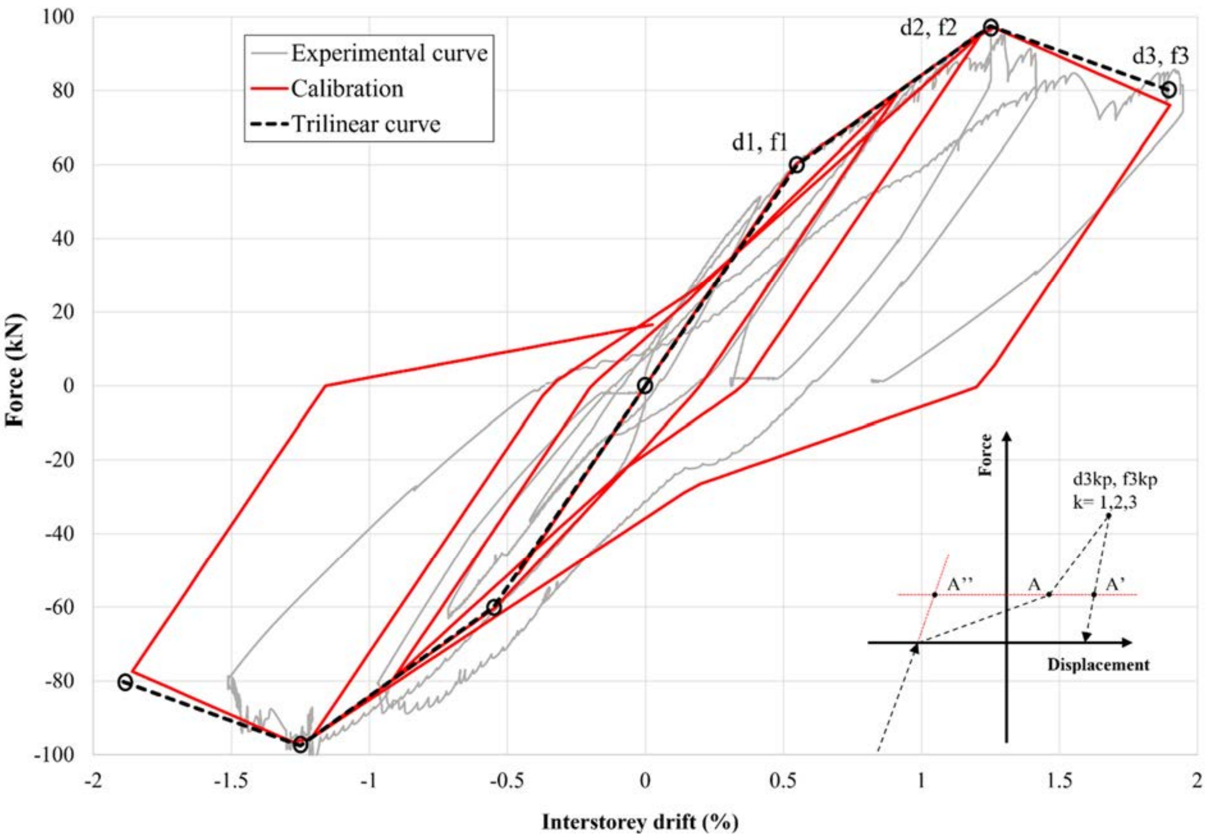


Figure 4. Calibration of the hysteretic response based on the in-plane experimental test conducted on a double-layer infill at the University of L'Aquila [12]. The bottom right graph shows the pinching rule.

The backbone curve of this macromodel is associated to both in-plane directions and it is fully described by five parameters that allow to describe the cycles of load/unload and the degradation as damage increases.

While the first three describe the points of the experimental curve where slope change, the last two allow to adjust the curve bottleneck and therefore are those subjected to calibration. The parameters *pichdisp* and *pinhforce* were therefore changed to minimize the gap between experimental data and the numerical simulation. Final set of the model parameters are listed in table 1.

	Parameter	Value
Backbone curve	*d1*	0.013
	f1	60
	d2	0.03
	f2	97.5
	d3	0.047
	f3	74.4
Pinching mechanism	*pinchdisp*	0.57
	pinchforce	0.28

Table 1. Setting parameters from calibration analysis.

3 MACROMODELLING IN SEISMIC ASSESSMENT OF INFILLED FRAME

3.1 Case study

In the present section the seismic assessment of a multi-storey frame representative of r.c. buildings of the 80's is shown.

More specifically, the study aims at both optimal modelling of the plastic joints system with a calibrated macroelement and comparing the performances with different frame configurations, as depicted in figure 5.

The considered structure is a frame consisting of three floors with height of 3.5 m each and two spans with width of 3.5 m each. Spacing among frames was assumed equal to 6 m.

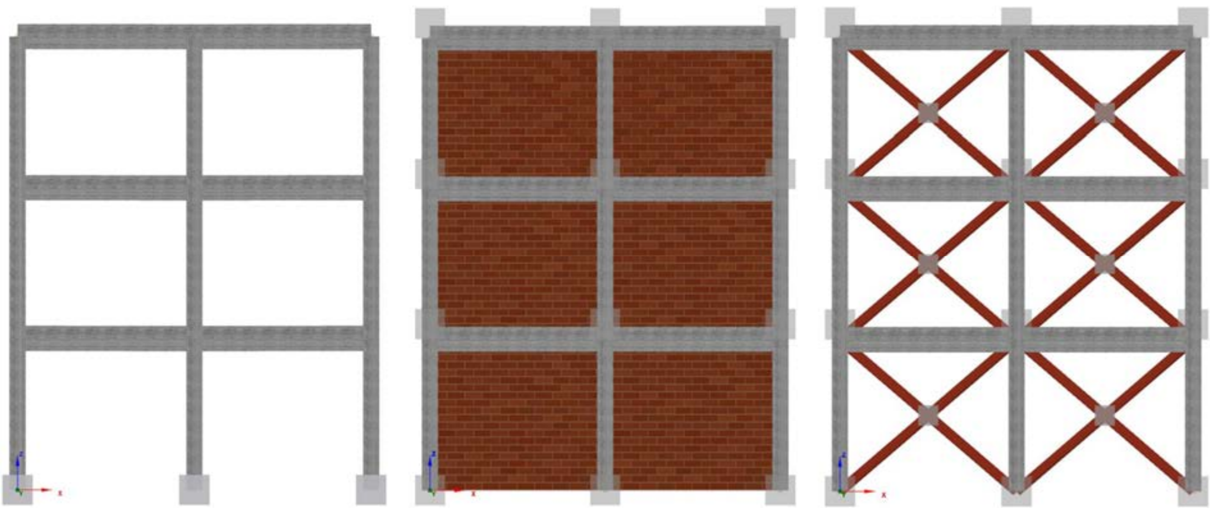
The columns have 0.32 m width and 0.6 m height, while the beams have 0.4 m width and 0.5 m height.

The positioning of the vertical rebars is the following: 4ϕ20 at the corners and 2ϕ20 along each of four sides for the beams, 4ϕ20 at the corners and 5ϕ20 along each of the two longer sides. Columns and beams have stirrups with 2 arms ϕ6/80 mm and ϕ10/100, respectively.

The infilled frame with Crisafulli macroelement is realized with single layer wall of 20 cm thickness, while the one with plastic joint macroelement, calibrated in the previous section, is realized with double-layer wall with outer and inner side of 12 cm and 8 cm thickness, respectively, and 12 cm intermediate hollow space.

From the analysis of dead and live loads - equal to 4 kN/m^2 each - and live load combination coefficient φ_{2j}= 0.6, were defined two lumped masses: the first of 7 tons was applied at each nodes of the external columns, while the second of 14 tons was applied at each nodes of the central column, for a total number of 6 and 3, respectively.

The mean strength values of beams and columns are f_{cm}= 25.2 MPa and f_{ym}= 374 MPa for concrete and steel rebars, respectively, while the compressive strength, orthogonal and parallel to the brick holes, is $f_{bm\perp}$ = 2.2 MPa and $f_{bm//}$ = 11 MPa, respectively. In professional practice these values are defined by onsite test as the average of the compressive and tension test conducted on a meaningful number of specimens, but not less than three.



a) **b)** **c)**

Figure 5. Multi-storey frame representative of r.c. buildings of the 80': a) bare frame, b) full infilled frame with Crisafulli macroelement, c) full infilled frame with plastic joints macroelement.

The three pairs of points that define the backbone curve of the double layer infill with plastic joints used in the numerical model of figure 5c, were defined as described in table 2. It should be notice that the computed forces refer to the bed-joint sliding mechanism, made the weakest by introducing the plastic joints. Further details on the equations adopted in computing of the parameters of table 2 are reported in [12].

	Parameter	Equation *	Reference code	Value
Elastic threshold	$f1$	$f_{1,120}+f_{1,80}$	FEMA 306	68375 kN
	$d1$	f_1/K_e	$K_e = \sum_{i=1}^{2} k_{e,i}$ (FEMA 273)	17.20 mm
Plastic threshold	$f2$	$f_{2,120}+f_{2,80}$ $f_{2,120} = 0.8 f_{1,120}$ $f_{2,80}$	$f_{2,80}$ (FEMA 306)	107190 kN
	$d2$	1.3% h	From experimental test	35.75 mm
Failure	$f3$	$0.8 f_2$	[15]	85750 kN
	$d3$	2.0% h	From experimental test	55 mm

* the subscript 120 refers to the layer of 120 mm thickness, than 80 refers to the layer of 80 mm thickness, f_i, d_i are the three pairs of backbone's points as depicted in figure 4, $k_{e,i}$ is the elastic stiffness of the layer i-th, h is the wall height

Table 2. Backbone's points of the double layer infill used in numerical simulation.

Infill force capacity for Crisafulli's macroelement was defined as the weakest between the bed-joint sliding and diagonal cracking, while the infill deformation capacity was defined as 1.3% of the wall height, according to [12] and [16], respectively, and equal to 45.8 kN and 30 mm.

3.2 Numerical analysis

Numerical analysis were carried out with 2D model under the hypothesis of large displacement in case of geometric and material nonlinearity, using SeismoStruct FEM software [15]. The earthquake effect were simulated with two accelerograms obtained with the Rexel software [17], according to the design spectra of the Italian code NTC-18 [18] for the town of Norcia (42.792 lat. and 13.093 long.). Figure 6 shows the spectrocompatibility for the selected accelerograms, Amatrice of the 24th August 2016 and L'Aquila 6th April 2009, while the figure 7 shows the two accelerograms used in time-domain non linear analysis. Columns and beams are modelled with fibers cross-section and nonlinear force-based elements, while Kent-Park and Menegotto-Pinto models are assigned to concrete and steel rebars, respectively. Joints are considered exclusively elastic as a consequence of their intrinsic weakness due to the absence of stirrups (widespread condition for buildings built according to pre-seismic codes).

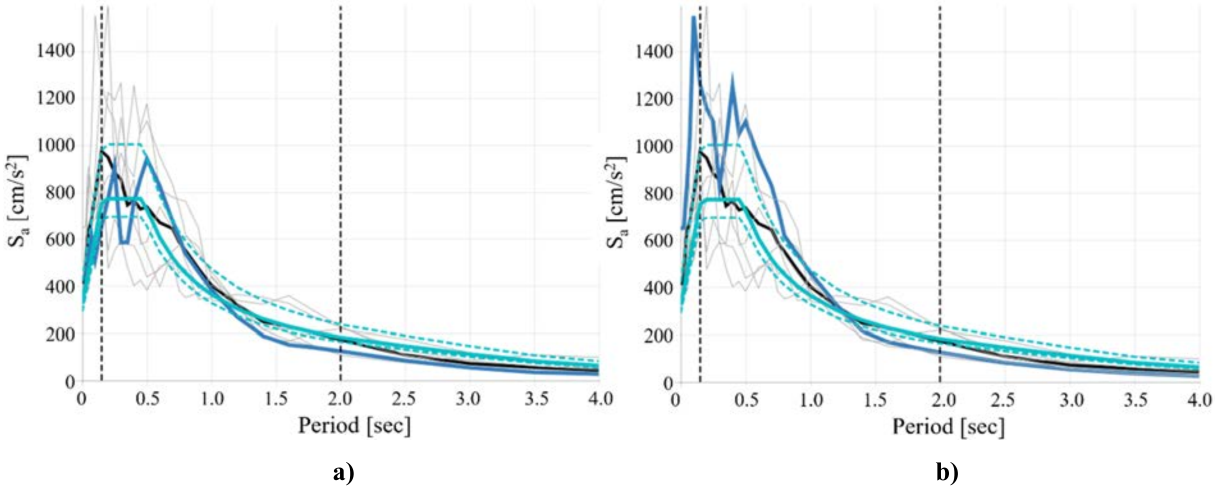


Figure 6. Highlighted in blue the response spectra of the selected accelerograms: a) Amatrice, b) L'Aquila.

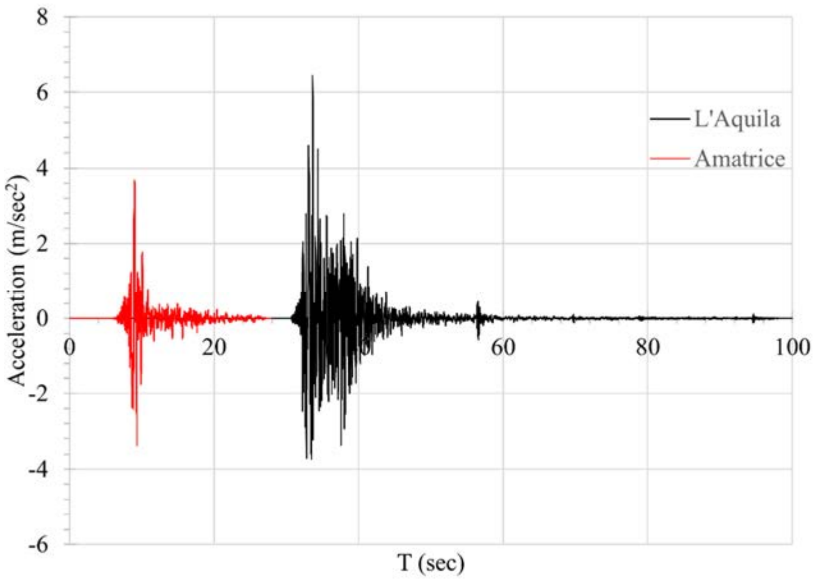


Figure 7. Accelerograms of Amatrice and L'Aquila for the time-domain non linear analysis.

The failure mechanisms are defined as performance criteria in three set, the first refers to the bare frame, while for infilled frames the set are coupled as follow: set 1 + set 2 for Crisafulli infilled frame, set 1 + set 3 for plastic joints infilled frame. The adopted values for the selected criteria are shown in table 3. Note that the first column on the right shows the assumption on the behavior of the elements after reaching the limit in force or deformation adopted in the corresponding criteria.

	Criterion name	Control on	Value	Strength degradation
SET 1 (Frame elements)	Crush_conf.	Concrete strain	-8 ‰	Keep strength
	Yield	Reinforcement strain	2.5 ‰	Keep strength
	Fracture	Reinforcement strain	10‰	Keep strength
	Chord_rot.	Frame element chord rotation capacity	(Variable)	Keep strength
	Shear	Frame element shear capacity	(Variable)	Keep strength
SET 2 (infilled Frame with Crisafulli macro-model)	Infill_strut_force	Infill force	45.9 kN	No residual strength
	Infill_shear_def.	Infill deformation	30 mm	No residual strength
SET 3 (infilled Frame with plastic joints macro-model)	Link_yield	Link deformation	17.2 mm	Keep strength
	Link_max		35.75 mm	Keep strength
	Link_ult		55 mm	No residual strength

Table 3. Performance criteria in defining of collapse mechanisms of the bare and infilled frames.

In resolving of non linear problem by means of time-domain non linear analysis, in addition to the hysteretic dissipation, the energy one was taken into account by using tangent stiffness proportional damping with damping ratio of 1% and first period of 0.48 s, 0.19 s and 0.35 s for bare frame, Crisafulli infilled frame and Plastic joints infilled frame, respectively.

4 RESULTS

4.1 Modal analysis

The analysis of natural frequencies of the three frames has confirmed the expectative. In fact, in case of infilled frame, the walls act increasing the global stiffness, and thus reducing the first period respect the bare frame. Comparing the two infilled frames, the one with Crisafulli macroelement shows higher stiffness respect the calibrated macroelement adopted for the plastic joints. By the fact that the mechanical properties of the materials are the same, the difference is likely due to different formulation of the macroelement. The following table 4 summarizes the results of the modal analysis for the three configuration of frames.

	Bare frame		**Crisafulli infilled frame**		**Plastic joint infilled frame**	
	T (Sec)	f (Hz)	T (Sec)	f (Hz)	T (Sec)	f (Hz)
1	0.48	2.08	0.19	5.26	0.35	2.86
2	0.16	6.25	0.07	14.60	0.12	8.33

Table 4. Modal analysis of the different frame configurations.

4.2 Non linear dynamic analysis

The results shown in the present section aiming at both proving the effectiveness of the strengthening technique with plastic joints through proper modelling with calibrated macroelement, and comparing results among different frame configurations, underlying pros and cons of the proposed modelling.

The analysis were carried out on the three frames using the accelerograms of Amatrice and L'Aquila, selected with disaggregation data analysis.

Figure 8 depicts two graphs in which the interstorey drift on the ground floor have been compared (the time axis were been truncated for a better yield of the curves).

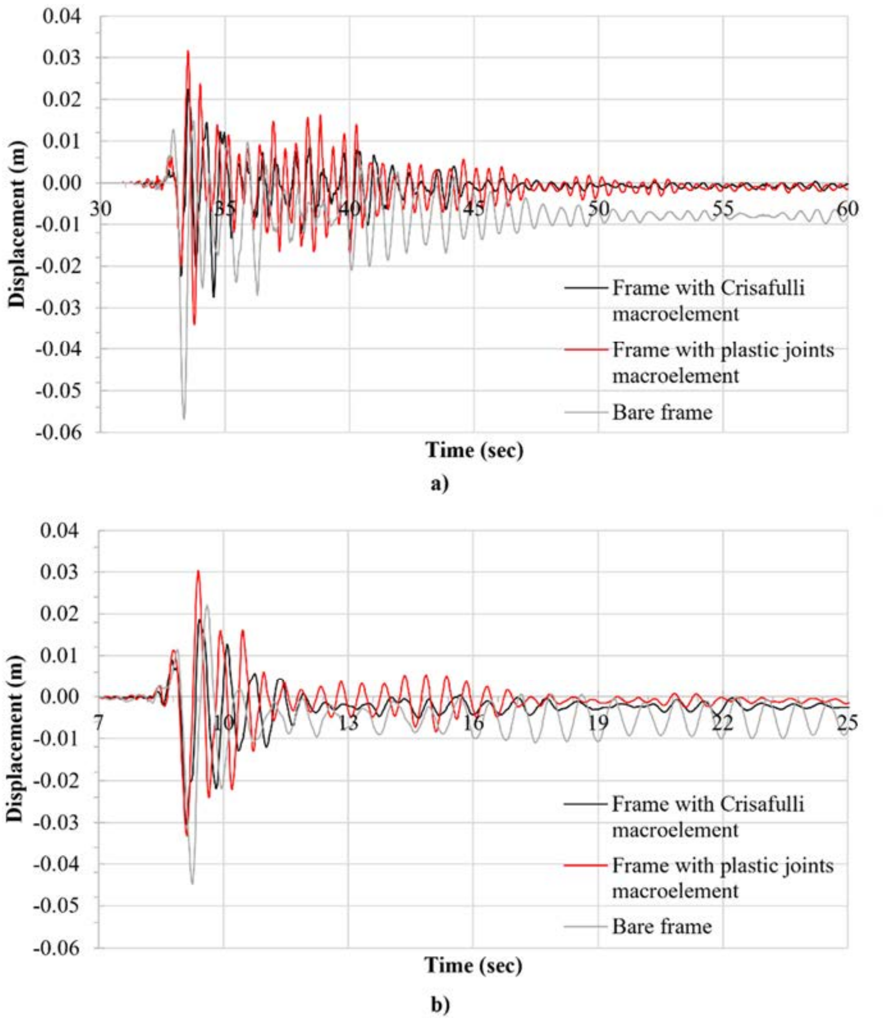


Figure 8. Interstorey drift on the ground floor for the three frame models undergoing to the effect of the selected natural accelerograms: a) L'Aquila, b) Amatrice.

Table 5 shows the maximum interstorey drift of all floor for both accelerograms, while the last row gives the average of the maximum. These displacement demands can be directly compared with the capacity of the infilled frame, according to the performance criteria listed in table 3.

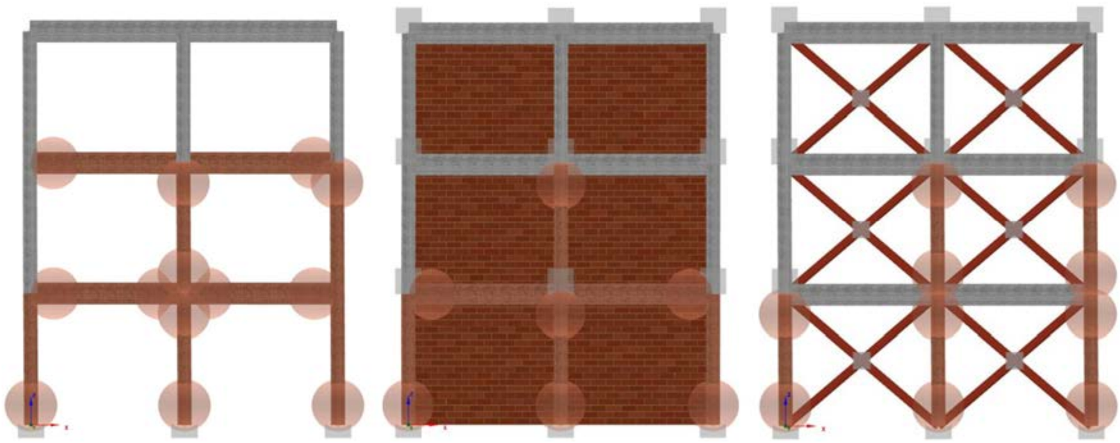
	Bare frame			**Crisafulli infilled frame**			**Plastic joint infilled frame**		
	First	Second	Top	First	Second	Top	First	Second	Top
L'Aquila	5.68	4.42	1.76	2.74	2.63	0.35	3.41	1.77	0.88
Amatrice	4.48	3.96	1.96	3.04	2.28	0.33	3.32	1.59	0.88
Average	5.08	4.19	1.86	2.89	2.46	0.34	3.36	1.68	0.88

Table 5. Maximum displacement at floors (given in cm).

The following figures 9 and 10 show the damaging of the three models of multistorey frame undergoing to the earthquakes of L'Aquila and Amatrice, respectively.

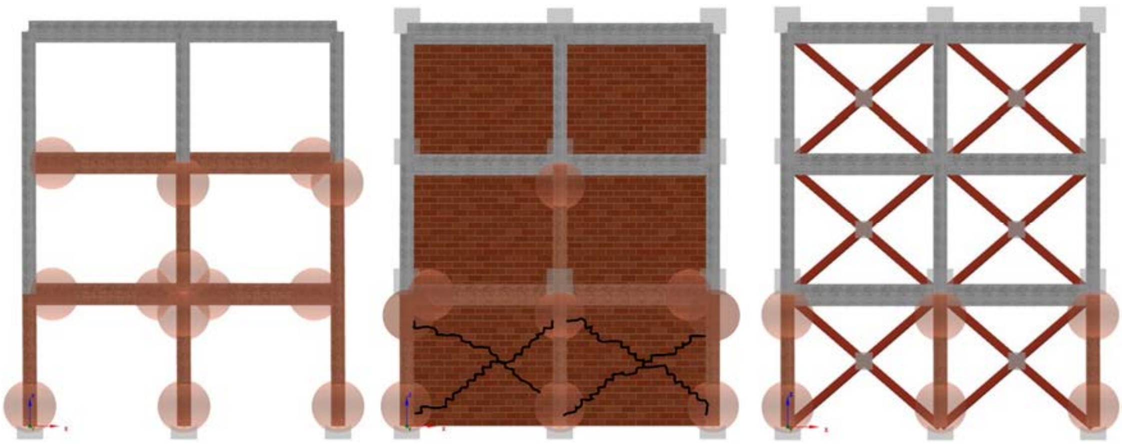
Analyzing the damage distributions, the bare frame reaches the same damaging level for both accelerograms, while the two infilled frames show opposite results: the one with Crisafulli macroelement suffers higher damage under the Amatrice earthquake, while the other with plastic joints reaches greater damage with L'Aquila earthquake. This is likely due to different energy content of the earthquakes: the armonics with higher energy are closer to Crisafulli infilled frame in case of Amatrice earthquake, respect to those of L'Aquila earthquake that are closer to the first period of the infilled frame with plastic joints. As general remark, the Crisafulli macroelement provide higher stiffness to the structures, shifting the first period toward the area of the response spectra where the acceleration are generally higher.

Beams and columns denote a widespread damaging due to rebars yielding in all three cases, while the infilled frame with Crisafulli macroelement shows brittle collapse of the infills on the ground floor in the case of Amatrice earthquake. The infilled frame with calibrated macroelements for plastic joints, shows only a few columns yielding - mostly placed on the ground floor - and the triggering of the bed-joint sliding of the infills on the ground and first floor, but without reaching the last displacement capacity, as shown in figure 11. The infills at the last level remain elastic. In case of bare frame we can observe the highest number of yielded beams and columns, as a consequence of a higher displacement demand due to lower stiffness.



a) **b)** **c)**

Figure 9. Damaging of the three frame subjected to L'Aquila earthquake: a) bare frame, b) infilled frame with Crisafulli macroelement, c) infilled frame with calibrated macroelement for walls with plastic joints.



a) **b)** **c)**

Figure 10. Damaging of the three frame subjected to Amatrice earthquake: a) bare frame, b) infilled frame with Crisafulli macroelement, c) infilled frame with calibrated macroelement for walls with plastic joints.

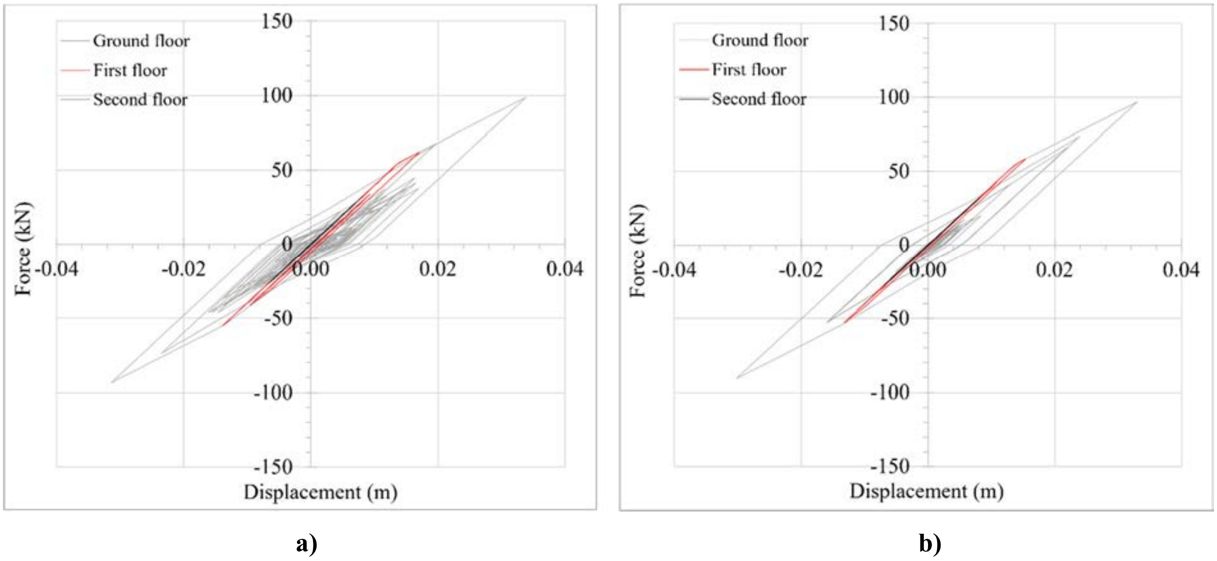


Figure 11. Hysteretic behavior of the walls strengthened with plastic joints at different floors for different accelerograms: a) L'Aquila, b) Amatrice.

5 CONCLUSIONS

In this study a macroelement available in literature has been calibrated on previous in-plane experimental test conducted at the University of L'Aquila on a double-layer masonry infill strengthened with an innovative technique based on recycled plastic joints.

The calibrated model was later used in seismic assessment of multistorey frames via numerical modelling by means of time-domain non linear analysis, using two natural accelerograms selected with Rexel software from disaggregation data of a location near Norcia. The research aims at both proving the effectiveness of the innovative strengthening technique with plastic joints through proper modelling, and comparing results among different configurations of multistorey frames representative of r.c. buildings of the 80's.

The considered structure is a frame consisting of three floors with total height of 10.5 m and two spans with total width of 7 m. Three configurations were studied: bare frame, infilled frame with Crisafulli macroelement and infilled frame with a calibrated macroelement based on the model proposed by Rodrigues et al. [14].

The calibration test have allowed to optimize the pinching mechanism parameters *pinchdisp*= 0.57 and *pinchforce*= 0.28 of the macroelement used in modelling of the bed-joints sliding mechanisms triggered by plastic joints.

The time-domain non linear analysis carried out on different multistorey frames, have shown a greater vulnerability of the infilled frame with Crisafulli macroelement, due to higher stiffness that shift the first modal shape toward the higher spectral accelerations. In particular, in the simulation with Amatrice earthquake, two beams and seven columns ends have reached the point of yielding, while two infills on the ground level have collapsed for the exceedance of shear deformation.

Regarding the infilled frame with plastic joints, the capacity of maintaining stable and large cyclic loading up to 2% of wall height, allows an high hysteretic dissipation that reduces the damaging on structural elements. Besides, the cantilever system behind bricks avoids the out-of-plane instability, thus preventing the early collapse of the infill.

Finally, the possibility to calibrate the triggering force of the bed-joints sliding mechanisms by means the increasing the number of joints per line, permits to reduce the main eigenfrequencies of the structure and thus decrease the inertia forces.

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