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ON THE SSI PERIOD AND DAMPING FOR BRIDGE PIERS ON CAISSON FOUNDATIONS

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Abstract

The vibrational period and damping of structure-foundation-soil systems are explored, with focus on the case of bridge piers on caisson foundations. To this end, a SDOF model resting on flexible supports is considered as reference model. A parametric study is then performed by considering different dimensions of the caisson, heights of the pier, mass of the deck and sub-soil. Rigorous results are compared with predictions of earlier solutions, obtained in the context of the "replacement oscillator" approach, to highlight the role of the double damping terms, foundation mass, reference system for the calculation of impedances and the cross swaying-rocking stiffness.

Keywords: Soil-Structure Interaction, caisson foundation, dynamic impedance, replacement oscillator.

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1 INTRODUCTION

Conventional modelling of structures considering the effects of Soil-Structure Interaction (SSI) involves a Single-Degree-Of-Freedom system resting on foundation impedances (i.e. springs and dashpots), as illustrated in Fig. 1. The above system is then converted into an equivalent SDOF system ([1], [2]), referred to as '*replacement oscillator*', having modified period and damping ratio, so that under a force *F*, equal to the mass *m* multiplied by the input acceleration *a*, it experiences the same response at resonance.

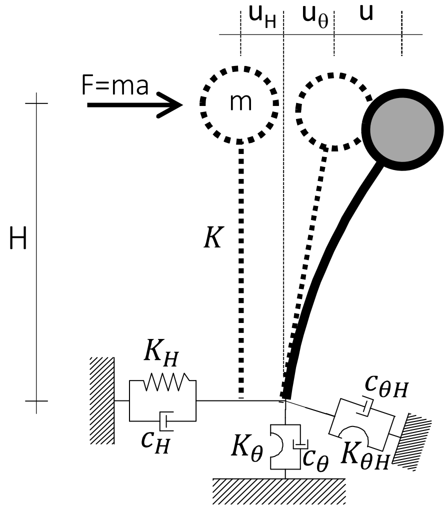

Figure 1: Single Degree-Of-Freedom system on foundation impedances to consider SSI.

Foundation impedances describe the compliance of the soil-foundation system along with its capability of dissipating energy by hysteresis and radiation of waves in the soil medium. Under harmonic force vibrations, these can be conveniently expressed as frequency-dependent complex functions in the following form:

$$K_H^* = K_H(1 + i2\xi_H)$$
$$K_\theta^* = K_\theta(1 + i2\xi_\theta) \quad (1)$$

where K_H, K_θ and ξ_H, ξ_θ are the stiffness and damping ratio of the foundation in swaying and rocking mode, respectively. It follows that the corresponding damping ratios can be back-calculated from the impedance functions as:

$$\xi_H(\omega) = \frac{\mathrm{Im}[K_H^*(\omega)]}{2\mathrm{Re}[K_H^*(\omega)]}$$
$$\xi_\theta(\omega) = \frac{\mathrm{Im}[K_\theta^*(\omega)]}{2\mathrm{Re}[K_\theta^*(\omega)]} \quad (2)$$

By imposing that the absolute displacement of the replacement oscillator, $\tilde{u}$, is equal to the displacement of the complete SSI system, $u + u_H + u_\theta$, it is possible to derive the ratio between the periods of the replacement oscillator, $\tilde{T}$, and the fixed-base SDOF system, T_0, as:

$$\frac{\tilde{T}}{T_0} = \frac{\omega_0}{\tilde{\omega}} = \sqrt{1 + \frac{K}{K_H} + \frac{KH^2}{K_\theta}} \quad (3)$$

where K is the stiffness of the original fixed-base SDOF system, while the symbol ω stands for the cyclic natural frequency (= $2\pi/T$). Since foundation stiffnesses are frequency-dependent, the calculation of the elongated period $\tilde{T}$ requires some iteration, the final stiffness values being calculated at $\omega = \tilde{\omega}$. With reference to the solution reported by Wolf (1985), the equivalence of the peak response between the replacement oscillator and the coupled system at resonance provides the following expression for the equivalent damping ratio of the replacement oscillator:

$$\tilde{\xi} = \left(\frac{\tilde{\omega}}{\omega_0}\right)^2 \xi + \left[1 - \left(\frac{\tilde{\omega}}{\omega_0}\right)^2\right] \xi_s + \left(\frac{\tilde{\omega}}{\omega_H}\right)^2 \xi_H + \left(\frac{\tilde{\omega}}{\omega_\theta}\right)^2 \xi_\theta \tag{4}$$

where ω_H and ω_θ are fictitious natural frequencies defined as:

$$\omega_H = \sqrt{\frac{K_H}{m}}$$
$$\omega_\theta = \sqrt{\frac{K_\theta}{mH^2}} \tag{5}$$

The hysteretic damping ratio of the soil (ξ_s) appears explicitly in Eq. (4). Therefore, proper treatment of the above solution requires that ξ_H and ξ_θ refer only to the radiation part of the foundation damping mechanism. On the other hand, the solution provided by Veletsos and co-workers, formulated for $\xi_s = 0$, reads:

$$\tilde{\xi} = \left(\frac{\tilde{\omega}}{\omega_0}\right)^3 \xi + \left(\frac{\tilde{\omega}}{\omega_H}\right)^2 \xi_H + \left(\frac{\tilde{\omega}}{\omega_\theta}\right)^2 \xi_\theta \tag{6}$$

where in this case the ratio of $\tilde{\omega}/\omega_0$ in the right-hand side of the equation is raised in the power of 3 due to the assumption of a viscously-damped replacement oscillator.

Eqs. (4) and (6) were developed for structures resting on shallow foundations. Therefore, direct application of these solutions to the case of embedded caisson foundations may lead to inaccurate results for two main reasons: (1) mass and inertia of the foundation are not accounted for, which could be relevant with increasing height of the foundation; (2) the formulation lacks of the cross-swaying coupled stiffness, which, contrary to the case of shallow foundations, is of paramount importance for embedded and deep foundations. A procedure to account for these aspects is illustrated in the following. A parametric study is then carried out to quantify the inaccuracy introduced by the above approximations.

2 REFERENCE MODEL FOR INERTIAL INTERACTION ANALYSIS

Owing to the finite mass and height of the caisson, the calculation of impedances requires the following nontrivial choices: (1) considering the foundation as massless or massive and (2) referring the impedance to the top or the bottom of the foundation. Despite the choice of considering a massive foundation could appear somehow obvious, in reality it is not, as explained below.

The models for SSI only refer to the so called 'inertial interaction' analysis, that is the structure-foundation-soil system excited at the top. However, under the assumption of (equivalent) linear behavior for all materials, a rigorous analysis of the three-component system should be carried out using the substructure method, *i.e.* first computing the Foundation Input Motion (FIM), consisting of both a translational and a rotational component, by means of a 'kinematic interaction' analysis. Then, the results of the inertial and kinematic interaction analyses must be combined to obtain the overall response of the structure-foundation system.

The substructure method could be in principle applied in different, yet consistent, manners to ensure that the sum of kinematic and inertial effects makes the correct total response. However, in its most widespread application, the kinematic interaction is performed with reference to the bottom of a massless foundation. Accordingly, the consistent inertial interaction analysis requires modelling both the structure and the foundation, the latter with its mass and height; moreover, the springs and dashpots attached to its bottom are evaluated with reference to the bottom of a massless foundation. Modelling the foundation height and mass is rather complicated and makes the replacement oscillator model hard to be defined, at least in a rigorous manner.

To overcome this issue, Conti & Di Laora [3] proposed an alternative, yet exact, substructure method, where the kinematic interaction analysis is performed considering a massive foundation, while the inertial interaction analysis requires modelling only the superstructure, attaching directly to its base springs and dashpots evaluated with reference to the top of a massive foundation. This way, the replacement oscillator model can be (easily and) rigorously adopted. Many works in literature provide simplified formulae for the impedances at the bottom of a massless foundation. Among these, the recent expressions in Conti & Di Laora [3] furnish very accurate results for a rigid cylindrical caisson, allowing for a more accurate prediction of the SSI system response over earlier solutions, when the impedances at the bottom of a massless foundation are considered in the context of the classical substructure approach. However, a simple transformation ([4], [5]), allows to obtain the corresponding impedances at the top of a massive foundation.

With these premises, a replacement oscillator can be derived including the coupled impedance term. Although the mathematical passages are herein omitted in the interest of space, it is possible to express the natural frequency of the equivalent SDOF as:

$$\frac{\tilde{T}}{T_0} = \frac{\omega_0}{\tilde{\omega}} = \sqrt{1 + K\frac{K_\theta + K_H H^2 - 2K_{\theta H} H}{K_H K_\theta - K_{\theta H}^2}} \tag{7}$$

where $K_{\theta H}$ is the coupled stiffness of the caisson. This expression, already derived by Zania [6], degenerates to the one by Veletsos and co-workers [1] and Wolf [2] for $K_{\theta H} = 0$.

The effective damping ratio is:

$$\tilde{\xi} = \xi\left(\frac{\tilde{\omega}}{\omega_0}\right)^2 + 2\xi_{\theta H}\frac{1-\left(\frac{\tilde{\omega}}{\omega_0}\right)^2-\left(\frac{\tilde{\omega}}{\omega_{\theta H}}\right)^2}{1-\left(\frac{\omega_H\omega_\theta}{\omega_{\theta H}^2}\right)^2} + \xi_H\frac{1-\left(\frac{\tilde{\omega}}{\omega_0}\right)^2-\left(\frac{\tilde{\omega}}{\omega_\theta}\right)^2}{1-\left(\frac{\omega_{\theta H}^2}{\omega_H\omega_\theta}\right)^2} + \xi_\theta\frac{1-\left(\frac{\tilde{\omega}}{\omega_0}\right)^2-\left(\frac{\tilde{\omega}}{\omega_H}\right)^2}{1-\left(\frac{\omega_{\theta H}^2}{\omega_H\omega_\theta}\right)^2} \tag{8}$$

where $\xi_{\theta H}(\omega) = \frac{\mathrm{Im}[K_{H\theta}^*(\omega)]}{2\mathrm{Re}[K_{H\theta}^*(\omega)]}$ and $\omega_{H\theta} = \sqrt{\frac{K_{H\theta}}{mH}}$.

Eq. (8) coincides with the expression by Zania [6] if in the latter the coupled damping terms $\xi_i\xi_j$ are neglected. Based on the above discussion, Eqs. (7) and (8) furnishes correct values for period and damping ratio at resonance only if impedances at the top of the massive foundation are employed.

3 PARAMETRIC STUDY

An extended parametric study was carried out to explore the influence of the impedance formulation and of the coupled stiffness on the natural period and damping of the replacement oscillator. The ideal case of a bridge pier with a caisson foundation embedded on a homogeneous clayey soil deposit was analyzed. Two soil deposits were considered, whose relevant mechanical properties are summarized in Table 1: the Plasticity Index, *PI*; the undrained shear strength, s_u; the small strain shear wave velocity, V_s; the hysteretic damping ratio, ξ_s. To take into account the nonlinear soil behavior (i.e.: the variation of the shear modulus *G* and damping with the mobilized strain level, γ), three values of V_s and ξ_s were considered, corresponding to G/G_0 = 1, 2/3 and 1/3.

The cylindrical caisson foundation has diameter D = 8 and 12 m, with slenderness ratio H_c/D = 0.5, 1 and 2, where H_c is the caisson depth. Three pier heights were considered (H = 20, 40 and 60 m). The hollow rectangular reinforced concrete cross section of the pier, as well as the mass of the deck, are representative of highway and railway bridges with span length of 20-150 m. Specifically, for each pier, the mass of the deck was designed taking into account three conditions: (i) to guarantee a minimum span length, $L/H \geq 0.5$; (ii) to avoid second order effects within the pier; and (iii) to provide a static global safety factor SF = 1.5 and 5 for the bearing capacity of the foundation against pure vertical loads.

The bridge pier was modelled as a single degree of freedom system with height *H*, structural damping ξ, stiffness *K* and lumped mass *m*, the latter including the mass of the deck and the mass of the upper half of the pier ($m = m_{deck} + 0.5 \cdot m_{pier}$). A total of 69 pier-foundation-soil systems were analyzed, comprising structural (fixed-base) periods T_0 in the range 0.3-2.8 s.

PI [%]	60		200	
s_u [kPa]	40		140	
	ξ_s [%]	V_s [m/s]	ξ_s [%]	V_s [m/s]
G/G_0 = 1	2	130	2	240
G/G_0 = 2/3	7	110	5	200
G/G_0 = 1/3	12	75	10	140

Table 1: Soil parameters considered in the parametric study.

4 SELECTED RESULTS

This section provides a comparison between different considerations in the evaluation of the SSI period, $\tilde{T}$, and damping, $\tilde{\xi}$, by applying the replacement oscillator model [Eqs. (7) and (8)] to the pier-foundation-soil systems analyzed in the parametric study.

The reference (rigorous) solution, which is mentioned in the graphs as $\tilde{T}_{ref}$ and $\tilde{\xi}_{ref}$, is obtained by computing the SSI period and damping using the formulas derived by Zania (2014), thus including also double damping terms in the equations, and evaluating the dynamic impedances at the top of the massive foundation.

The effect of double damping terms is shown in Fig. 2. Considering that double damping terms have been omitted in Eq. (8), their effect can be directly explored by the comparison between $\tilde{\xi}$ and $\tilde{\xi}_{ref}$, as specified above. As expected, the effect is more pronounced on the effective damping rather than on the effective period, which in turn can be safely calculated by neglecting the products $\xi_i\xi_j$. The effective damping is generally not sensitive to this approximation, except few cases which, however, are difficult to detect a priori.

The role of the coupled stiffness is assessed in Fig. 3. Neglecting the cross swaying-rocking stiffness, K_{HM}, results in an underestimation of the effective period, leading to stiffer SSI systems with respect to the actual ones (Fig. 3b). Such an underestimation may be as high as 50%, depending on the properties of the SSI system. This is because, in the case of the actual SSI system with non-zero coupled stiffness, which also corresponds to a non-zero coupled compliance, the shear force produces an additional rotation and the moment produces an additional displacement. The effect of the coupled impedance term on the effective damping is more critical (Fig. 3a): neglecting this term may lead to a severe overestimation of damping ratio (by a factor of 2 or larger) and thereby a severe underestimation of structural response.

Figure 4 shows the effect of neglecting the mass of the foundation: except for few cases, the mass of the foundation has a minor influence with respect to a proper choice of the reference system for the evaluation of the dynamic impedances.

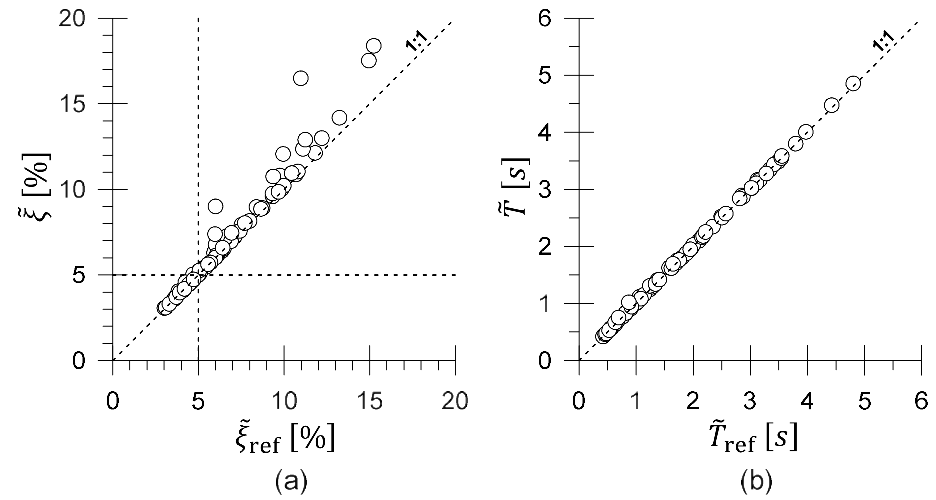

Figure 2: Effect of double damping terms on effective period and damping ratio.

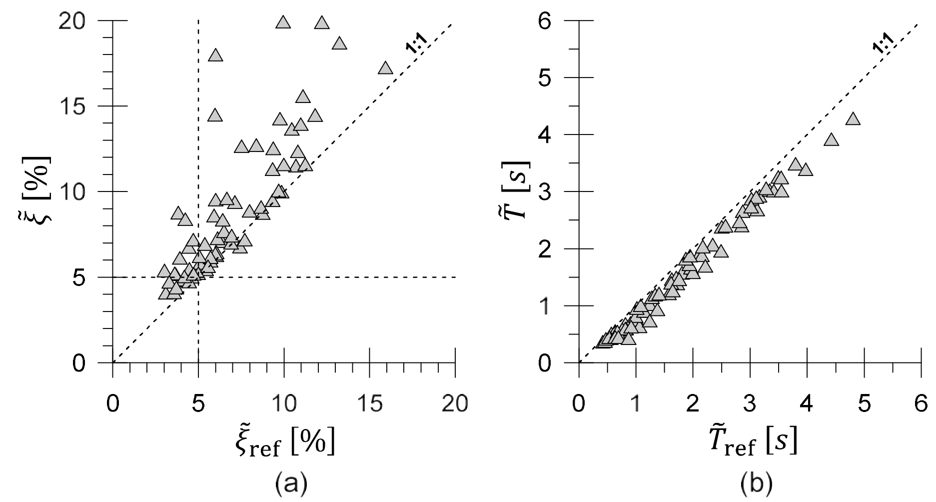

Figure 3: Effect of coupled impedance on effective period and damping ratio.

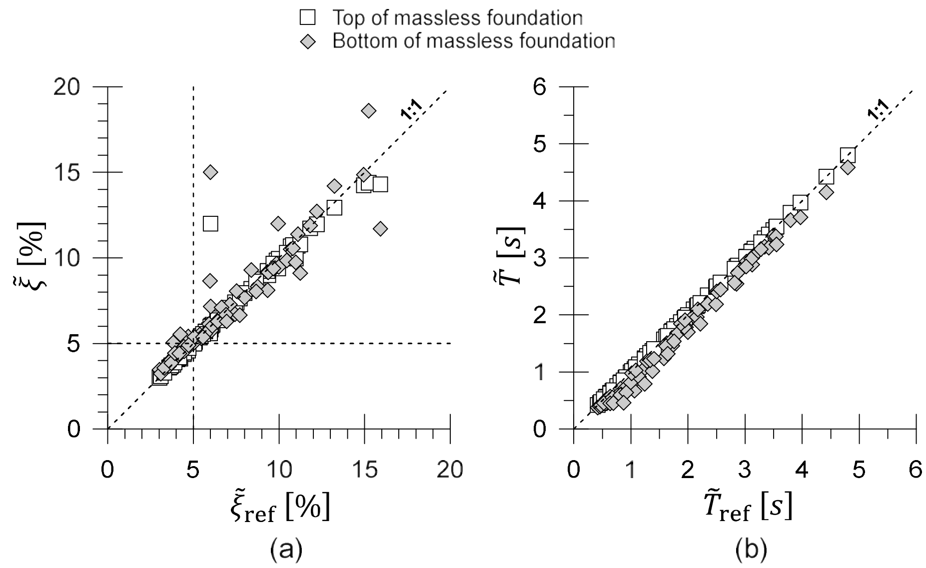

Figure 4: Effect of impedance reference system on effective period and damping ratio.

5 CONCLUSIONS

The natural period and damping of soil-foundation-structure systems, referring to bridge piers founded on caisson foundations, were explored by means of a SDOF model resting on flexible supports. The problem is treated numerically by implementing a hybrid substructure approach of the complete SSI system, where the foundation mass is introduced directly in the foundation input motion and caisson impedances. In light of the above considerations, closed-form expressions for the effective natural period and damping are reported in Equations 7 and 8, respectively, which allowed investigation of the importance of some critical components on the vibrational characteristics of the SSI system. The main findings of the study can be summarized as follows:

- The effect of double damping terms (i.e. products of $\xi_i\xi_j$) is more pronounced on the SSI damping of the system than on its natural period. The latter can thus be derived with sufficient accuracy if double damping terms are omitted.

- The role of the cross swaying – rocking stiffness of the caisson foundations on the vibrational characteristics of the replacement oscillator is critical. Neglecting the foundation impedance of this particular mode of vibration may lead to a strong underestimation of the effective natural period as high as 50% and a strong overestimation of the SSI damping by a factor of 2 or even larger, depending on the properties of the soil-caisson-structure system.

- Except for few cases, the foundation mass has a minor influence with respect to a proper choice of the reference system for the evaluation of the dynamic impedances.

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