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COMPARATIVE STUDY OF OPTIMUM FORMULATIONS FOR TUNED MASS DAMPERS IN SMALL WIND TURBINE TOWERS

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Abstract

In the field of wind turbines, supplemental dampers have been utilized to control vibration and improve the response of slender structures. Although studies on vibration control in large wind turbines are common, studies on small structures are scarce. This article aims to analyze the dynamic behavior of a metallic hollow tower used as a horizontal small wind turbine support and design a vibration control device, namely a tuned mass damper (TMD), which is one of the most commonly used systems in large wind turbines due to its simplicity and effectiveness. The study investigates the influence of different formulations and parameters on the performance of TMDs in wind turbine towers. The results reveal that the selection of optimum parameters significantly influences the performance of the TMD system, and different formulations can lead to significant differences in the structure's response. The Single Degree of Freedom (SDOF) method accurately represents the main structure's behavior under harmonic excitation if the correct modal parameters are used. The study also finds that the maximum amplification factor for the main structure can be significantly reduced by the use of TMD systems, by up to 43% even for a mass ratio of 0.01 of the modal mass. However, using inadequate parameters can lead to a reduction of only 2.5%. Therefore, it is crucial to carefully evaluate and select the optimal parameters for the TMD system, which depends on the specific characteristics and requirements of the system.

Keywords: Wind turbines, Tuned mass damper, Vibration control, Small wind turbines, Dynamic behavior, Structural response.

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1 INTRODUCTION

Wind turbine towers are subjected to significant dynamic loads due to the fluctuating wind loads that they experience during their lifetime. These dynamic loads can cause undesirable vibrations in the tower, which can lead to fatigue and ultimately failure of the structure. To mitigate these vibrations, various vibration control techniques have been developed, and among them, tuned mass dampers (TMDs) have emerged as a promising solution.

TMDs stands out as a reliable and effective structural vibration control device that can be attached to a primary system to suppress undesirable vibrations induced by wind and earthquake loads. Typically, the TMD consists of a mass, damping component, and spring. The natural frequency of the TMD is tuned to resonate with the fundamental mode of the primary structure, allowing it to absorb a significant amount of the structural vibrating energy and dissipate it through the damping component when the primary structure is subjected to external disturbances [1].

Over the past century, several researchers have analyzed the optimal parameters of TMD systems for different types of structures. Many of these studies have been carried out using numerical analysis, and more recently, with the advent of advanced computational techniques, finite element analysis (FEA) has become a popular tool for studying TMD systems. However, despite the extensive research, there is no consensus on the optimal TMD parameters for a given structure.

In wind turbine engineering, TMDs are recognized as the standard for vibration absorption. Installing a TMD in a wind turbine tower can improve the dynamic response and reliability of the structure by increasing its damping. The effectiveness of the TMD depends on its mass and point of installation. Tuning its mass and damping to match the frequency of the tower's fundamental vibration mode allows the TMD to absorb and dissipate a significant portion of the structural vibrating energy. Proper placement of the TMD is also crucial for optimal energy transfer.

The first vibration absorbers were introduced by Frahm in 1909 [2], and over the following years, several authors proposed optimal design theories for TMDs according to different types of excitation. The first optimal frequency tuning formulation was given by Ormondroyd and Den Hartog in 1928 [3]. They minimized the steady-state response of the undamped main system subjected to a harmonic external force. According to Den Hartog, the optimum TMD parameters are found when the dynamic amplification curves cross two fixed points independent of the damping ratio. The main objective is to obtain a flat curve near the resonant frequency and suppress the dynamic response of the structure. Meanwhile, Brock derived the optimal damping for the same system [4]. He also derived solutions for the case in which the amplitude of the disturbing force varies with the square of its frequency [5], which is the same result obtained by [6] when the optimized parameter is the acceleration of the main mass. Following the same procedure, Neubert [7] proposed a formulation where the optimum parameters are obtained to minimize the velocity response.

The work of Warburton [6] presented several optimum absorber parameters for various excitations and response quantities, including force (main mass) and acceleration excitation (base), for harmonic and random white noise. The parameters are optimized by displacement, velocity, and acceleration of the main mass, as well as for the relative displacement between the main mass and ground. However, all the proposed formulations were made under the assumption of an undamped main mass. In the absence of structural damping, the optimal frequency tuning of the TMD can be determined based solely on the mass ratio between the TMD and the structure. But for real structures, some structural damping is always present, and the above formulations may not be adequate. Ioi and Ikeda proposed some optimization parameters for damped structures using numerical calculation in 1978 [8].

The work of Fujino and Abé in 1993 derived analytical expressions for the frequency response function of the structure with the TMD and used a perturbation technique to obtain the design formulas [9]. In the same edition, Tsai and Lin derived analytical expressions for the frequency response function of the structure with the TMD and used the energy method to obtain the optimal mass, damping, and stiffness of the TMD using series representations [10]. Asami and Nishihara obtained a closed-form exact solution for the $H\infty$ optimization, which can be applied to a range of transfer functions and damped systems [11].

In 2005, Krenk proposed a simplified analytical model to estimate the optimal parameters of a tuned mass damper (TMD) system, following a theoretical study focused on frequency analysis [12]. Another closed-form optimization criterion, proposed by Ghosh and Basu in 2007 [13], is based on the minimization of a cost function derived from the energy of the system under consideration. The optimization is subject to constraints on the maximum displacement and acceleration of the primary structure, using the equal peak theory employed by Den Hartog.

Krenk and Høgsberg (2008) demonstrated that the classical frequency tuning for harmonic loads of a system with both structural and applied damping, under the effects of random white noise, can be simplified by making an approximation that involves a small term in the structural damping coefficient. This approximation reduces the complexity of the formula, making it easier to apply in practice. The authors also found that the magnitude of the optimal applied damping depends only on the mass ratio, but is independent of the structural damping [14].

Anh and Nguyen (2013) utilized the Equivalent Linearization Method and a Dual Criterion of Equivalent Linearization Method to optimize a TMD, and also found that the expression of the optimal tuning ratio is independent of the TMD's damping [15].

Puzyrov and Awrejcewicz (2017) revised the classical formulations and proposed an alternative method for determining the optimum absorber parameters that takes into account the frequency response of the absorber and the frequency response of the primary system. The proposed formula does not rely on the existence of invariant points [16].

While the general formulations are proposed under the assumption that the system can be represented by a single degree of freedom (SDOF) system, when this assumption is not true, the optimum parameters may be found by numerical analysis. In this scope, Kordi and Alamatian (2019), based on the complex stiffness theory, proposed closed-form solutions that are applicable to both single and multiple-degree-of-freedom systems [17].

Numerous researchers have employed numerical analysis to investigate the optimal parameters of tuned mass damper (TMD) systems [18]–[20]. In the context of wind turbine towers, studies have primarily focused on numerical and finite element analyses [21]–[26].

The objective of the present study is to compare the outcomes of various TMD parameter formulations described in the literature when applied to a metallic tapered tower functioning as a support for a wind turbine. Specifically, the maximum dynamic amplification factor for both the main structure and TMD will be compared across several formulations. Furthermore, the maximum displacement of both the structure and TMD will be evaluated by numerically integrating the 2-degree-of-freedom (2DOF) system under harmonic excitation. The findings of this study will help determine the optimal TMD parameters for metallic tapered towers supporting wind turbines.

2 ANALYTICAL PROCEDURE

2.1 Formulation of the problem

Considering the structure represented as a SDOF system the damper system Kelvin-Voigt model [27] can be represented according to Figure 1. In the context of structural engineering, when the primary structure is represented as a single degree of freedom system (SDOF), only the dominant mode of energy dissipation is considered and involves the integration of a mass element (m_s) with a spring element (k_s) and a viscous damper element (c_s). The SDOF system offers a straightforward yet effective approach to analyzing the dynamic behavior of structures under external loads. The effects of TMD are accounted for within the SDOF model, by incorporating the mass of the TMD (m_d), its spring stiffness constant (k_d) and damping coefficient (c_d).

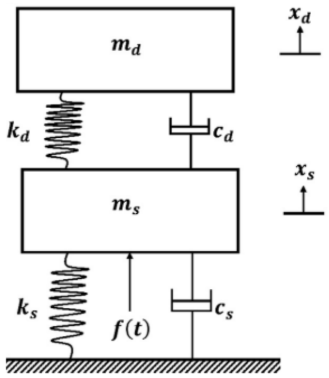


Figure 1 : Theoretical model of a damped main structure and TMD under force excitation f(t) [15].

Equation (1) presents the dynamic equilibrium equation for the 2DOF system illustrated in Figure 1.

$$\begin{gathered} m_s\,\ddot{x}_s + (c_s + c_d)\dot{x}_s - c_d\dot{x}_d + (k_s + k_d)x_s - k_d x_d = f(t) \\ m_d\,\ddot{x}_t - c_d\dot{x}_s + c_d\dot{x}_d - k_d x_s + k_d x_d = 0 \end{gathered} \tag{1}$$

The amplitude magnification of the steady-state response of a system, also referred to as the frequency transfer function, is determined as Equation (2) and (3) for the main structure and TMD, respectively. Appropriate substitutions are made in accordance with Equations (4-5) to obtain these equations. Utilizing Equations (2) and (3), it is feasible to evaluate the response of the system in relation to the dimensionless parameters outlined in Equation (5).

$$\left|\frac{x_s}{f_0/k_s}\right| = \sqrt{\frac{(\gamma^2-\beta^2)^2 + (2\gamma\beta\xi_d)^2}{[\gamma^2-(1+4\gamma\xi_s\xi_d+\gamma^2+\mu\gamma^2)\beta^2+\beta^4]^2 + [2\beta\xi_s(\gamma^2-\beta^2)^2 + 2\gamma\beta\xi_d(1-\beta^2-\mu\beta^2)]^2}} \tag{2}$$

$$\left|\frac{x_d}{f_0/k_s}\right| = \sqrt{\frac{\gamma^4 + (2\gamma\beta\xi_d)^2}{[\gamma^2-(1+4\gamma\xi_s\xi_d+\gamma^2+\mu\gamma^2)\beta^2+\beta^4]^2 + [2\beta\xi_s(\gamma^2-\beta^2)^2 + 2\gamma\beta\xi_d(1-\beta^2-\mu\beta^2)]^2}} \tag{3}$$

$$\omega_s = \sqrt{\frac{k_s}{m_s}}, \qquad \xi_s = \frac{c_s}{2m_s\omega_s}, \qquad \omega_d = \sqrt{\frac{k_d}{m_d}}, \qquad \xi_d = \frac{c_d}{2m_d\omega_d}, \tag{4}$$

$$\mu = \frac{m_d}{m_s}, \qquad \gamma = \frac{\omega_d}{\omega_s}, \qquad \beta = \frac{\omega}{\omega_s} \tag{5}$$

2.2 Optimum parameters

Based on the previous research outlined in Section 1, Table 2 presents the optimal values for the frequency ratio (γ) and damping of the damper system (ξ_d). To assess the efficacy of the proposed formulations, Equations (2) and (3) were solved using each of the 22 different configurations. The value of μ was held constant at 0.01 throughout the analysis.

Code	Author	Frequency ratio γ_{opt}	Damping TMD ξ_{dopt}
DHUH	Den Hartog (1928) [3]	$\frac{1}{1+\mu}$	$\sqrt{\frac{3\mu}{8(1+\mu)}}$
NUUHVEL	Neubert (1964) [7]	$\frac{\sqrt{1+0.5\mu}}{1+\mu}$	$\sqrt{\frac{3\mu\left(1+\mu+\frac{5\mu^2}{24}\right)}{8(1+\mu)(1-0.5\mu)^2}}$
WBUH	Warburton (1982)[6]	$\frac{1}{\sqrt{1+\mu}}$	$\sqrt{\frac{3\mu}{4(2+\mu)}}$
WBUHB	Warburton (1982)[6]	$\frac{\sqrt{1+0.5\mu}}{1+\mu}$	$\sqrt{\frac{3\mu}{8(1+\mu)(1-0.5\mu)}}$
WBUR	Warburton (1982)[6]	$\frac{\sqrt{1+0.5\mu}}{1+\mu}$	$\sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$
WBURVEL	Warburton (1982)[6]	$\frac{1}{\sqrt{1+\mu}}$	$\frac{\mu}{4}$
WBURB	Warburton (1982)[6]	$\frac{\sqrt{1+0.5\mu}}{1+\mu}$	$\sqrt{\frac{\mu(4-\mu)}{8(1+\mu)(2-\mu)}}$
IKDH	Ioi and Ikeda (1978) [8]	$\frac{1}{1+\mu} - (0.241 + 1.7\mu - 2.6\mu^2)\xi_s - (1 + 1.9\mu + \mu^2)\xi_s^2$	$\sqrt{\frac{3\mu}{8(1+\mu)}} + (0.13 + 0.12\mu + 0.4\mu^2)\xi_s - (0.01 + 0.9\mu + 3\mu^2)\xi_s^2$ $with\ 0.1 \leq \mu \leq 0.3\ and\ 0 \leq \xi_s \leq 0.125$
IKDHB	Ioi and Ikeda (1978) [8]	$\frac{\sqrt{1+0.5\mu}}{1+\mu} - (0.33 + 0.28\mu - 0.5\mu^2)\xi_s + (0.8 - \mu)\xi_s^2$	$\sqrt{\frac{3\mu}{8(1+\mu)}} + (0.135 - 0.28\mu - 0.2\mu^2)\xi_s + (0.05 + 1.8\mu - \mu^2)\xi_s^2$
FAUL	Fujino & Abé (1993) [9]	$\frac{1}{1-\mu}$	$\sqrt{\frac{\mu}{(1+\mu)}} = \xi_{bif}$
FADL	Fujino & Abé (1993) [9]	$\frac{1}{1+\mu} - \frac{\sqrt{\mu}\xi_s}{(1+\mu)\sqrt{1+\mu-\xi_s^2}}$	$\frac{\xi_s}{1+\mu} + \frac{\sqrt{\mu}\sqrt{1+\mu-\xi_s^2}}{1+\mu}\ with\ \xi_s^2 \leq \mu$
TGDH	Tsai & Lin (1993) [10]	$\left(\frac{\sqrt{1+0.5\mu}}{1+\mu} + \frac{1}{\sqrt{1-2\xi_s^2}} - 1\right) - (0.288 - 0.611\sqrt{\mu} + 1.12\mu)\sqrt{\mu}\xi_s - (2.298 - 6.739\sqrt{\mu} + 8.316\mu)\sqrt{\mu}\xi_s^2$	$\sqrt{\frac{3\mu}{8(1+\mu)}} + 0.151\xi_s - 0.187\xi_s^2 + 0.238\xi_s\mu$

TGDHB	Tsai & Lin (1993) [10]	$\left(\frac{\sqrt{1+0.5\mu}}{1+\mu}+\sqrt{1-2\xi_s^2}-1\right)-(2.375-1.034\sqrt{\mu}-0.426\mu)\sqrt{\mu}\xi_s-(3.73-16.903\sqrt{\mu}+20.496\mu)\sqrt{\mu}\xi_s^2$	$\sqrt{\frac{3\mu}{8(1+\mu)(1-0.5\mu)}}+(0.151\xi_s-0.170\xi_s^2)+(0.163\xi_s+4.98\xi_s^2)\mu$
ANUH	Asami & Nishihara (2003) [11]	$\frac{2}{1+\mu}\sqrt{\frac{2\left(16+23\mu+9\mu^2+2(2+\mu)\sqrt{4+3\mu}\right)}{3(64+80\mu+27\mu^2)}}$	$\frac{1}{4}\sqrt{\frac{\left(8+9\mu-4\sqrt{4+3\mu}\right)}{1+\mu}}$
KRUL	Krenk (2005)[12]	$\frac{1}{1-\mu}$	$\sqrt{\frac{\mu}{2(1-\mu)}}$
GB	Ghosh & Basu (2007) [13]	$\sqrt{\frac{1-4\xi_s^2-\mu(2\xi_s^2-1)}{(1+\mu)^3}}$	
KHDR	Krenk & Høgsberg (2008) [14]	$\frac{1}{1-\mu}$	$\frac{\sqrt{\mu}}{2}$
KHDRB	Krenk & Høgsberg (2008) [14]	$\frac{1}{1-\mu}$	$\frac{1}{2}\sqrt{\frac{\mu}{(1-\mu)}}$
AND1	Anh & Nguyen (2013) [15]	$\frac{\sqrt{1+\frac{\pi^2}{(\pi^2-2)^2}\xi_s^2}-\frac{\pi}{\pi^2-2}\xi_s}{1+\mu}$	
AND2	Anh & Nguyen (2013) [15]	$\frac{1}{(1+\mu)\left(\sqrt{1+\frac{4}{\pi^2}\xi_s^2}+\frac{2}{\pi}\xi_s\right)}$	
PAUH	Puzyrov & Awrejcewicz (2017) [16]	$\sqrt{\frac{8\left(16+23\mu+9\mu^2+2(2+\mu)\sqrt{4+3\mu}\right)}{3\left((1+\mu)^2(64+80\mu+27\mu^2)\right)}}$	$\sqrt{\frac{8+9\mu-4\sqrt{(4+3\mu)}}{16(1+\mu)}}$
KAUL	Kordi & Ala-matian (2019)[17]	$\frac{\sqrt{1+\mu}}{1+\mu}$	$\frac{\sqrt{\mu}}{1-\mu}$

Table 2: Optimum frequency ratio and damping of TMD according to Authors.

2.3 Time history analysis

The present study involved the analysis of the response of a system subjected to a harmonic force, where $f(t)=f_0.\sin(\omega t)$. This was accomplished through numerical integration of the equations of motion of the system, as presented in equation (1). In accordance with the optimal formulations detailed in Table 2, the parameters were varied while keeping μ constant at 0.01. The harmonic force was standardized at 1kN and applied over 10 seconds after that the structure are in free vibration, while the frequency excitation was set at $\omega=\omega_s$.

2.4 Finite element analysis

To obtain the modal parameters for the first mode of the wind turbine tower structure, an eigenvalue analysis was performed using Finite Element Method (FEM). The tower was represented as a multi-degree of freedom system, wherein the rotor and nacelle masses were modeled as a lumped mass situated at the hub height. To ensure the accuracy of the FEM model, it was evaluated using the SAP2000 software as a beam model. The accuracy of the model was then validated against experimental results obtained from Operational Modal Analysis [28].

Furthermore, a time history analysis was conducted on the beam model according to section 2.3. The resulting response was then utilized to validate the response of the SDOF model, which was obtained through time history integration.

3 DESCRIPTION OF THE TOWER

The focus of this investigation is a metal tower with a height of 17.8 meters, which has been constructed utilizing S275 steel. The tower features a hexadecagonal cross-section, exhibiting an outer diameter of 0.5890 meters at the base and tapering down to 0.1954 meters at the top, with a uniform wall thickness of 4 millimeters. Further significant details related to the key characteristics of the tower are provided in Table 3.

The dynamic characteristics of the tower, comprising of the initial frequency and damping ratio of the primary structure, were determined via in-situ vibration testing using operational modal analysis techniques [28]. The outcomes have been tabulated in Table 3 for easy reference. Additionally, Table 3 also includes the modal mass of the first mode obtained through modal analysis, where the mode shape was normalized to unity at maximum displacement.

Density	7850.000	[kg/m³]
Young's modulus	210.000	[GPa]
Poisson's ratio	0.300	--
Yield strength	275.000	[MPa]
Height	17.800	[m]
Top diameter	0.195	[m]
Bottom diameter	0.589	[m]
Nacelle mass	75.000	[kg]
Tower mass	631.355	[kg]
1st frequency	1.601	[Hz]
$\xi_{s1^{st}}$	0.034	--
1st modal mass	165.113	[kg]

Table 3: Tower properties.

4 RESULTS

In this section, we present the results of our analysis of the optimum TMD parameters for a metallic tapered tower supporting a SWT. The analysis involved comparing the results obtained using several formulations for estimating the TMD parameters, as described in the previous section. Specifically, we compared the maximum dynamic amplification factor and maximum displacement of the main structure and TMD, as determined through numerical integration under a harmonic excitation.

Figure 2 shows the comparison between the time history analysis of the main structure, comparing the MDOF system solved by FEM and the SDOF solved by numerical integration under harmonic excitation force where $\omega = 2\pi$. The correct modal parameters were used to accurately represent the behavior of the main structure. The graph plots the top displacement of the tower over time, highlighting the agreement between the FEM and SDOF methods. These results provide evidence that the behavior of the structure can be effectively captured through a SDOF system.

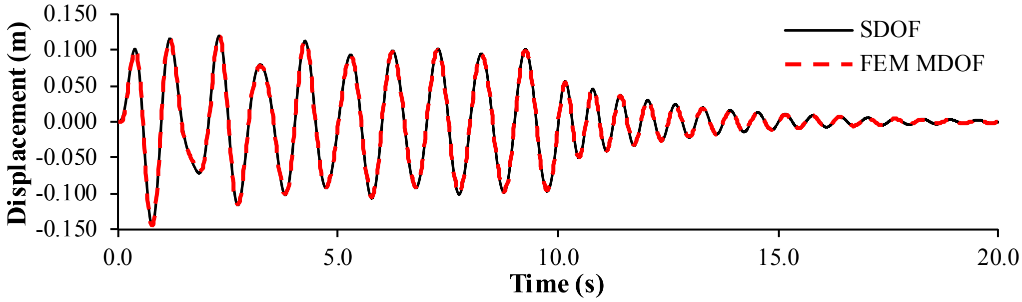


Figure 2 : Comparison of top displacement of main structure under harmonic excitation force

The results of the amplification response of the structure and the TMD system are illustrated in Figure 3, for all formulations listed in Table 2. The figure clearly shows that different formulations provide vastly different responses, with some of them proving to be unsuitable for the analyzed structure. Specifically, some formulations exhibit lower amplification factors around the resonance frequency, but at the same time, exhibit high amplification in the vicinity of this frequency, thereby leading to non-flat AM curves.

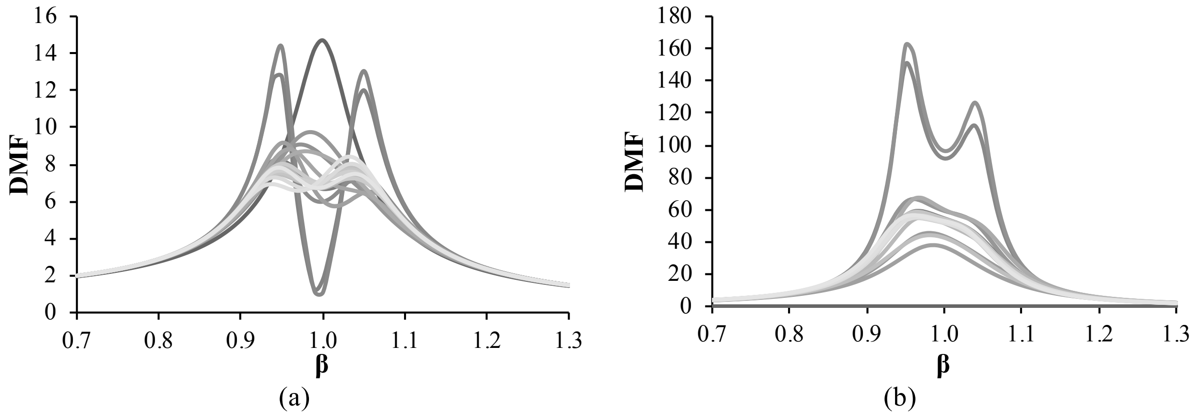


Figure 3 : Amplification magnification curves μ= 0.01 (a) main structure, (b) TMD

Figure 4 shows that, in general, all maximum amplification factors for the main structure do not exhibit high variation. The original structure amplification factor of 14.7 can be reduced by approximately 40% in almost all formulations, except for the codes WBUHB and WBUR VEL, which represent the two AM curves with two peaks around the resonant frequency. When these two values are excluded, the reduction of the DMF maximum factor when the TDM is used is 43 ± 3% for a μ=0.01.

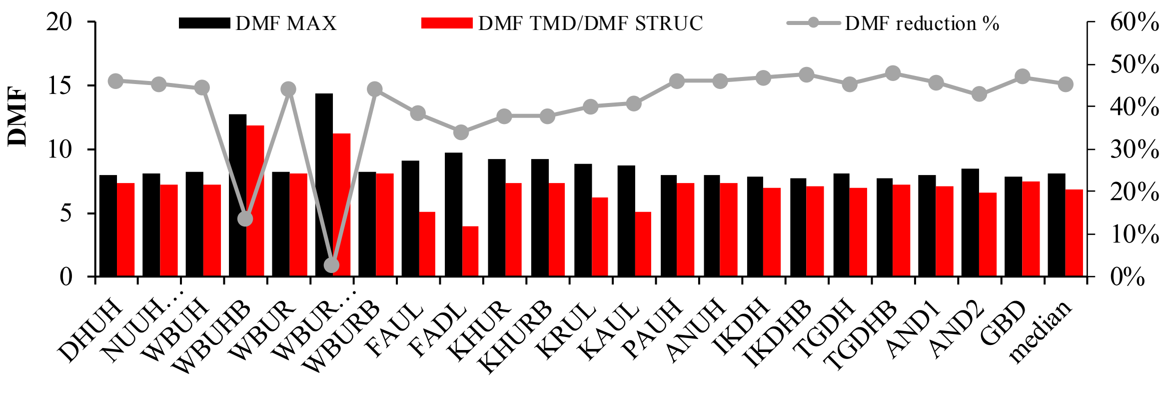


Figure 4 : Comparison DMF for the authors with μ= 0.01

In Figure 5, it is noteworthy that for some cases, the DMF MAX and the attenuation of the Xs exhibit an inverse relation, as in the case of FAUL and FADL, where the DMF MAX exhibits the highest values obtained by the steady-state response, while the Xs reduction is at lower values in the time history. This implies that the DMF curve has peaks near the resonant frequency.

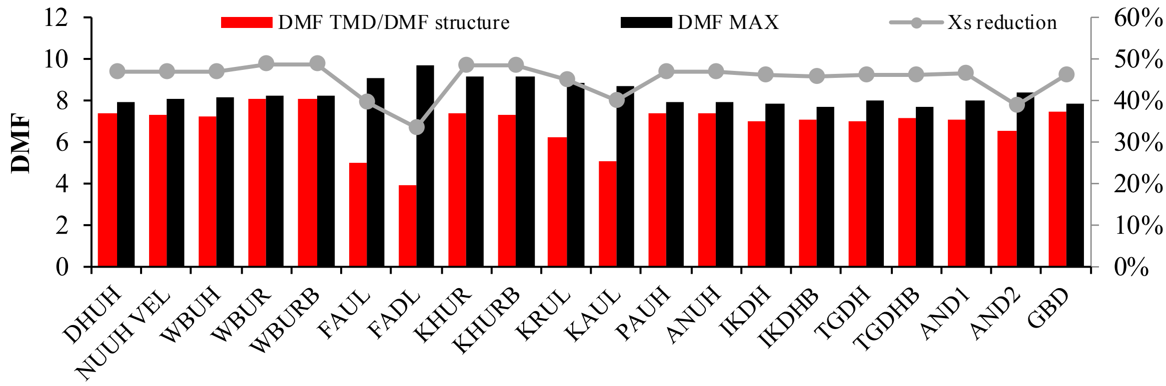
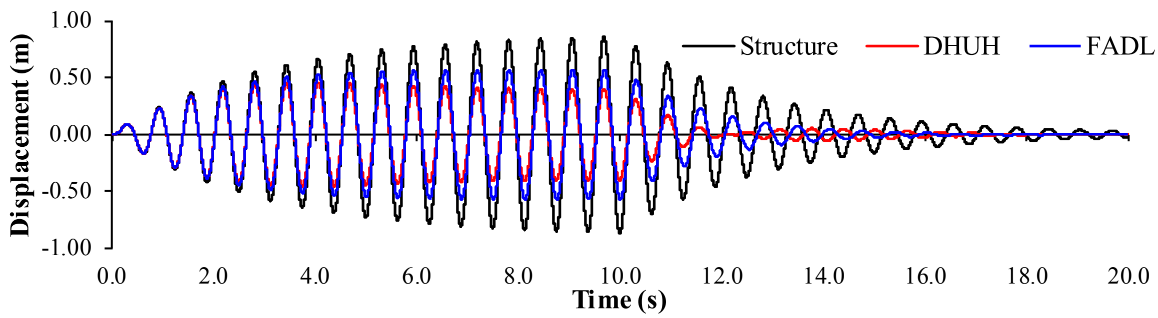
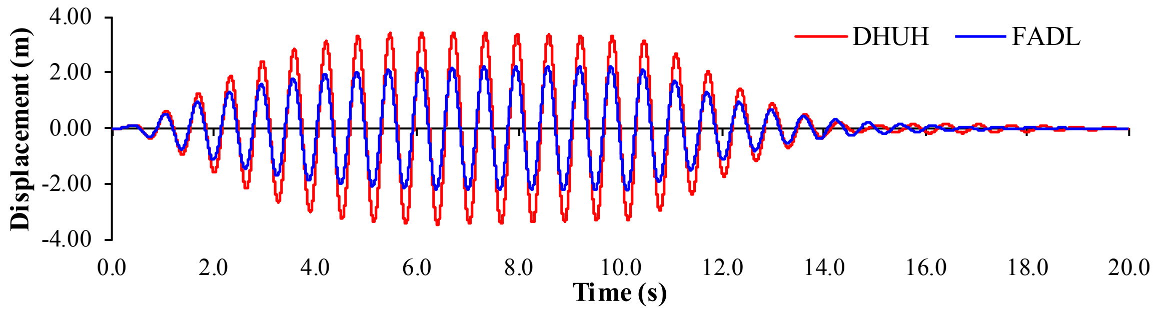


Figure 5 : Comparison DMF and the % reduction of Xs in the time history analysis for $\omega = 2\pi 1.601$

Regarding the time history response presented in Figure 6, it is evident that for the main structure, the classical DHUH formulation exhibits better performance than the FADL formulation. However, for the TMD system, the opposite is true, and FADL performs better. This result highlights the fact that optimal parameters depend on the system's specific requirements. In the case of wind towers, where space for TMD movement is limited, it is essential to find a solution that balances the main structure and TMD displacements.



(a) Structure



(b) TMD

Figure 6: time history response of the system subjected to harmonic force with $\omega = 2\pi 1.601$

5 CONCLUSIONS

Based on the findings of this investigation, it can be inferred that the efficiency of the Tuned Mass Damper (TMD) system in wind turbine towers is significantly impacted by the choice of optimal parameters, and distinct formulations can result in significant variations in the structure's response. In general, for the metallic tapered tower considered in this study, all optimal formulations yielded comparable outcomes. Among these, the classical DHUH formulation demonstrated superior performance for the main structure, whereas the FADL formulation exhibited better performance for the TMD system. Nonetheless, the selection of optimal parameters must be tailored to the specific characteristics and demands of the system, and should therefore be evaluated for each case.

It is also demonstrated that the SDOF method is capable of accurately depicting the dynamic response of the main structure subject to harmonic excitation, assuming that the appropriate modal parameters are employed. Moreover, the results revealed that the utilization of TMD systems can result in a considerable reduction in the maximum amplification factor for the main structure, even at a relatively low mass ratio of 0.01 of the modal mass. Specifically, in the presented case, the use of a mass of 1.65 kg proved to be effective in significantly diminishing the primary vibration of the system, resulting in a reduction of up to 43%.

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