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EQUAL DISPLACEMENT CONCEPT FOR INELASTIC ANALYSIS OF STRUCTURES

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Abstract

In this paper, the equal displacement concept is applied to inelastic analysis of structures subjected to symmetric and asymmetric excitation. Numerical analyses of inelastic single-degree-of-freedom (SDOF) systems were conducted to determine the effect of symmetric and nonsymmetric excitation on the nonlinear response history of the structure. The results show that for elastoplastic structures, the equal displacement rule holds for symmetric loading. For nonsymmetric loading, the rule breaks down for lateral systems with a yield strength that is less than half of the corresponding elastic system. The results of this study explain why the displacements are approximate in earthquake engineering: application of the equal displacement rule for asymmetric excitations generally does not hold and depends on the strength reduction compared to the elastic system.

Keywords: Structural Dynamics, Earthquake Engineering, Wind Engineering.

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1 INTRODUCTION

1.1 Equal Displacement Rule

A fundamental notion in modern earthquake engineering is that ductility, not strength, is the key to economically design a structure to withstand a strong earthquake. The importance of ductility was determined by Veletsos and Newmark in a landmark study [1] which showed that the displacement of an inelastic structure subjected to a seismic ground motion is approximately equal to the displacement of the same structure responding elastically.

To illustrate, consider the response of a single-degree-of-freedom (SDOF) system with a period of vibration (T) of 1.0 s and 2% viscous damping (ζ) subjected to the horizontal ground acceleration recorded during the 1940 El Centro earthquake at the Imperial Valley Irrigation District station [2], shown in Figure 1(a). The displacement history, normalized by the peak elastic displacement (Δ_e), is shown in Figure 1(b) for an elastic system. The effect of the yield strength (F_y) for an elastoplastic system on the displacement history is shown for F_y equal to 1/2, 1/4, and 1/8 of the minimum yield strength (F_e) required for the system to remain elastic.

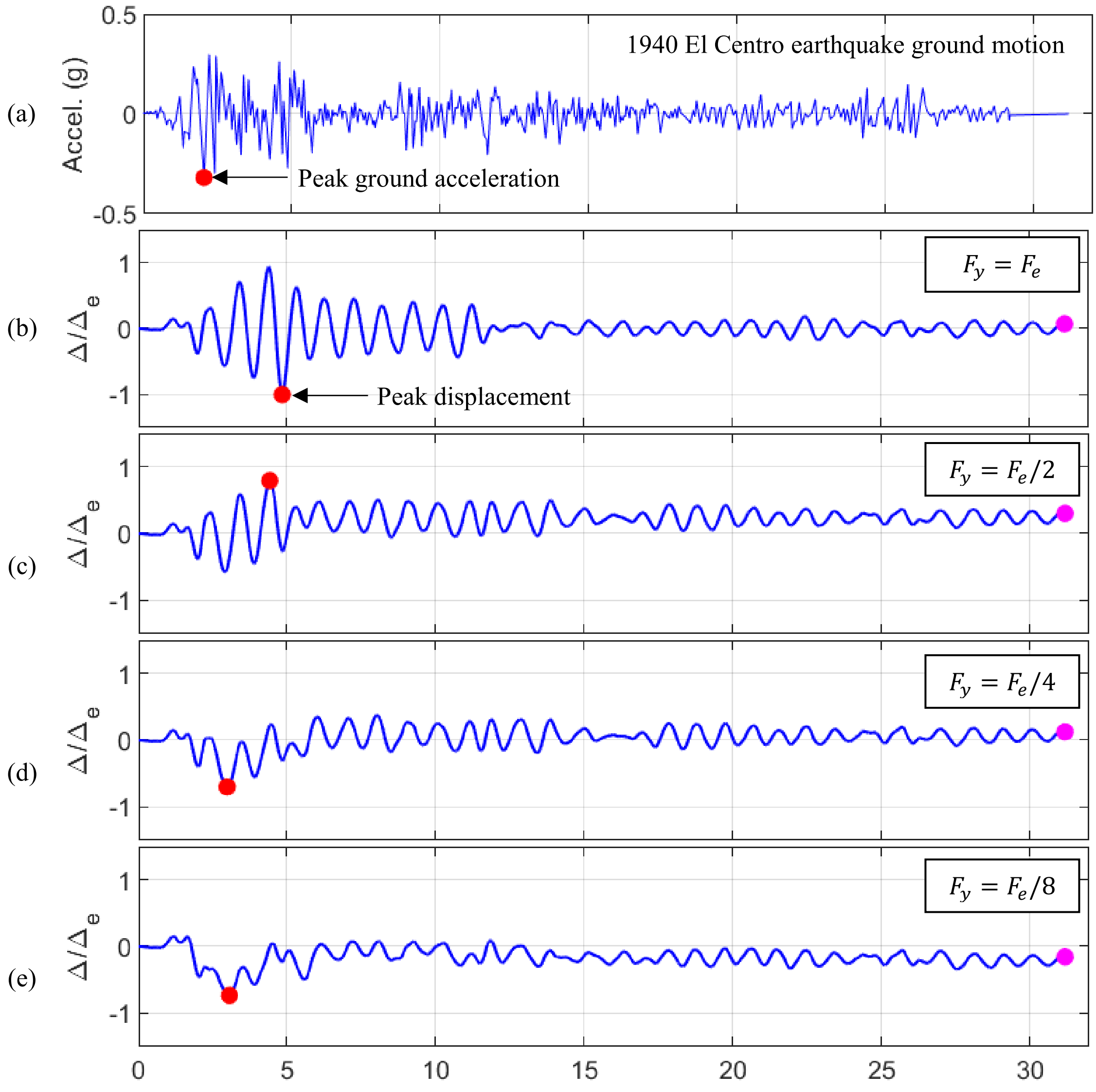

Figure 1: Displacement history (s) for SDOF system (T = 1.0, ζ = 2%): (a) ground acceleration record, (b) normalized displacement for elastic system, and (c), (d), and (e) normalized displacements for elastoplastic systems.

The response in Figure 1 shows that as the yield strength of the system decreases, the peak displacement is somewhat constant. Thus, the displacement of an elastoplastic structure is essentially independent of the yield strength of the lateral system. This concept is commonly known as the "equal displacement rule."

The equal displacement rule has had a major impact on seismic design of buildings worldwide because it postulates that a structure can be designed for significantly reduced forces, provided that (1) the lateral system has the capability to deform (ductility) proportional to that reduction, (2) the lateral system can undergo a high number of inelastic cycles without fatigue, and (3) the strength and stiffness of the lateral system is preserved (no degradation of stiffness and strength). Additionally, the rule is conditioned on a large-strain but small displacement response, such that second-order ("P-Delta") effects are negligible.

It is well-known that the peak inelastic displacement is only roughly equal to the peak elastic displacement. For example, compare the peak displacement in Figure 1(b) with the peak displacements in Figure 1(c). It is also well known that the peak inelastic displacement is not constant. For example, note the variation in peak displacements in Figure 1(c) to Figure 1(e). However, to the author's knowledge, the approximate and variable nature of the equal displacement rule in earthquake engineering has not been explained previously.

1.2 Application to Wind Engineering

The equal displacement rule is also important for wind design of buildings. Since the 1960s, inelastic structural dynamics has evolved beyond earthquake engineering to consideration of other hazards, windstorms in particular [3]. One reason for this is that rare, large-magnitude windstorms, such as thunderstorms, tornadoes, derechos, and hurricanes cannot always be mitigated economically using elastic resistance. For example, consider a structure in Chicago. The 3-s average wind speed used for design ranges from 47.8 m/s for ordinary buildings up to 53.2 m/s for buildings that are considered essential facilities (https://asce7hazardtool.online/ [4]). However, if tornados are considered in the design, the wind speed jumps to 116 m/s, depending on the effective plan area of the structure and the mean recurrence interval. Since wind pressure is proportional to the square of the wind speed, an elastic design could require a lateral system with nearly five times the strength to resist tornado wind pressures. That would be similar to requiring a building in San Francisco to remain elastic under the Maximum Consider Earthquake (MCE) ground motions. In both cases, an elastic approach may not be economical. Nevertheless, although several frameworks for performance-based wind design have been proposed, e.g., [5,6,7], application of inelastic behavior to wind loads is still undeveloped.

Lateral wind loads differ from seismic loads in two key aspects:

- First, lateral wind loads are generated by air pressures exerted on the structure (force excitation). Seismic loads are generated by inertial forces as the structure moves relative to the ground (ground excitation). As a result, wind loads depend on the building shape, but seismic loads depend on the building mass. Moreover, seismic excitation is essentially symmetric (it moves the building in both positive and negative directions), whereas wind loads can be either asymmetric (i.e. along-wind direction) or symmetric (cross-wind direction).
- Second, the temporal variation of wind pressure varies greatly on the type of wind event. Synoptic wind events caused by pressure differences over large distances, such as hurricanes, tend to be very long duration excitations measured in hours. Non-synoptic wind events, such as thunderstorms and tornadoes, tend to be long duration events measured in minutes. Both types of wind events are at least an order of magnitude longer in duration than earthquakes, which are generally last a matter of seconds. As a result, wind records are longer and involve many more cycles compared to ground motion records.

1.3 Objective

The objective of this study is to the examine the approximate nature of the equal displacement rule and determine if the rule is applicable to wind engineering. The hypothesis is that the equal displacement rule depends primarily on the symmetry of the excitation. To test the hypothesis, inelastic SDOF systems were analyzed using excitations with varying degrees of symmetry (equal motion in the positive and negative directions).

2 METHODOLOGY AND RESULTS

Wavelet-based excitations were used to determine the effect of symmetry. Unlike a sine wave, which is symmetric but has unlimited duration, a wavelet is a symmetric or asymmetric waveform with a limited duration. A wavelet provides a way to emulate both the degree of symmetry and the non-stationary characteristics (duration) of an "ideal" ground motion, defined in this paper as a ground motion without stochastic features.

For this study a wavelet-based excitation was generated using a complex frequency wavelet that was constructed using a basis-spline (B-Spline) function [8]. The complex frequency B-Spline wavelet used in this study is defined in Equation 1:

$$\psi(t) = \sqrt{f_b}\left[\sin\left(\frac{f_b t}{m}\right)\right]^m e^{2i\pi f_c t} \tag{1}$$

where ψ is the complex wavelet of order m, f_c is the center frequency of the wavelet, and f_b is the bandwidth parameter. A ground acceleration history was generated by taking the real component of the wavelet, as defined in Equation 2:

$$a_g(t) = Re(\psi) \tag{2}$$

For purposes of comparison with the El Centro ground motion, the ground acceleration in Equation 2 was normalized to match the peak ground acceleration in Figure 1. The resulting excitation is shown in Figure 2 for a wavelet center frequency of 2 Hz, a bandwidth parameter of 0.5, and a wavelet order of 3. In Figure 2, the upper and lower bounds of the waveform duration are 10 s. The wavelet was computed using wavelet toolbox in MATLAB [9].

The response of a an elastic SDOF system with T equal to 1.0 s and ζ equal to 2% is shown in Figure 3. The displacement history is normalized by the peak elastic displacement (Δ_e), and the force is normalized by the minimum yield strength ($F_e = F_y$) that is required for the system to remain elastic.

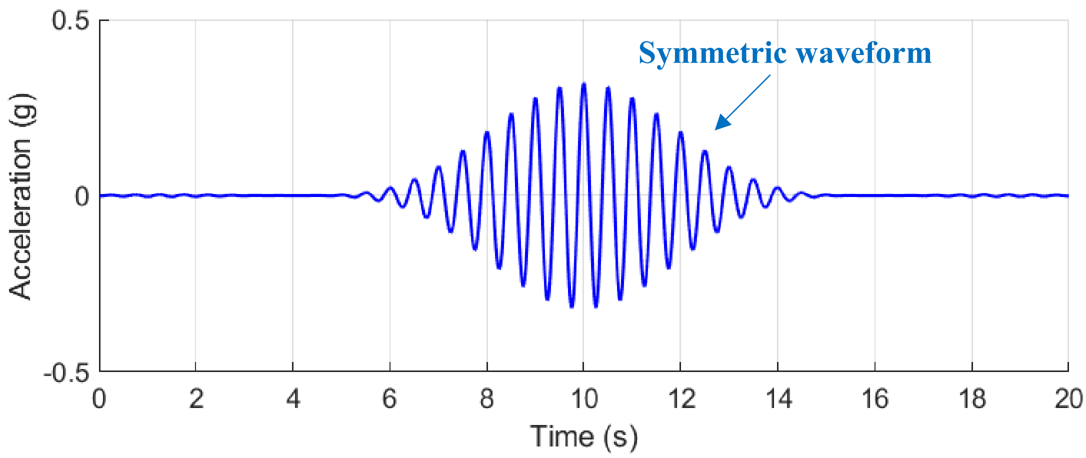

Figure 2: Symmetric ground excitation generated using a B-Spline function complex wavelet.

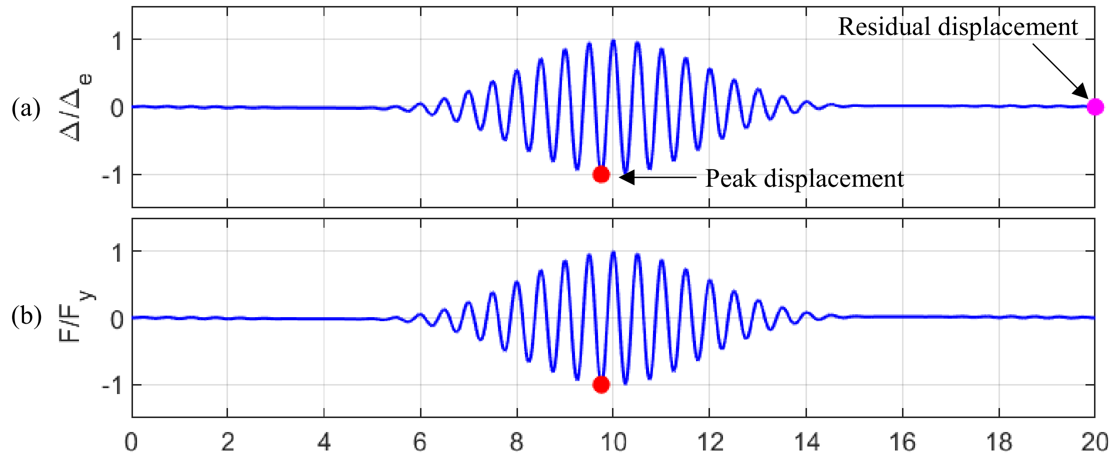

Figure 3: Response history (s) for elastic SDOF system (T = 0.5, ζ = 2%) subjected to wavelet excitation: (a) normalized displacement, and (b) normalized force.

The nonlinear equation of motion was solved iteratively using implicit direct-time integration. A constant average acceleration (Newmark's method) was used with Newton-Raphson iterations until force convergence within a tolerance of 10^{-8}. Newmark's method is relatively efficient compared to explicit integration methods, unconditionally stable for linear analysis, and is generally effective for nonlinear analysis of simple constitutive models, including the elastoplastic model used in this study.

Figure 4 shows the nonlinear response history for the elastoplastic SDOF system with F_y equal to 1/2 of F_e when subjected to a symmetric wavelet excitation. Figure 4(a) and Figure 4(b) show the displacement and force histories. Note that the peak inelastic displacement is exactly equal to the peak elastic displacement. Figure 4(c) shows yield events (in this case, 18 inelastic excursions).

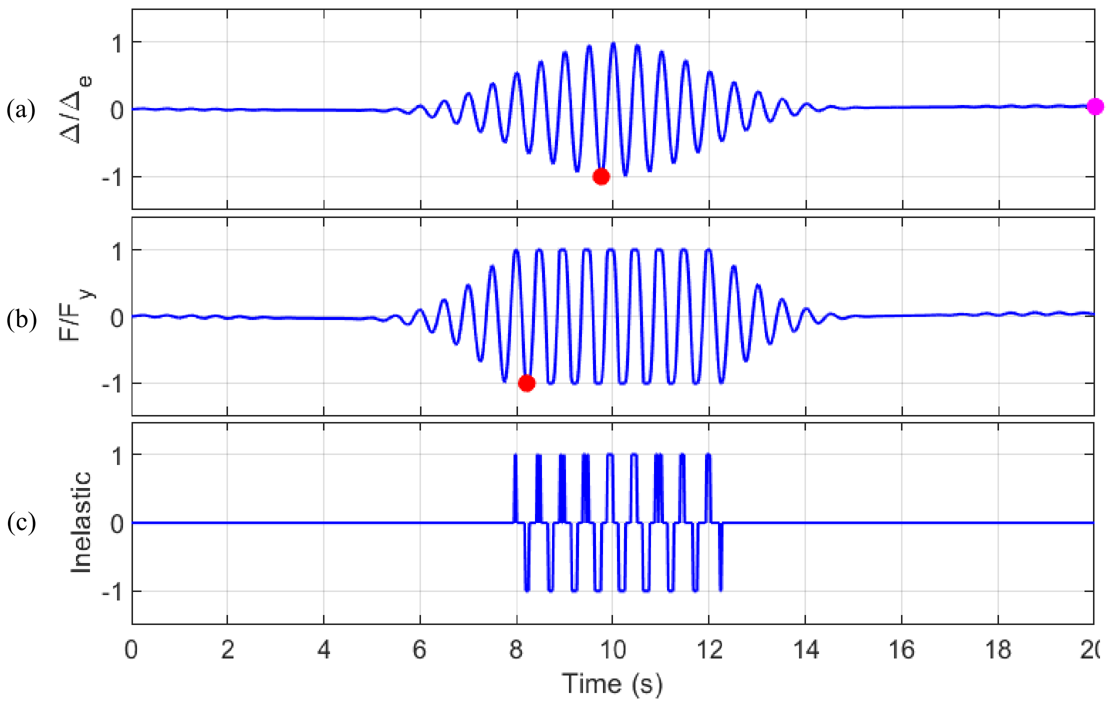

Figure 4: Response history (s) for elastoplastic SDOF system (T = 0.5, ζ = 2%) subjected to wavelet excitation: (a) normalized displacement, (b) normalized force, and (c) yield events (1 = inelastic excursion).

Nonlinear response history analyses were then run using the symmetric waveform applied across a range of strength reduction ratios (F_e/F_y). The resulting ratio of the normalized peak inelastic displacement (Δ/Δ_e) versus the strength reduction ratio is shown in Figure 5. For purposes of comparison, nonlinear response history analyses were also run using the El Centro ground motion. The results clearly show that for an ideal symmetric excitation, the peak displacements are constant, but for a ground motion record, the peak displacements are only approximately constant. To explore the effect of other analysis parameters, additional analyses, not shown here, were run using a range of periods of vibration, damping, and other symmetric waveforms. The results confirm the observation that the equal displacement rule holds for symmetric excitation for other periods, damping, and waveforms.

The effect of the waveform symmetric was examined by subjecting the system to an asymmetric complex frequency B-Spline wavelet excitation. Asymmetry was introduced by amplitude-scaling the positive half of the signal by 50%. The resulting excitation is shown in Figure 6. The nonlinear response history analyses were then run again using the asymmetric waveform. Figure 5 shows that the asymmetric waveform has a large effect on the peak displacements. The results confirmed the hypothesis that the equal displacement rule depends primarily on the symmetry of the excitation.

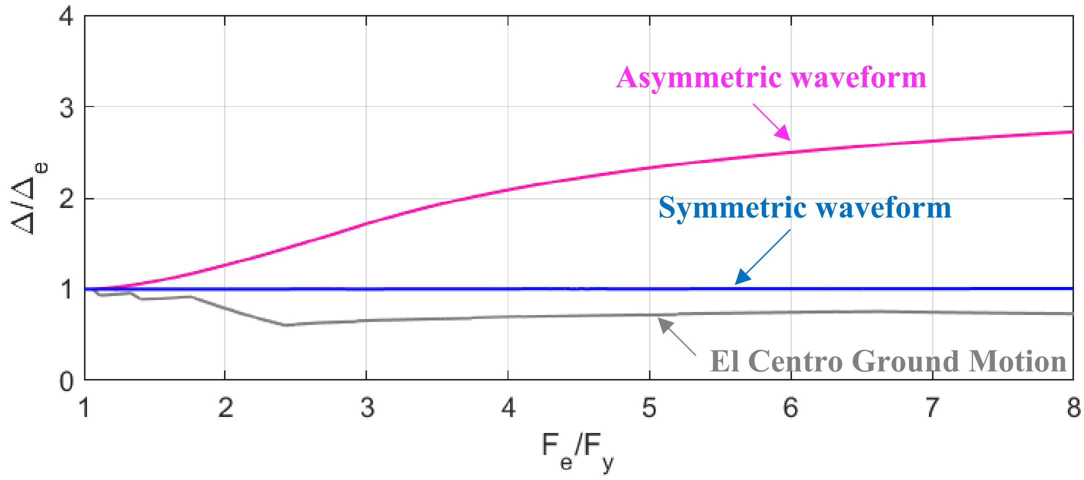

Figure 5: Ratio of normalized peak inelastic displacement versus strength reduction ratio.

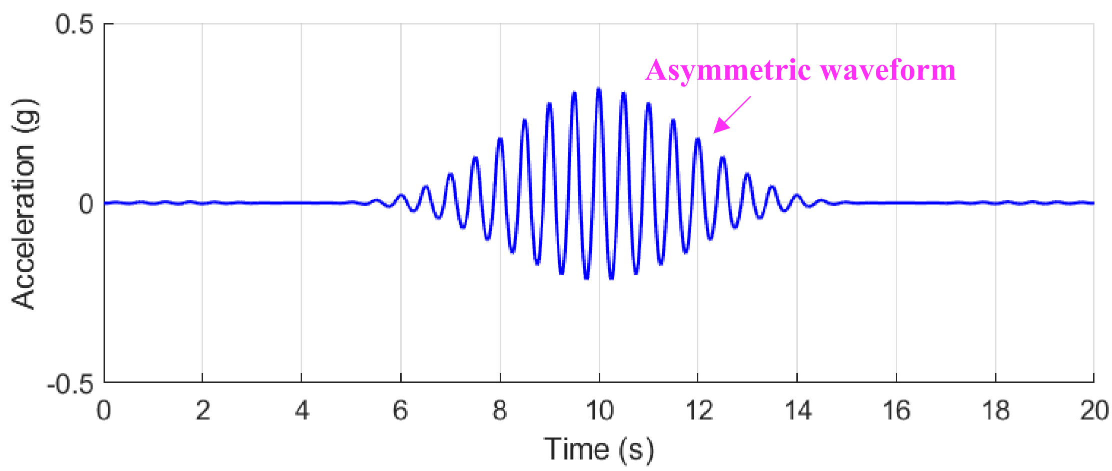

Figure 6: Asymmetric ground excitation generated using a B-Spline function complex wavelet.

3 CONCLUSIONS

Elastoplastic SDOF systems were subject to both symmetric and asymmetric excitations. The nonlinear response history results show that the peak inelastic displacement is exactly equal to the peak displacement for the corresponding elastic system if the excitation is symmetric. The equal displacement rule does not hold for asymmetric excitations. Peak inelastic displacements increase for elastoplastic systems with larger strength reduction ratios. For pseudo-symmetric excitations, including typical ground motion records, the equal displacement rule is approximate.

The results indicate that the equal displacement concept is conditional upon the symmetry of the excitation. As a consequence, the peak inelastic displacements for asymmetric excitations, including non-stationary stochastic wind loads, are not constant and depend on the strength reduction. The implication for wind engineering is that only a limited strength reduction may be justified for rare large-magnitude windstorms, and additional studies are warranted to determine the effect of duration on the response of non-elastoplastic systems and the application of the equal displacement rule to multi-degree of freedom systems and systems with gravity load effects.

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