

## **CROSS INTERACTION FOR A SMALL GROUP OF SHALLOW FOUNDATIONS**

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### **Abstract**

*A few literature studies have highlighted the dynamic interaction that occurs through the underlying soil among closely spaced structures (SSSI). This interaction, although neglected in most technical codes, could affect the seismic response of structures located in densely urbanized areas. Extensive studies on this topic are still limited and more refined approaches accounting for soil continuity are needed. In this context, the work illustrates a three-dimensional numerical study aimed at quantifying the effect of SSSI on the dynamic impedance of a shallow rectangular rigid foundation in the presence of two lateral neighbors. The key factors accounted in the parametric study are: (i) the relative distance between three nearby foundations, (ii) the position of the foundation in the group and (iii) the input frequency of the harmonic loads exciting the three foundations. At this stage of the work, a viscoelastic linear behavior was assumed for the soil. Due to SSSI, a decrease of the static stiffnesses of the reference foundation in the group was computed. This reduction depends on the distance among the foundations and its position in the group (central or lateral). Lateral foundations are less affected by the mutual interaction.*

**Keywords:** SSSI, Impedance function, Shallow foundations, 3D numerical modelling.

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## 1 INTRODUCTION

Even though in current design buildings are analyzed as isolated structures (i. e. without other structures nearby), the seismic response of a structure located among others can be different from that of the same structure alone. This is the case, for example, of large metropolises or the historic quarters of small villages, where buildings are very close to each other and, hence, may interact through the underlying soil. In literature, this phenomenon is referred to as Structure-Soil-Structure Interaction (SSSI). From the first studies dating back to the 70s [1-3] to the most recent ones, it emerged that in some cases the effects of through-soil interaction are not negligible and depend on various factors, such as the distance between the foundations, their plan layout, the dynamic features of the structures, the frequencies of the input motion and the subsoil characteristics.

An oscillating foundation generates surface and body waves that spread through the soil, reaching the neighboring passive foundations. These latter ones may experience additional displacement and stress fields, which in some cases may be comparable to that of the active foundation. For a given structure [4], this additional cross coupling could affect soil compliance with the consequent modification of the fundamental period of the overall soil-structure system [5].

If SSSI occurs, in addition to the soil reactions in the direction of the applied load, further stresses perpendicular to it are generated at the soil-foundation contact [6-9]. The displacement (or rotation) components of two or more nearby foundations can be mutually in phase or out of phase with respect to each other but also to the ground as evidenced by Betti for different types of incident waves [10]. Interaction phenomena generally produce higher effects when the number of foundations increases and their clear spacing decreases [11-13]. The SSSI effects may be emphasized or not depending on the specific soil properties [14-17], the plan layout of the building stock and the dynamic features of the buildings (fundamental period, etc.) [18-21].

Despite the knowledge gained so far, it is still not easy to predict when through-soil interaction effects may affect the dynamic response of a structure, in consideration of several factors involved. For this reason, to date there is a gap in design code or guidelines on SSSI; hence, to improve current standards more extensive numerical and experimental investigations are needed. In this vein, the paper aims to find a simple but practical procedure to account for SSSI in the design (or verification) of buildings in highly urbanized context. Since the well-known substructure (impedance) approach is still the fastest and most manageable analysis method to handle classical soil-structure interaction (SSI) problems, this approach with all related advantages will be proposed to solve even the more complex problem of structure-soil-structure interaction (SSSI). In short, the idea is to include the presence of other neighboring foundations in the impedance function of the target foundation (and structure).

Using a continuous 3D finite difference approach, a simple scheme of three shallow rigid foundations on a homogeneous linear elastic halfspace under harmonic excitation was analyzed. In the parametric study, the distance between the foundations and the input frequency were varied and the different components of the soil-foundation impedance matrix were rigorously calculated. In a previous works of the same authors [22], the impedance functions for two closely-spaced foundations have already been obtained, and suitable interaction coefficients proposed. These interaction coefficients allow the static stiffness of the single foundation to be modified in order to take into account for the presence of its neighbor. In this way, the target foundation can still be modelled as “isolated” but equipped at its base with modified stiffnesses accounting for the presence of the close foundation.

In this study, the same procedure already validated for the couple of footings, will be extended to the case of three independent foundations. These could represent, for example, the

foundations of ancient masonry buildings without a proper connection in the transversal direction.

## 2 FEATURES OF THE NUMERICAL MODEL

The reference scheme considered in this numerical study is illustrated in Figure 1. It consists of three infinitely rigid and equally-spaced foundations with their length parallel to the horizontal global axis  $x$  and placed on the surface of a linear, isotropic, elastic half-space (Figure 1a). The geometric features of the foundations are shown in Figure 1b: the three foundations are assumed to be rectangular, with length,  $2L$ , and width,  $2B$ . The free distance between the foundation edges,  $S$ , is varied between  $0.5B$  and  $4B$  and the properties assigned to the soil, assumed linear elastic, are listed in Table 1. It is worth emphasizing that at this stage of the work a very low  $V_s$  value was intentionally assigned to the soil to maximize the interaction effects among the neighboring foundations.

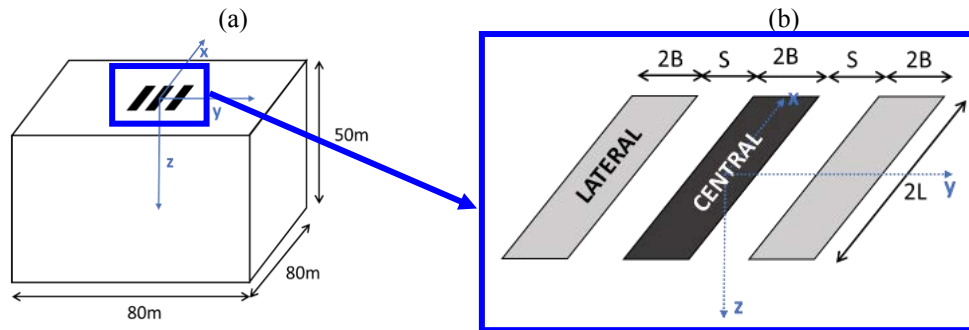


Figure 1. Analysis domain (a) and layout (b) of the three foundations considered in this study

Since it is needed to model the soil as a continuum to properly analyze the cross interaction between adjacent foundations, the 3D soil domain shown in Figure 1a was implemented in FLAC3D [30] and discretized through the finite difference technique (Itasca, 2004). A grid spacing,  $\Delta$ , respecting the rule of Kuhlemeyer and Lysmer (1973) ( $\Delta < V_s/8f_{\max}$ ) was chosen, so that input frequencies up to a maximum value  $f_{\max} = 25$  Hz can reliably be propagated in the soil with shear wave velocity,  $V_s$ .

	$\gamma$ [kN/m <sup>3</sup> ]	K [MPa]	G [MPa]	$V_s$ [m/s]
Halfspace	19	12.2	5.6	54

Table 1. Soil parameters

The three foundations were simultaneously loaded through a harmonic displacement associated with the stiffness component to be calculated. The frequency of the applied harmonic oscillation ( $f = \omega/2\pi$ ) was alternately set equal to the odd frequencies in the range 1-13 Hz to cover a probable frequency band of interest in the engineering field. A further case, corresponding to a zero-excitation frequency, was simulated to obtain the static stiffness that appears in the real part of

the impedance function in Equation 2. Addinational features of the model and its validation may be found in [23].

### 3 NUMERICAL RESULTS

In Figure 2, the static stiffnesses associated to the translational and rotational modes of the single foundation were compared with those of the same foundation in the presence of its twins, loaded equally and simultaneously. The translational stiffnesses ( $K_{zz}$ ,  $K_{yy}$  and  $K_{xx}$ ) were normalized with respect to the soil shear modulus,  $G$ , and the half-width of the foundation,  $B$ , while the rotational stiffnesses ( $K_{rx}$  and  $K_{ry}$ ) were divided by  $G$  and  $B$  cubed. The clear horizontal (edge-to-edge) distance  $S$  between the three foundations was also normalized to the foundation half-width,  $B$ .

It emerged that when the target foundation is adjacent to the other two (clones), its static stiffnesses decrease compared to the values corresponding to the same foundation isolated and the reduction in stiffness decreases as the distance between them increases. On the other hand, for  $S/B=4$ , the cross interaction between the two foundations is still found to be non-negligible. Moreover, the reduction also depends on the position of the target foundation in the group. As expected, the reduction is greater if the foundation is central. When the distance is short (e.g.  $S=0.5B$ ), a percentage difference with respect to the value of the individual foundation more than 50% for the central foundation and more than 30% for the lateral ones is achieved. For larger distances (e.g.  $S=4B$ ), the above differences decrease, being around 40% for the central foundation and more than 20% for the lateral ones.

For the oscillation around the x-axis (Figure 1), the neighboring foundations exert a kind of mutual constraint on the central one, which causes an increase in its rotational stiffness. Regardless of the position of the foundation (central or lateral), the interaction effect on the rocking motion is more sensitive to the distance. For small values of the foundation-foundation spacing, the difference is still around 50% and 30%, while it decreases to 2% and 10% for  $S/B=4$

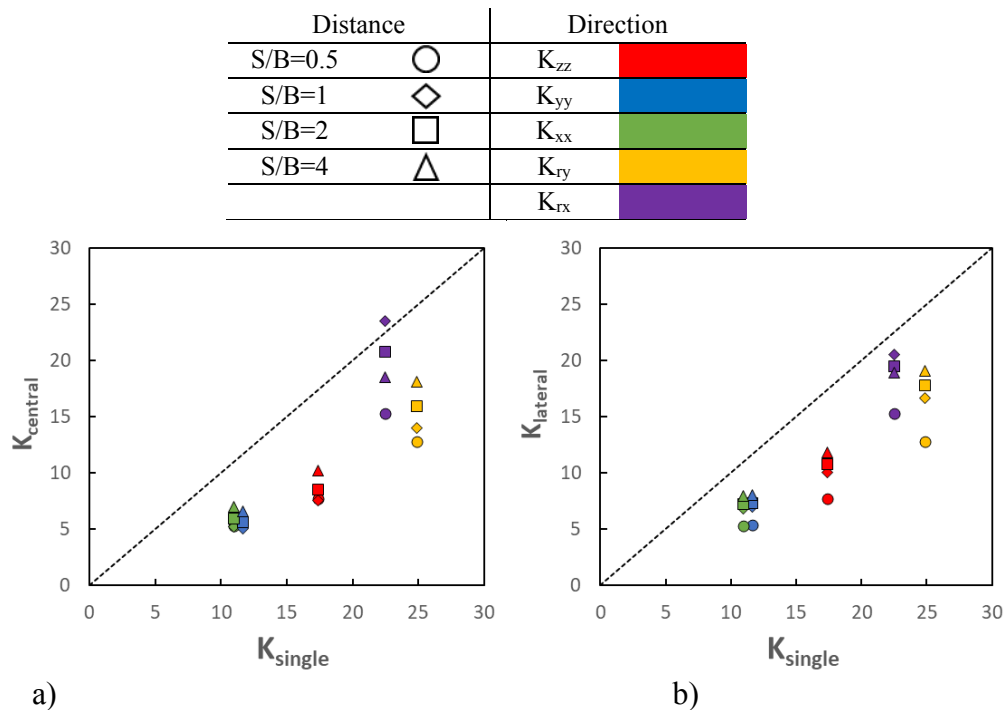


Figure 2. Dimensionless static stiffness of the central (a) or lateral (b) foundation of the group vs the static stiffness of the single footing.

#### 4 COMPARISON WITH ANALYTICAL SOLUTIONS

For the couple of foundations, the authors defined interaction coefficients that allow the static stiffnesses of the single foundation to be modified in presence of its twin foundation:

$$\alpha_{ij} = (K_{ij,single} - K_{ij,group})/K_{ij,single} \quad (2)$$

where  $K_{ii,single}$  and  $K_{ii,group}$  are the stiffnesses of the target foundation in case it is alone or in couple. The cross-interaction modifiers  $\alpha_{ij}$  are provided in an exponential law:

$$\alpha_{ij} = a_{ij} \exp [b_{ij} (S/B)] \quad \text{with } S/B \in [0.5; \infty[ \quad (3)$$

where  $a_{ij}$  and  $b_{ij}$  are the coefficients for each foundation oscillation mode, given in Table 2 in the case of an elastic halfspace. Other subsoil configurations can be found in [23].

Assuming that the stiffness component linking the forces below the master footing to its displacement is the same if the foundation is in a couple or in a triplet, the analytical stiffness of the lateral foundation was computed by:

$$K_{ij,lateral} = K_{ij,single}(1 - \alpha_{ij}) \quad (4)$$

For the central foundation, the presence of the two equally-spaced lateral neighbors was expressed by:

$$K_{ij,central} = K_{ij,single}(1 - 2\alpha_{ij}) \quad (5)$$

<b>a<sub>zz</sub></b>	<b>a<sub>yy</sub></b>	<b>a<sub>xx</sub></b>	<b>a<sub>ry</sub></b>	<b>a<sub>rx</sub></b>
0.30	0.30	0.30	0.30	-0.48
<b>b<sub>zz</sub></b>	<b>b<sub>yy</sub></b>	<b>b<sub>xx</sub></b>	<b>b<sub>ry</sub></b>	<b>b<sub>rx</sub></b>
-0.18	-0.07	-0.13	-0.62	-0.32

Table 2. Coefficients of the exponential law in Eq.3 for the case of two interacting foundations on halfspace

Figure 4 shows the ratio between the numerically and analytically-computed stiffnesses versus the dimensionless spacing,  $S/B$ , for both the central and the lateral foundation of the trio. From Figure 4-a it could be observed that for the swaying and rocking modes of the central foundation the ratio is always greater than one for the  $S/B$  considered and tends to one for larger spacing (greater than  $4B$ ). This means that when the foundations are very close together and placed in the centre, the stiffness predicted by Eq. 5 underestimates the numerical value. This is due to the fact that the analytical approach neglects the stiffening effect provided by the lateral foundations on the central one. As the clear distance increases, the mismatch decreases and the analytical predictions are in good agreement with the numerical ones provided by the rigorous 3D analyses with FLAC3D. Conversely, for the lateral foundations of the trio (Figure 4-b), the mismatch between numerically and analytically predicted static stiffnesses does not exceed 5-10%.

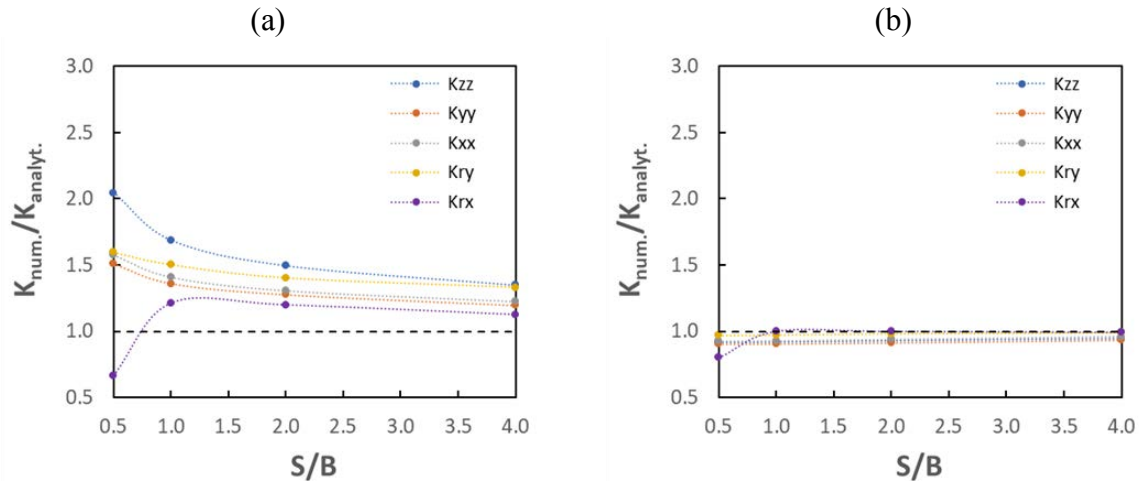


Figure 4. Ratio between numerical and analytical stiffnesses for the central (a) and lateral (b) foundation of the trio.

## 5 CONCLUSIONS

The static interaction among three closely-spaced shallow foundations through the underlying soil was investigated by means of a 3D continuum approach. In the performed study, a simple scheme of three shallow foundations placed on an ideal halfspace was considered and a parametric study was carried out considering different values of the foundation-foundation spacing.

Due to soil continuity, the phenomenon of cross interaction among multiple footings produces a reduction of the low-frequency (static) stiffness terms of the impedance matrix for all degrees of freedom of the foundation. For smaller values of the foundation-foundation spacing, the cross-interaction revealed to be not negligible, either for the central foundation of the group or for the lateral one.

Suitable coefficients, which reduce the static stiffnesses of the reference foundation in presence of its neighbors, were formerly derived by the authors for a group of two foundations [23]. In this work, the same modifiers have been adopted to estimate the static stiffnesses of the foundation placed in a small group of three footings. The closed-form solutions found for the foundation couple also proved reliable for the foundation trio in the case of a lateral foundation. For a central foundation, the same solutions are applicable for clear distances among the foundations greater than 4 times the half-width of the foundation. For shorter distances, an underestimation of the stiffness modifiers occurs.

## 6 ACKNOWLEDGEMENTS

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