

OUT-OF-PLANE RESPONSE OF MASONRY CHURCH FACADES INCLUDING P-DELTA EFFECTS

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Abstract

Out-of-plane rocking mechanisms represent one of the major causes of damage and failure for unreinforced masonry buildings, monuments and churches leading to significant economic and social losses. Due to their slenderness, masonry walls subjected to rocking mechanisms can show large displacements before the complete overturning. Therefore, seismic analyses should consider geometric nonlinearities, requiring more complex formulations and increasing the computational effort. This paper applies a recently-proposed P-Delta formulation of the discrete macro-element method (DMEM) to analyse the seismic behaviour of masonry façades interacting with lateral walls. The model accounts for geometric nonlinearities considering the P-Delta effects by updating the load vector at each step of the analysis, avoiding assembling and updating the geometric stiffness of the system. The presented study aims at quantifying the role of the P-Delta effects in conjunction with the cohesive-friction connections between the façade and lateral walls on the ultimate rocking of the external façade. These effects are here investigated through pushover analyses on a church façade. The analyses are conducted on a global model, accounting for brick interlocking, and on a simplified model, including only the façade where ad hoc calibrated non-linear links simulate the interaction with the lateral walls. The results show that geometric nonlinearities affect the façade's response even at relatively low-magnitude displacements, with increasing influence as the quality of the façade-to-lateral-walls connections reduces.

Keywords: Seismic analyses, Pushover analyses, Discrete models, P-Delta effects, Masonry churches, Historical construction, Cultural heritage.

1 INTRODUCTION

Several events have evidenced the vulnerability of historical and monumental unreinforced masonry constructions to the out-of-plane rocking mechanisms of external facades [1]. Once activated, these failure mechanisms can lead the system to large displacements, significantly reducing the stabilizing effects of the wall self-weights. For this reason, geometric nonlinearities should be considered in seismic analyses to assess these mechanisms. However, this represents a non-straightforward task since rigorous formulations may lead to computationally expensive models and a robustness reduction of the numerical simulations.

In this framework, detailed non-linear finite element (FEM) and discrete element (DEM) methods, characterized by different levels of accuracy, can explicitly account for the actual masonry bond and geometry ([2]–[4]), also in the presence of irregular masonry arrangements [5]. However, FEM and DEM approaches require a significant computational effort and many parameters to calibrate the models in the non-linear field; the analyses can be time-consuming and computationally expensive when assessing large structures. Due to this, reliable numerical models effectively used in research are rarely compatible with practical seismic assessments [6].

Simplified approaches are instead represented by macro-block limit-analysis models ([7], [8]) also accounting for the stabilising contribution of friction, lateral walls and slabs ([9]–[12]), which have been mainly adopted in combination with force-based approaches to evaluate the safety level of the considered structure [13] and, more recently, with displacement-based methods [14].

Belonging to relatively simplified approaches is also the discrete macro-element method (DMEM), recently upgraded to include P-Delta effects [15]. The DMEM was proposed initially for the in-plane response of masonry walls [16] and subsequently extended to 3D kinematics to account for the coupled in-plane and out-of-plane responses of masonry walls when subjected to earthquake loadings [17]. According to this strategy, the wall is discretised by shear-deformable macro-elements interacting with the adjacent ones by means of discrete or continuous interfaces ([16], [18]), allowing for low computational cost while enabling a straightforward model calibration. In this spatial formulation, the model has been adopted to assess monumental structures subjected to a non-box behaviour, and thus vulnerable against out-of-plane mechanisms ([19], [20]). The model has been implemented within the HiStrA (Historical Structure Analysis) software [21] and has been validated in [15] against analytical rigid-body solutions and quasi-static experimental tests.

Aiming to evaluate the capability of the P-Delta DMEM model in real cases, this paper addresses pushover analyses of a real masonry church in Italy, chosen as a representative case study, considering its global and local behaviour. The analyses are conducted explicitly considering the presence of lateral walls interacting with the façade, and accounting for brick interlocking. Then, a simplified model in which the effects of lateral walls is concentrated in non-linear links is analysed for comparison. Solutions including and neglecting P-Delta effects are compared to each other to evaluate the role of geometric nonlinearities on the wall response and the accuracy of the simplified approach.

2 THE DISCRETE MACRO-ELEMENT MODEL

According to the DMEM approach, a masonry wall is discretised by a mesh of shear-deformable spatial macro-elements interacting with the other elements through non-linear zero-thickness interfaces. Each macro-element is characterised by four vertexes (v_1, \dots, v_4), and its kinematics is governed by six Lagrangian parameters ($U, V, W, \Phi, \Theta, \Psi$) describing the rigid motion of the element, and one additional parameter (γ) describing the element shear

deformation. Each interface is composed of a set of non-linear mono-dimensional links calibrated following a straightforward fiber procedure ([18], [20]) oriented in the normal and tangential directions (Figure 1b) of the interfaces. The number n of rows of orthogonal links is chosen according to the desired level of accuracy to be reached for the interface integration. It is worth noting that no additional Lagrangian parameters are needed to describe the kinematics of interfaces. Therefore, the total degrees of freedom of the model are directly given by $7 \times N$ being N the number of macro-elements; this aspect strongly contributes to contain the computational burden of the overall model.

According to the P-Delta formulation proposed in [15], P-Delta effects are considered by updating the current positions of the external and along-interface internal forces applied to the macro-elements. This simplified procedure avoids assembling and updating the geometric stiffness matrix according to the current system configuration, ensuring a good efficiency of the model. More details on the model formulation and validation can be found in [15].

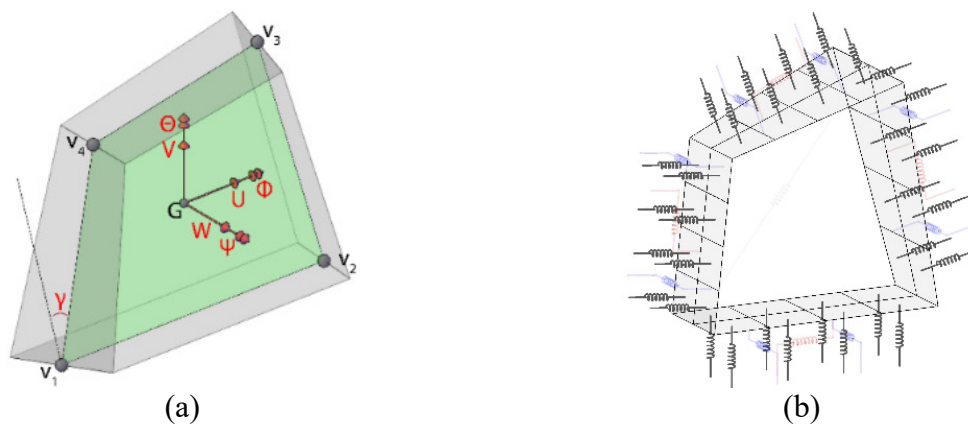


Figure 1: a) Lagrangian parameters of the macro-element; b) interfaces non-linear links.

3 NUMERICAL APPLICATIONS

This section considers the Church of San Nicolò di Capodimonte case study, which has been numerically investigated in the literature ([22], [23]). In the following, the response of the main façade of the church is evaluated by performing pushover analyses considering a distribution of lateral forces proportional to the masses.

The out-of-plane (OOP) behavior of masonry walls, especially for tall and slender walls such as church facades, is significantly influenced by the boundary conditions, especially horizontal restraints, such as orthogonal walls, flexible roofs, timber beams, tie-rodes, or a combination of them. For these reasons, there is the need to have global models capable of simulating the complex interactions between all the structural elements. Nevertheless, these global models may be difficult to calibrate, computationally expansive and time-consuming. In order to overcome this limit, simplified models have been introduced which simulate the structural horizontal restrained by means of horizontal links. In 2017 Giresini et al. ([24], [25]) introduced “compressive” links in order to simulate the impact and the rebound effect, which in some cases may affect the seismic response in a negative way so considering a free-standing wall. However, assuming zero tensile stiffness, these studies did not consider the stabilising effects of the horizontal links in tension. Afterwards, Casapulla et al. [26] introduced a horizontally distribution of orthogonal links to simulate the effect of retaining walls, possessing non-zero compression and tensile stiffnesses, so employed to perform incremental dynamic analyses. A similar approach is used in the present paper to assess its effectiveness in the static field. More specifically, the behaviour of the church involving the OOP overturning

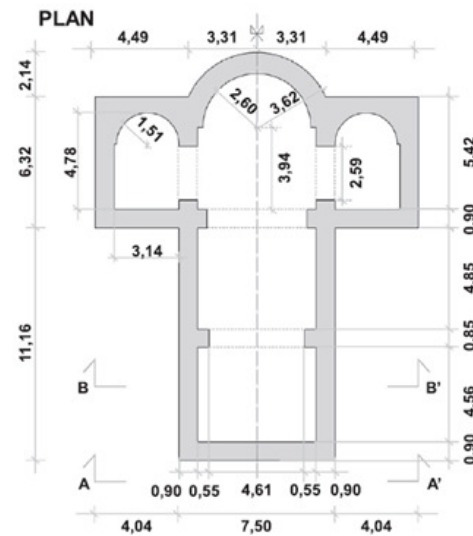
of the façade is assessed by adopting two different models: one global model (DMEM-G) including the entire church and one comprising only the façade (DMEM-F). In the latter model, the retaining walls are accounted for by equivalent non-linear links working in tension and compression with the same elastic stiffness, with nonlinear mechanical parameters evaluated considering a cohesive-friction strength criterion, describing the brick interlocking between the façade and the retaining walls, and considering the axial load due to the masonry self-weight. The P-Delta effects are alternatively considered and neglected to evaluate their effects on the ultimate strength and displacement capacity of the façade.

3.1 The case study of San Nicolò di Capodimonte

The Church of San Nicolò di Capodimonte is located in Camogli (Genoa, Italy) and dates from the twelfth century. This ancient masonry building has been first investigated by Malena et al. [22] by adopting both continuous FEM and DEM approaches and, more recently, by Funari et al. [23] by combining limit analysis with non-linear dynamic (rocking) analyses. A view of the main façade of the church is reported in Figure 2a, while the plan geometrical layout of the church is reported in Figure 2b. The two computational models have been developed in the structural code HiStrA (Historical Structural Analysis) [21]. The global model (DMEM-G) comprises 1433 macro-elements and 37 triangular (vertex) elements, 3719 interfaces with a total of 10235 degrees of freedom. The façade (DMEM-F) model comprises 141 macro-elements, 5 triangular elements, 34 non-linear links, and 318 interfaces with 017 degrees of freedom, corresponding to approximately 10% of the DMEM-G's model.



(a)



(b)

Figure 2: Church of San Nicolò di Capodimonte: a) external view; b) plan geometrical dimension [22].

The deformability properties, namely the Young's and shear modules, the compressive strength and the self-weight, are chosen according to the Italian standards [27] for natural-stone masonry. In addition, the tensile strength and cohesion are assumed to equal 0.01MPa, to simulate a quasi-no-tension friction material consistently to Malena et al. [22] and Funari et al. [23]. The tensile fracture energy is assumed to equal 0.001N/mm. The mechanical parameters of masonry adopted in the analyses are summarized in Table 1.

3.2 Numerical modelling and results

A 3D view of the global model of the church (DMEM-G) is reported in Figure 3a. The mesh is automatically generated, maintaining the average shape ratio of units reported in [22]. In the models, the average in-plane dimensions of macro-elements are 1600mm x 800mm, while the actual dimensions of units can be assumed to be 800mm x 400mm. Therefore each macro-element represents a masonry volume corresponding about to 2x2 units.

Young's modulus	Shear modulus	Compressive strength	Tensile strength	Tensile fracture energy	Cohesion	Friction coefficient
E	G	σ_c	σ_t	G_t	c	μ
[MPa]	[MPa]	[MPa]	[MPa]	[N/mm]	[MPa]	[-]
1500	500	3.8	0.01	0.01	0.01	0.6

Table 1: Mechanical masonry parameters.

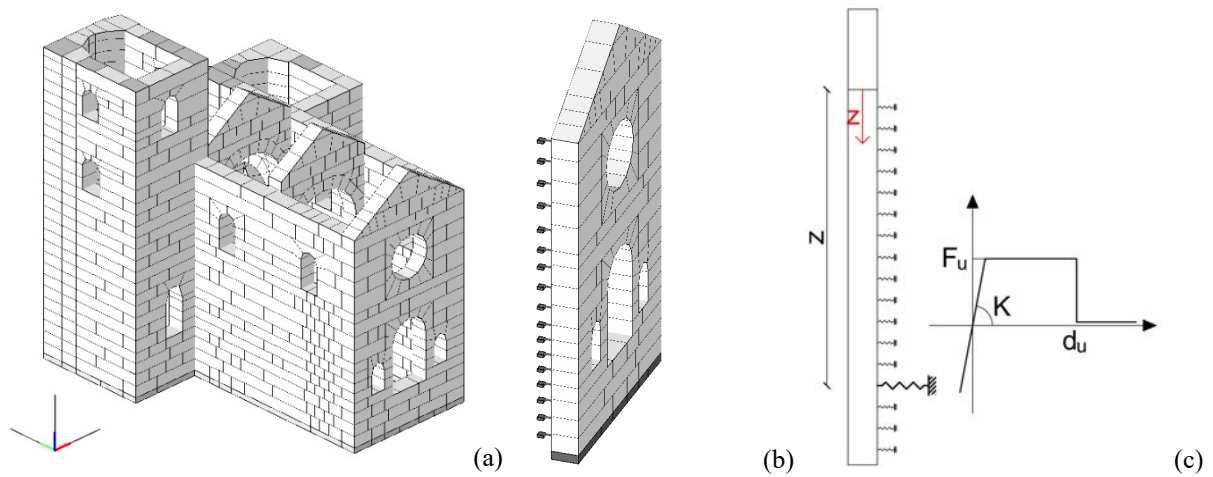


Figure 3: a) 3D model of the church (DMEM-G) and b) simplified model of the façade (DMEM-F).

Figure 3b shows the simplified model (DMEM-F), including only the façade and the discrete distribution of non-linear links accounting for the interaction with the lateral walls. For the latter an elasto-plastic constitutive law is considered with a fixed ultimate displacement. The elastic stiffness (K), both in tension and compression, is calculated assuming 1.5 mm as a yielding displacement and a ultimate force (F_u) equal to:

$$F_u = N \cdot \mu + c \cdot A \quad (1)$$

being N the axial force and A the trasvenrsal section of a single horizontal block-to-block interlocking surface. Finally, the ultimate displacement d_u is assumed equal to 50% of the average length of the stone blocks. The results are summarised in Table 2. For the sake of simplicity, the value of N is evaluated considering the vertical loads and is kept constant during the push-over analysis. Figure 4 shows the load multiplier vs. displacement curves for the global model and the facade model considering and neglecting the P-Delta effects. The difference between the two models is due to the different collapse mechanisms shown in Figure 5. In particular, with the DMEM-F model, the collapse mechanism involving portions of the orthogonal walls cannot be caught.

In order to investigate the differences between the global and simplified models, and the role of lateral walls on the response of the façade, a parameter α “bond coefficient”, ranging from 0 to 1, and multiplying the strength and ductility capacity of the non-linear links that simulate the presence of lateral walls, is considered (the results reported in Figures 4 and 5 can be seen associated with the value $\alpha=1$).

Quote of the link z [mm]	Tributary area Δz [mm]	Axial force N [kN]	Elastic stiffness K [kN/mm]	Ultimate force F_u [kN]	Ultimate displacement d_u [mm]
692	692	3.19	2.987	4.48	285
1505	813	6.94	4.489	6.73	285
2329	824	10.75	6.011	9.01	285
3148	819	14.53	7.523	11.28	285
4110	962	18.97	9.300	13.95	285
5365	522	21.38	10.264	15.39	285
4632	733	24.77	11.618	17.42	285
6108	743	28.20	12.990	19.48	285
6850	742	31.62	14.360	21.54	285
7398	548	34.15	15.372	23.05	285
7946	548	36.68	16.384	24.57	285
8494	548	39.21	17.3967	26.09	285
9042	548	41.74	18.408	27.61	285
9590	548	44.27	19.420	29.13	285
10228	638	47.22	20.599	30.89	285
10868	640	50.17	21.781	32.67	285
11508	640	53.13	22.962	34.44	285

Table 2: Calibration of the lateral non-linear links according to Eq. (1).

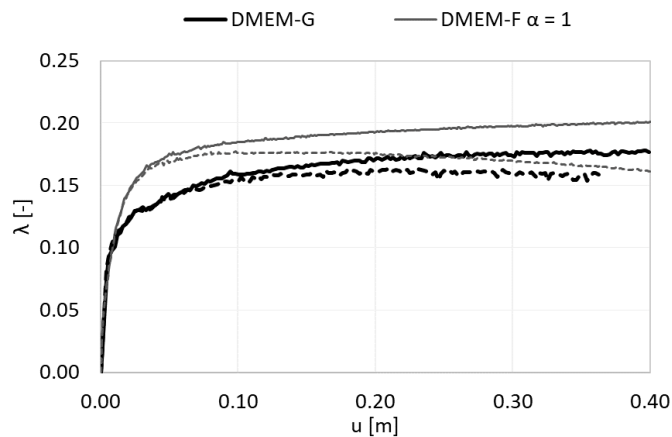


Figure 4: Pushover curves considering (continue lines) and neglecting (dashed lines) P-Delta effects.

Figure 6 shows the pushover curves obtained varying the value of α . In Figure 5 and 7 the color map shows the ratio between the maximum plastic deformation over the ultimate plastic deformation at the last step of the analyses for the two investigated models. It is worth noting that for $\alpha \geq 0.5$ (Figure 7a and 7b) the collapse mechanism does not involve the failure of the

links, and the façade fails by activating a vertical central crack. Conversely, values of $\alpha < 0.5$ (Figure 7c) lead to the yield of a significant number of links, consistent with the damage observed in the lateral walls of the global model (Figure 5a) and a façade damage pattern similar to that observed in the façade of the DMEM-G model. These results may be justified by the fact that the global model allows for combined shear-flexural mechanisms involving the lateral walls due to the low axial load level. In contrast, the facade model considers a pure cohesive-friction mechanism. Comparing the curves obtained by including and neglecting P-Delta effects, it can be noted that they start diverging at about 0.1m of displacement (9% of the wall thickness). After that level of displacement, the capacity curves of the analyses with P-Delta effects show a softening behaviour, while the analyses without P-Delta effects show a hardening response, leading to a significant overestimation of the load capacity of the system, with higher errors observed in the case of weak connections (low values of α). Finally, Figure 8 shows the axial stresses of the normal nonlinear links. It can be observed that the highest links (with the smallest abscissa z) are those subject to the greatest deformations, reaching their limit of ductility capacity in the case of $\alpha < 0.5$, where the axial stress goes to zero (indicated with the white colour in Figure 8).

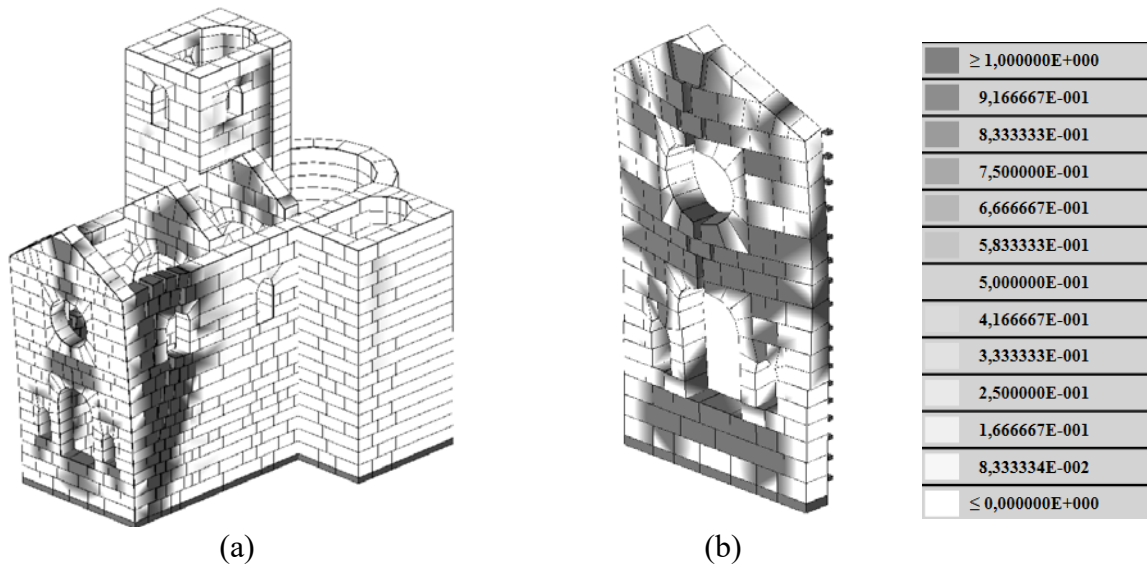


Figure 5: Distributions of plastic deformations at the last step of the analysis for the a) DMEM-G model and b) DMEM-F model.

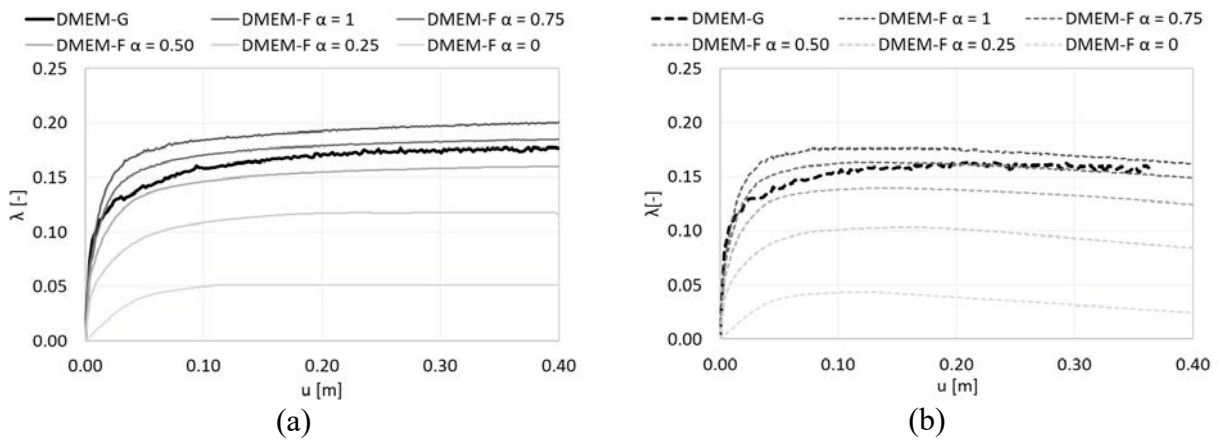


Figure 6: Pushover curves for the DMEM-G model and the DMEM-F model varying the quality factor α : a) considering and b) neglecting P-Delta effects.

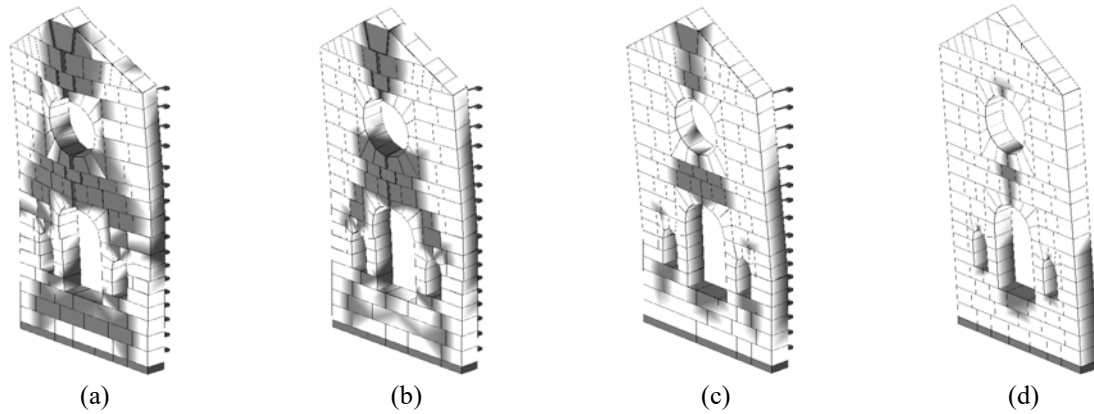


Figure 7: Collapse mechanisms of simplified models for a) $\alpha = 0.75$; b) $\alpha = 0.50$; c) $\alpha = 0.25$; d) $\alpha = 0$.

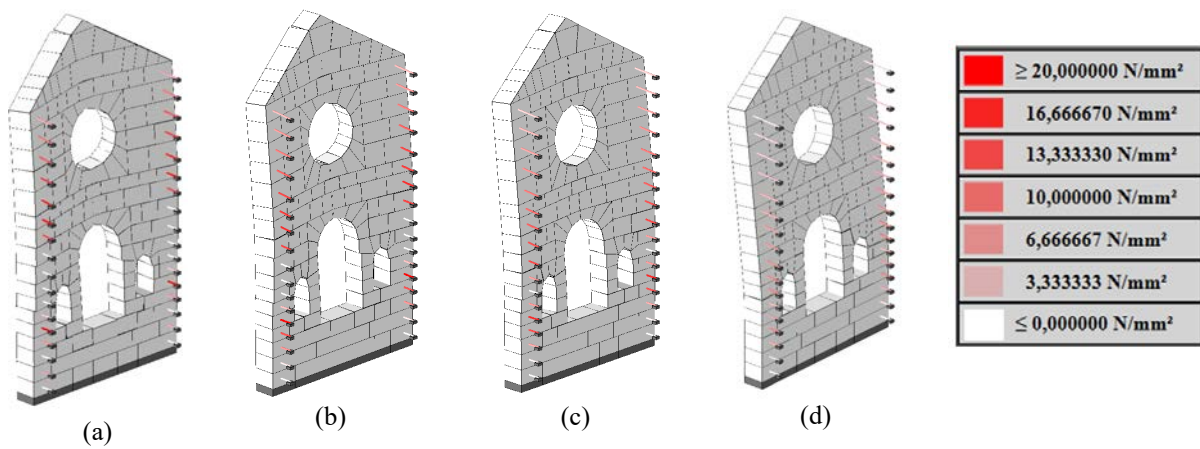


Figure 8: Stresses in the links of the simplified model for a) $\alpha = 1$; b) $\alpha = 0.75$; c) $\alpha = 0.50$; d) $\alpha = 0.25$.

4 CONCLUSIONS

The paper investigates the non-linear response of masonry church façades subjected to seismic rocking mechanisms, accounting for the stabilising contribution of retaining walls and the effects of geometric nonlinearities in the analyses. With reference to the case study of the *San Nicolò di Capodimonte* Church (Italy), a recently-proposed P-Delta discrete macro-modelling strategy, is used to develop two models, namely a 3D model of the entire church accounting for the unit interlocking between the façade and retaining walls, and a simplified model of the façade in which a discrete distribution of non-linear links with an hoc calibration accounts for the contribution of the retaining walls. Both models are analysed through push-over analyses performed along with the direction orthogonal to the façade to: *i*) assess the role of lateral walls; *ii*) validate the simplified model of the façade; *iii*) assess the role of P-Delta effects on the façade response. With this aims, the analyses are performed considering and neglecting the P-Delta effects and considering different strength and ductility values for the facade-to-lateral-wall connections. For the investigated case study, the P-Delta effects become significant at a level of displacement magnitude of the control point of the façade approximately of 10% of the masonry thickness, with larger discrepancies between the two models observed in the presence of weaker connections. These results confirm that neglecting the P-Delta effects may lead to significant errors in the ultimate limit checks, for which the displacement threshold mentioned above is easily reached or passed by the seismic demand. Finally, the comparisons between the global model and the simplified one show a reasonable

accuracy of the simplified model, with the introduction of a bond coefficient, requiring a significantly lower computational effort, so confirming its applicability for assessing large facades.

5 ACKNOWLEDGEMENTS

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