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MULTI-SCALE TOPOLOGY OPTIMIZATION FOR INNOVATIVE 3D-PRINTED WALLS AND SHELL STRUCTURES

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Abstract

Topology optimization is a computational design tool that allow to optimize specific properties in a design domain imposing a priori conditions. A common topology optimization formulation adopted for civil engineering problem is the minimization of compliance, which is equivalent to maximize the stiffness. In this work, we propose a homogenization-based multiscale approach with a compliance minimization formulation for large-scale 3D printing of innovative in-plane loaded walls and shells for building engineering. This approach entails a two-dimensional structural optimization scheme, that accounts for the presence of predefined microstructures and different material properties. Afterwards, the three-dimensional layout of the optimized structure is reconstructed at the micro-scale starting from the obtained optimal layout by means of a specifically tailored 2.5-D post-processing algorithm. The proposed multiscale topology optimization approach is demonstrated by several meaningful numerical examples.

Keywords: 2D topology optimization, multiscale approach, spinodal cells architectures

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1 INTRODUCTION

Topology optimization is a mathematical technique used in engineering to identify optimal material layout. It uses a combination of an optimization algorithm and finite element analysis to determine the most efficient distribution of materials within a defined domain subject to a set of performance constraints. The objective of this method, in general, is to obtain light-weight, reduced cost structures. In addition, the modularity of topology optimization algorithms allows to integrate other specific functional requirements, such as stiffness, thermal conductivity or acoustic absorption. This method, recently, has gained interest in civil engineering with the diffusion of Large-Scale Additive Manufacturing (LSAM) technologies, thus reducing the gap between theory and applications. Particularly interesting and challenging for large-scale structures is the so-called multiscale topology optimization, which has been revived due to the development of 3D printing. Indeed, this method finds its origin in the work of Bendsøe and N. Kikuchi [1] and actually has been applied end extended a numerous kind of application. Such multi-material formulation has been proposed to allow the optimization of structures to account for the presence of cellular micro-architectures. Moreover, the optimization of this kind of structures with transitions between different microstructures allow the design of functionally graded materials (FGMs) structures [2], which have heterogenous features through the structure and the smooth transition between cells increases bonding strength and reduces stress concentration [3]. In the field of civil engineering, such methodology has been proposed considering both periodic [4] and non-periodic cell [5] architectures. Periodic cell architectures involve repeating the same unit cell pattern throughout the structure, while non-periodic cell architectures allow for more flexibility in the design. Both approaches can generate hierarchical structures with smooth transitions between each part, resulting in FGMs with tailored material properties. Here, a 2D multiscale topology optimization approach is proposed for in-plane-loaded walls and shell structures equipped with non-periodic spinodal cell micro-architectures [6], while suitably post-processing the results to create a specifically tailored 2.5-D structures [7].

2 METHODOLOGY

The proposed homogenization-based multiscale topology optimization approach [8] relies on on a multi-material formulation compliance-based with volume constrained. The topology optimization problem can be characterized as follows:

$$\begin{aligned} &\min_{\mathbf{Z}\in\left[\underline{\rho},\bar{\rho}\right]^{Nxm}} C = \mathbf{F}^T\mathbf{U} \\ &\text{such that } \mathbf{KU}=\mathbf{F};\ g_j \leq \bar{v}_j \end{aligned} \tag{1}$$

where C is the compliance, which is the objective function of this problem, and g_j is the volume constrain. The constraint g_j is defined for each material $j = 1,...,K$ as follows:

$$g_j = \frac{\sum_{i\in G_j}\sum_{l\in\mathcal{E}_j} A_l m_V}{\sum_{l\in\mathcal{E}_j} A_l} \tag{2}$$

which through the sets $\mathcal{E}_j$ (element indices) and G_j (material indices) control the selection of the candidate material j in a specific sub-region of the domain and the constraint is verified through the volume fraction $\bar{v}_j$. Additionally, the constraint is given by the area A_l of each element, $l = 1,...,N$ and the interpolation function $m_V = y_{li}$, where y_{li} appoint the existence of the discretized element.

The solution of the state equation $\mathbf{KU} = \mathbf{F}$, computing the load-vector $\mathbf{F}$ and the stiffness matrix $\mathbf{K}$, provides the displacement field $\mathbf{U}$. The local stiffness matrix $\mathbf{k}_l$ is defined as:

$$\mathbf{k}_l = m_M = \sum_{i=1}^{m} y_{li}^{P} \prod_{\substack{j=1 \\ j \neq i}}^{m} \left(1 - \gamma y_{lj}^{p}\right) \mathbf{k}_{li}^{0}, l = 1,...,N \tag{3}$$

which allocated the materials properties through the material interpolation m_M [9] defined as $m_W = y_{li}^{p}$, where p penalizes the intermediate densities, the mixing parameter γ and the element stiffness matrix $\mathbf{k}_{li}^{0}$:

$$\mathbf{k}_{li}^{0} = \int_{\Omega_l} \mathbf{B}_j^T \mathbf{D}_i^H \mathbf{B}_k d\mathbf{x} \tag{4}$$

where $\mathbf{B}$ describes the strain-displacement matrix, Ω_l is the domain of each element and $\mathbf{D}_i^H$ is the elasticity matrix in Voigt notation of each material. This tensor is provided through computational homogenization [10] and contains information about the effective properties of the microstructure.

The topology optimization problem is solved through a gradient-based method with the Zhang-Paulino-Ramos Jr (ZPR) scheme [11]. This update scheme is capable of handling a wide range of candidate materials with different unit cell volume $\hat{v}_i$, i.e. material porosity, properties and microstructure. It also has the ability to handle multiple constraints, which is particularly necessary for solving multi-material problems. The sensitivities to handle this update scheme are reported here:

$$\frac{\partial f}{\partial \mathbf{x}_i} = \frac{\partial \mathbf{y}_i}{\partial \mathbf{x}_i} \frac{\partial \mathbf{w}_i}{\partial \mathbf{y}_i} \frac{\partial f}{\partial \mathbf{w}_i} = \mathbf{P}^T \frac{\partial \mathbf{w}_i}{\partial \mathbf{y}_i} \frac{\partial f}{\partial \mathbf{w}_i} \tag{5}$$

$$\frac{\partial g_j}{\partial x_i} = \frac{\partial \mathbf{y}_i}{\partial \mathbf{x}_i} \frac{\partial \mathbf{V}_i}{\partial \mathbf{y}_i} \frac{\partial g_j}{\partial \mathbf{V}_i} = \mathbf{P}^T \frac{\partial \mathbf{V}_i}{\partial \mathbf{y}_i} \frac{\partial g_j}{\partial \mathbf{V}_i} \tag{6}$$

where (5) and (6) are respectively the sensitivities of objective and constraint function. Moreover, $\mathbf{P}$ is the density filter matrix [12], and the individual components of sensitivities are:

$$\frac{\partial w_{kj}}{\partial y_{li}} = \begin{cases} p y_{li}^{p-1}, & \text{if } l = k \text{ and } j = i \\ 0, & \text{otherwise} \end{cases} ; \frac{\partial f}{\partial w_{li}} = -\mathbf{U}^T \frac{\partial \mathbf{K}}{\partial w_{li}} \mathbf{U} \tag{7}$$

$$\frac{\partial v_{kj}}{\partial y_{li}} = \begin{cases} \hat{v}_i, \text{ if } l = k \text{ and } j = i \\ 0, \text{ otherwise} \end{cases} ; \quad \frac{\partial g_j}{\partial v_{li}} = \frac{\mathrm{A}_l}{\sum_{l \in \mathcal{E}_j} \mathrm{A}_l} \tag{8}$$

$$\frac{\partial \mathbf{k}_k}{\partial w_{li}} = \begin{cases} \prod_{\substack{j=1 \\ j \neq i}}^{m} \left(1 - \gamma w_{lj}\right) \mathbf{k}_{li}^0 - \sum_{\substack{p=1 \\ p \neq i}}^{m} \gamma w_{lp} \prod_{\substack{r=1 \\ r \neq p \\ r \neq i}}^{m} \left(1 - \gamma w_{lr}\right) \mathbf{k}_{lp}^0, \text{ if } l = k \\ 0, \qquad \text{otherwise} \end{cases} . \tag{9}$$

With these quantities, it is possible to determine the update of the design variable by using the candidate design variable:

$$x_{li}^* = \underline{\mu} + \left(\frac{\left.\frac{\partial f}{\partial x_{li}}\right|_{\mathbf{Z}=\mathbf{Z}^0}}{+\lambda_j \left.\frac{\partial g_j}{\partial x_{li}}\right|_{\mathbf{Z}=\mathbf{Z}^0}} \right)^{\frac{1}{1+\alpha}} \left(\sum_{k=1}^{N} P_{lk} x_{ki}^0 - \underline{\mu} \right) \tag{10}$$

where the Langrange multipliers λ_j is obtained through bisection method:

$$g_j\left(x_i^k\right) + \sum_{i \in M_j} \frac{\partial g_j}{\partial x_i}\left(x_i^k\right)^T \left(x_i\left(\lambda_j\right) - x_i^k\right) = 0 \tag{11}$$

Finally, the candidate design variable is verified as follows:

$$x_{li}^{new} = \begin{cases} x_{li}^+, \; x_{li}^* \geq x_{li}^+ \\ x_{li}^-, \; x_{li}^* \leq x_{li}^+ \\ x_{li}^*, \text{ otherwise} \end{cases} . \tag{12}$$

where $x_{li}^- = \max\left(x_{li\,min}, x_{li}^0 - M\right)$ and $x_{li}^+ = \min\left(x_{li\,max}, x_{li}^0 + M\right)$ with M as move limit.

3 RESULTS

The spinodal topologies defined in [6] are implemented through an homogenized elasticity tensor. These cells architectures are characterized to be non-periodic with tunable anisotropy and they are described mathematically through a Gaussian random field (GRF) [13]:

$$f\left(\mathbf{x}\right) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} cos\left(\kappa \mathbf{n}_i \cdot \mathbf{x} + \gamma_i\right) \tag{13}$$

as $\mathbf{n}_i \in U\left[\mathbb{S}^2\right]$, $i = 1,...N$, the wave vectors, N the number of waves, κ the wavelength and γ_i the phase shift randomly sampled. The anisotropy is tuned the restriction of wave vectors through cone angles [6] $\theta_1, \theta_2, \theta_3 \in \left[0, \frac{\pi}{2}\right]$. The microstructures considered in this contribution are isotropic, columnar and lamellar. These non-periodic cells are implemented in the 2D-case study depicted in Fig. 1 considering only the components of the homogenized elasticity matrix that exist in the correspondent plane of the macrostructure. The 2D structure (*L=2*; *H=1)* is loaded with a vertical distributed *q* in a X-Y plane as shown in Fig. 1.

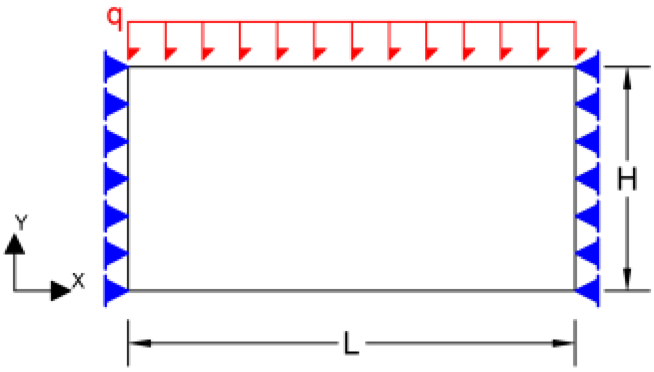


Figure 1: Geometry, load and boundary conditions

The homogenized mechanical features are obtained as the average of 20 realizations of the phase field to account for the randomness of this type of microstructures. The elastic surface of the relevant components for the 2D problem are reported in Table 1. The smooth transition between each candidate material is enforced by means of the following relation:

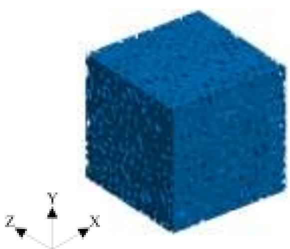
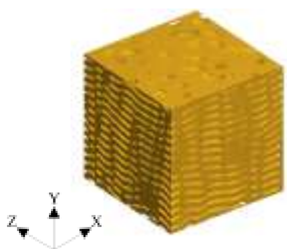
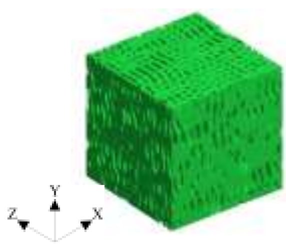
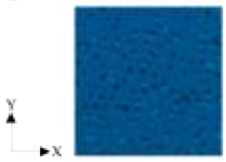
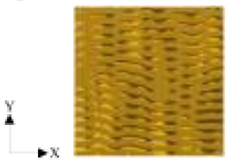
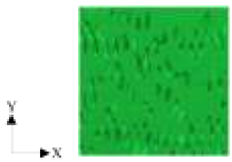
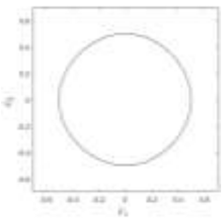
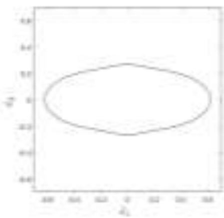
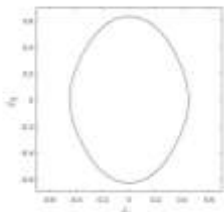
Isotropic	Lamellar	Columnar
		
		
		

Table 1: 3D view, 2D view (X-Y) and elastic surfaces of spinodal topologies

$$f_l(\mathbf{p}_l)=\frac{\sum_{i=1}^{m} max\left[0,1-\frac{d_{min}(\mathbf{p}_l,S_i)}{R_\phi}\right]^{1/2} f_{li}^0(\mathbf{p}_l)}{\sum_{i=1}^{m} max\left[0,1-\frac{d_{min}(\mathbf{p}_l,S_i)}{R_\phi}\right]^{1/2}}, \tag{14}$$

where f_l is a function that interpolate the phase fields $f_{li}^0(\mathbf{p}_l)$ of each material. Every phase field is defined through a discrete version of (13), where $\mathbf{p}_l$ is a point of a set S_i, which contain all points of a candidate material. The proposed approach has been applied considering one global volume constraint, that controls all elements and candidate materials, and three volume constraints, one for each of three candidate materials. The optimal layouts are respectively represented in Figg. 2-3. Subsequently, the optimal layouts are post-processed in 3D to visualize the spinodal architectures:

Figure 2: Optimal layout with one global volume constrain

Figure 3: Optimal layout with three global volume constrains

Subsequently, the optimal layouts are post-processed in 3D to visualize the spinodal architectures, as shown in Figg. 4-5.

4 CONCLUSIONS

This work serves as a starting point for the development of a multi-scale topological optimization approach for structures endowed with predefined microstructures, with the goal of producing hierarchical components for civil structures through large-scale additive manufacturing techniques. Moreover, this approach is meant to be applied to more complex structures, in particular posing attention to structures that are mainly under compression, e.g. vaults and shells.

Figure 4: Optimal layout with one global volume constrain: a) front view (XY) b) lateral view (XZ-YZ)

Figure 5: Optimal layout with three global volume constrain: a) front view (XY) b) lateral view (XZ-YZ)

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