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CASE STUDY APPLICATIONS OF PERFECTLY MATCHED LAYERS IMPLEMENTED IN THE ANALYSIS OF VIBRATIONS INDUCED BY TRAIN PASSING

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Abstract

Two case study applications of perfectly matched layers (PML) combined with finite elements are demonstrated. The first case concerns a vibration analysis of an office building to be constructed in close vicinity to rail tracks. A rigorous finite element model, including solid elements for representation of the soil, was built and used for estimating vibrations transferred to the building from a passing train. The size of modelled soil domain was kept within reasonable limits thanks to the wave absorbing properties of the PML's. The second case is a part of a train-bridge interaction analysis related to the train's transition from an embankment to a bridge. Due to the unusually high maximum operational velocity foreseen (360 km/h), vibrations in the train's body and displacements in the transition zone required special attention. Modelling of the soil within the transition zone was again achieved with the above described approach. A finite element model combining both the bridge and transition was built. This allowed the estimation of response quantities, which otherwise cannot be obtained with a more simplified modelling approach.

Keywords: Vibration control, Train-track interaction, Wave propagation, Non-reflecting boundaries.

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1 INTRODUCTION

In civil engineering practice, the challenge of modeling vibration propagation through an infinite soil domain is a recurring issue. Typically, numerical modeling is used to investigate the interaction between engineered structures and infinite soil domains, with a particular focus on soil-structure interaction during seismic events. As a result, there are numerous standards and research studies dedicated to this problem. In contrast, modeling traffic-induced vibrations is not as commonly encountered, leading to a smaller number of standards and recommendations and a less well-established procedure for numerical modeling.

This paper aims to shed more light on the issue by presenting two case studies that involve numerical modeling of traffic-induced vibrations in engineered structures. The first case study involves modeling the vibrations induced by a passing train on a currently designed office building. Vibrations are generated at the wheel-rail interface and transmitted via the track structure into the adjacent soil and nearby office building. The ultimate goal of the vibration assessment is to identify if the potential impacts on sensitive receivers exceed the acceptance criteria and consequently recommend mitigation measures. The second case-study deals with the dynamic analysis of a high-speed train going over a transition from earthworks to a bridge structure. Such transitions along a rail track are usually accompanied by a change in the track stiffness. Special measures have to be undertaken both in the design and construction phases in order to avoid a sharp change in the track stiffness. A sharp increase or a decrease in the stiffness might lead to passenger discomfort and increased dynamic loading on the track.

In both of the above described analyses a finite element model is built combining the civil structure of interest and the surrounding soil domain. For the modelling of the structures beam and shell elements are implemented, whilst the soil is represented entirely by solid elements. More details on the modelling approach are provided in Sections 3 and 4, but here it is important to mention that perfectly matched layers (PML) are used for the efficient representation of unbounded soil domains as bounded ones. With the introduction of PML's at the boundaries of the finite element soil models the incident waves are absorbed effectively simulating an unbounded domain at a relatively low computational cost. Further details can be found in Section 2.

The paper has the following structure:

(ii) Introduction to the theory behind perfectly matched layers
(iii) Case-study 1 - Vibration assessment in an office building near train tracks
(iv) Case-study 2 - Dynamic analysis of a high-speed train transitioning from earthworks to a bridge
(v) Conclusions

All analyses presented herein are performed with the finite element software LS-DYNA. Therefore the theoretical background and model definitions will be described with reference to the functionalities of LS-DYNA.

2 PERFECTLY MATCHED LAYERS

2.1 Backround

PMLs are a numerical method used to simulate wave behavior in unbounded domains. The technique involves adding a layer of material around the domain of interest that can absorb waves attempting to pass through it, creating a boundary condition that mimics an infinite domain. The key to PMLs is ensuring that the absorbing layer matches the wave impedance of the surrounding medium, which facilitates a smooth transition between the PML and the domain of interest. This is crucial because any mismatch in the impedance could result in wave energy being reflected back into the domain of interest, leading to unrealistic estimations of the response. The absorbing layer is typically implemented as a complex-valued material with a variable elastic modulus that gradually increases towards the boundary of the domain. The complex nature of the material allows it to absorb waves with different frequencies, while the variable elastic modulus ensures that the absorption is gradual and doesn't cause any unwanted wave reflections.

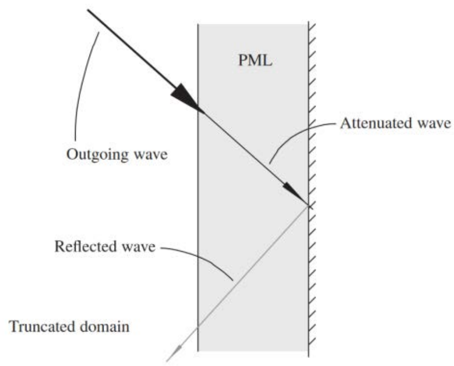


Figure 1: A PML adjacent to a truncated domain attenuates and reflects back an outgoing wave [5]

PML was initially developed for electromagnetic waves in seminal works by Bérenger and Chew [7]. Since then, extensive research has been conducted on electromagnetic PMLs by numerous researchers, with extensions to other fields such as elastic waves for seismic applications. However, most of the formulations and implementations used finite-difference split-field methods to implement the PML, which had two disadvantages. Firstly, the finite-difference methods could not be easily used with finite-element models for structures, and secondly, the split-field formulation often led to long-time instability. To overcome these shortcomings, Başu and Chopra [5] developed a displacement-based finite-element implementation for elastic PMLs. This allowed for explicit analysis, enabling realistic analysis of three-dimensional soil-structure systems. Exactly this numerical procedure is applied in the current paper. A brief theoretical explanation of the main concept behind the implemented numerical solution is provided hereafter.

2.2 Basics

Consider a simple model of a semi-infinite rod unbounded on the right side (Figure 2). The equations describing the elastic medium of the rod can be transformed into equations for a perfectly matched layer (PML). The PML is designed mathematically to dampen waves by utilizing a damping function *f(x)* that increases in the unbounded direction. The PLM is placed next to the bounded rod and absorbs all incident waves in the direction of the unbounded medium.

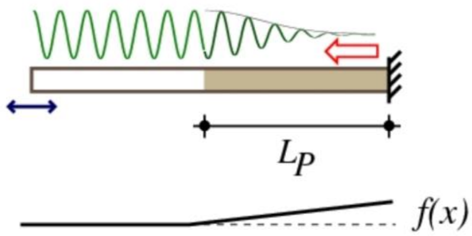


Figure 2: PML concept [6]

The PLM is truncated once the outgoing waves are sufficiently damped. A partial reflection of the wave will occur with the amplitude of the reflected wave given by:

$$|R| = \exp[-2F(L_P)], \quad F = \int f\,\mathrm{d}x \tag{1}$$

The amplitude is controlled by f and L_p and can be made as small as desired. The attenuation function is according to the equation:

$$f(x) = f_0\left(\frac{x}{L_P}\right)^m \tag{2}$$

$m = 2$ works best for finite element analysis and f0 is chosen from a simplified discrete analysis. An automatic estimation of f0 is performed in LS-DYNA according to the depth of the layer. The depth Lp of the layer may be chose so that the layer is about 5-8 elements deep, with the mesh density of the PML similar to that of the bounded medium. The PML material is forms a parallelepiped around the bounded domain, which is aligned to the coordinate axes. The outer boundary of the PML is fixed.

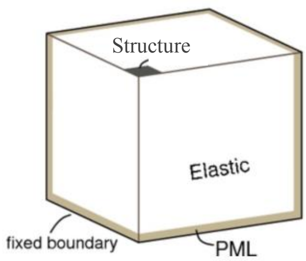


Figure 3: Unbounded domain representation [6]

3 CASE-STUDY 1 - VIBRATION ASSESSMENT IN AN OFFICE BUILDING NEAR TRAIN TRACKS

3.1 General

Noise & Vibration (N&V) are an important aspect to be assessed when building in near proximity to rail tracks. N&V are generated at the wheel-rail interface and are transferred via the track to the soil and the adjacent buildings. Inside those buildings vibrations could be perceptible to the inhabitants and cause discomfort and/or induce structure borne noise radiated by surrounding walls and ceilings. The acceptance criteria for N&V are defined according to the local regulations. A detailed analysis for vibrations and ground borne noise generally uses all available tools to accurately predict the vibration impact on specific sites. The procedure includes defining the forces generated by the vibration source and defining how the local geology affects vibration propagation. Furthermore, details of structural dynamics of the affected buildings need to be assessed to predict the impact on objects inside the building. In this case study the analysis of an office building to be built in Köln, Germany nearby existing rail tracks is presented. The main objective of the numerical analysis was to conclude if mitigation measures related to N&V are required. Project restraints allowed only for mitigation measures applied on the building. After a close cooperation with the designer of the building it was concluded that the only feasible mitigation measure for the specific project is a trench along the perimeter of the building facing the tracks. It was decided to perform the analysis first with no trench included and then with the trench. Although, as concluded later, no mitigation measures were required the analysis with a trench was performed as a helpful reference for future projects.

3.2 Validation of the numerical model and determination of the train loading function

Since the dynamic loads induced in the track by the train are unknown, the following approach is applied:

1) Estimation of the transfer function between the load application points (Figure 8) and a node corresponding to a channel *X* (*X* is an example value) from the model. The transfer function describes the behavior of the system in the frequency domain:

$$UF_X(f) \tag{3}$$

The transfer function is calculated by applying an impulse load, solving the response with explicit time-integration and then transferring the obtained response in the frequency domain.

2) The in-situ measured response for channel *X* ($A_{X,in-situ}$) is divided by the transfer function (*UF*) to give an estimate of the load function (L) in the frequency domain:

$$L(f) = \frac{A_{in-situ}}{UF_X} \tag{4}$$

3) For the validation of the model, the transfer function UF_Y is calculated between the load application points on the track and a node, which in turn corresponds to a channel in

the model. The load function (L) is multiplied by UF_Y and compared to the in-situ measured response $A_{Y,in-situ}$.

If the multiplication approximates well the in-situ measured response ($L * UF_Y \approx A_{Y,in-situ}$), it can be assumed that the numerical model is validated. If $A_{Y,in-situ}$ deviates from the numerical estimation a calibration of the model has to be performed.

4) The dynamic response of the building is obtained by multiplying $A_{Z,Neubau} = L * UF_Z$. UF_Z is the transfer function between the load application points on the track and the newly constructed building.
The above described procedure was implemented for model validation using several field measurements as references. Results for point MP6 (see Figure 4) are shown on Figure 5.The load *L(f)* is applied according to Figure 6 as an area load (force) in the vertical direction.

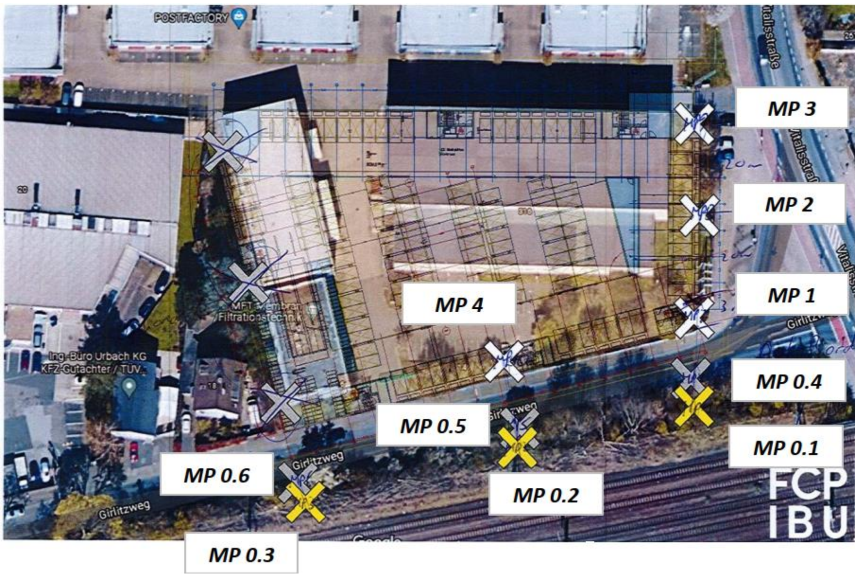


Figure 4: Measurement points on the free filed before construction of the building

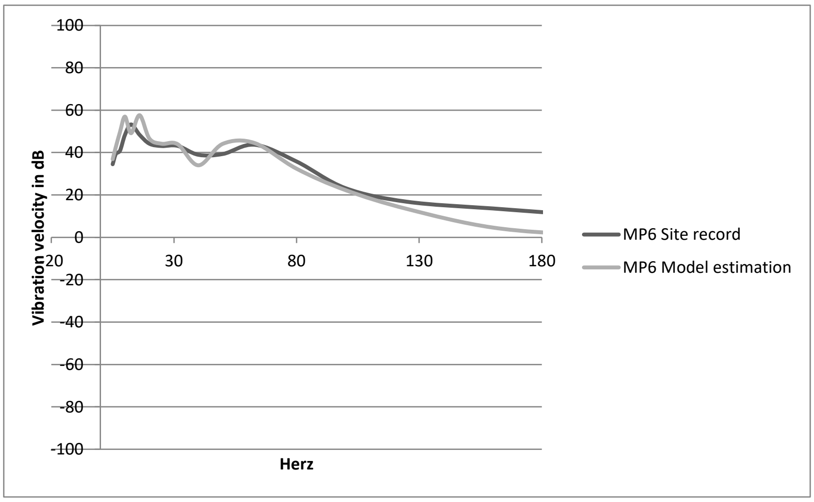


Figure 5: Comparison between estimated and measured vertical vibration velocity at the free field near the rail tracks

3.3 Estimating the vibrations in the building

By applying the already deduced in Section 3.2 train loading function L(f) along the length of the track (see load application on Figure 6) the vibration velocity spectra in the building are predicted.

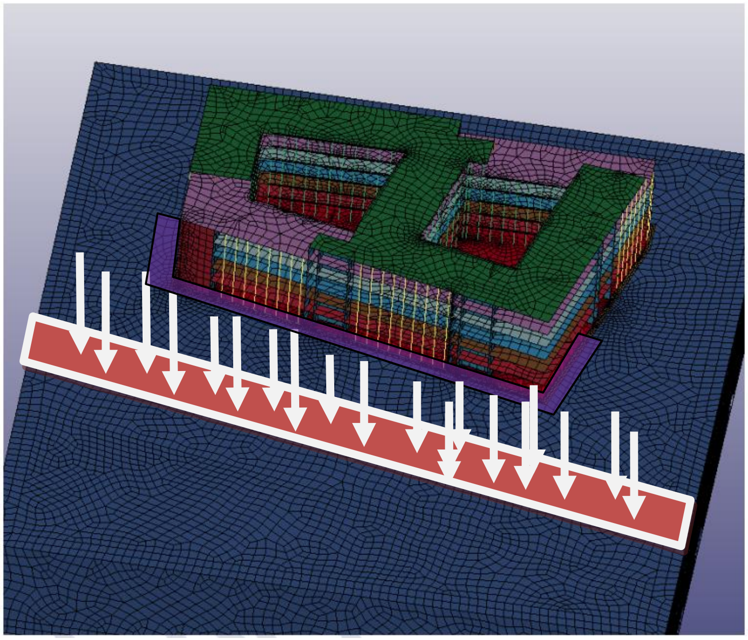


Figure 6: Finite element model of the analyzed building

Figure 7 presents the results **with** and **without** the presence of a trench for the second floor of the building.

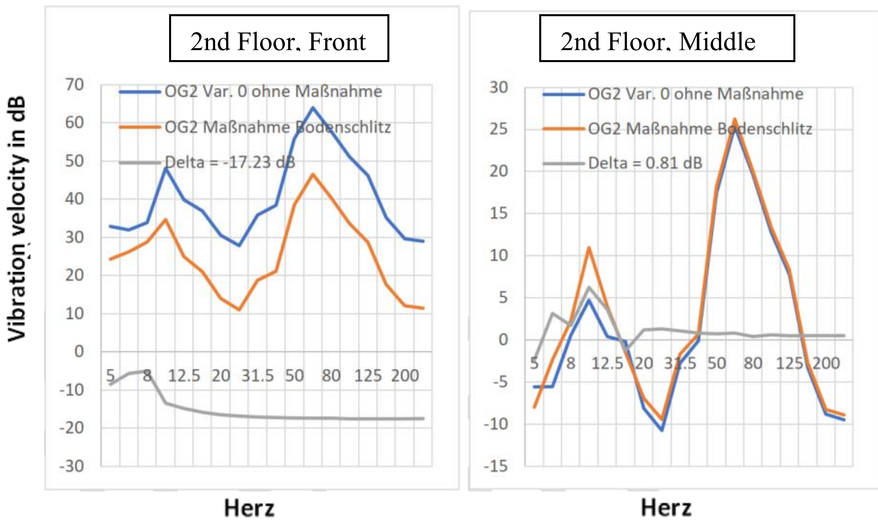


Figure 7: Vibration velocity spectra

From the above figure it can be concluded that the trench has a significant effect along the perimeter of the building facing the track and less so towards the middle of the building. This is mainly because in the middle of the building a very small part of the vibrations is transmit-

ted. The vibration is also reduced at the higher floors, which is expected due to wave distance attenuation.
Once the vibration velocity within the building is obtained an estimation of vibration and structure borne noise as perceived by the inhabitants has to be performed. The procedure is according to DIN4150 [2]. This part of the analysis is not included, as the focus of the paper is on the practical application of the PML's.

4 CASE STUDY 2 - DYNAMIC ANALYSIS OF A HIGH-SPEED TRAIN TRANSITIONING FROM EARTHWORKS TO A BRIDGE

4.1 General

As already elaborated on the transition zones between parts of a rail way track with pronounced changes in the vertical stiffness are usually problematic zones and require special attention. If the change in stiffness is too sharp this would lead to an increase of the dynamic contact forces acting at the wheel-rail interface. As a result passenger comfort might be compromised accompanied with an increased maintenance demands on the track. To avoid these undesired scenarios a smooth transition in the track stiffness has to be assured.

In this case-study the transition of a high-speed train between a slab track system placed on earthworks and a slab track system placed on a bridge is analyzed. Due to the exceptionally high foreseen operational velocity between 360 and 400 kph a detailed finite element analysis was deemed necessary.

The analysis is dynamic and performed in the time-domain. It includes in an explicit manner the stiffness, mass and damping of earthworks train and bridge. For the modelling of the earthworks part at the transition a solid element model is built. As the earthworks, or soil domain is again infinite PML had to be implemented. The application of the PMLs is according to the above described approach and shown on Figure 11 The bridge is modelled using beams, spring, dampers and constraints according to the description in Section 4.2. The train is modelled according to the multi-body dynamics approach with a detailed description in Section 4.4.

The main purpose of the analysis is to include a more detailed representation of the track component and potentially capture resonant effects, which cannot be captured by conventional modelling techniques. Such effects could be resonance due to the gap at the track slab and base layer, or exceptionally high dynamic amplifications at the transition between embankment and bridge. Further, the track slabs are supported on a polymer elastic layer used for noise mitigation. Analyzing the dynamic behavior with the elastic layer included for an operational velocity of 400 kph was also deemed as useful.

4.2 Bridge model

As previously described the bridge model consists entirely of beam elements, springs, dampers, masses and rigid constraints.

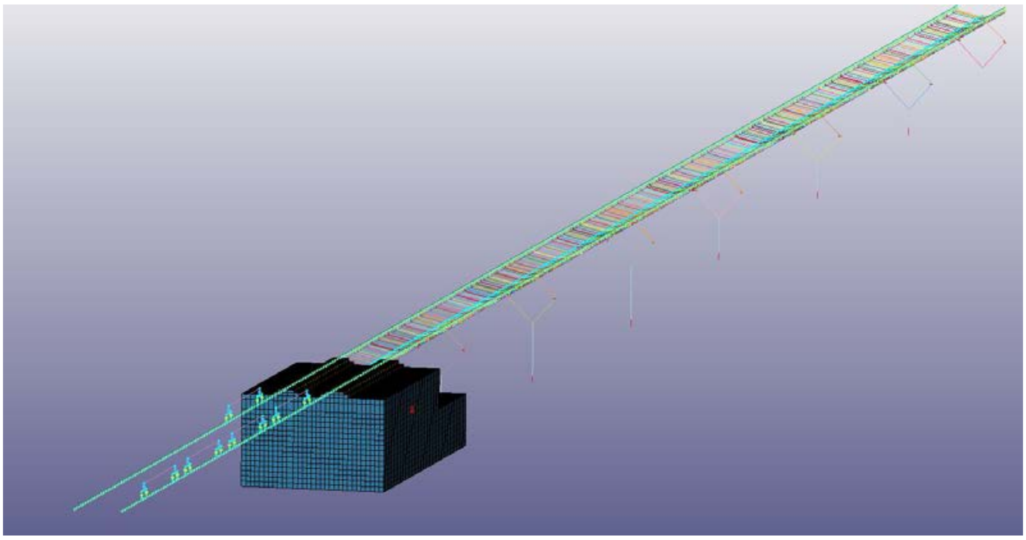


Figure 8: Finite Element Model define in LS-DYNA

A schematic view of the model definition is shown below:

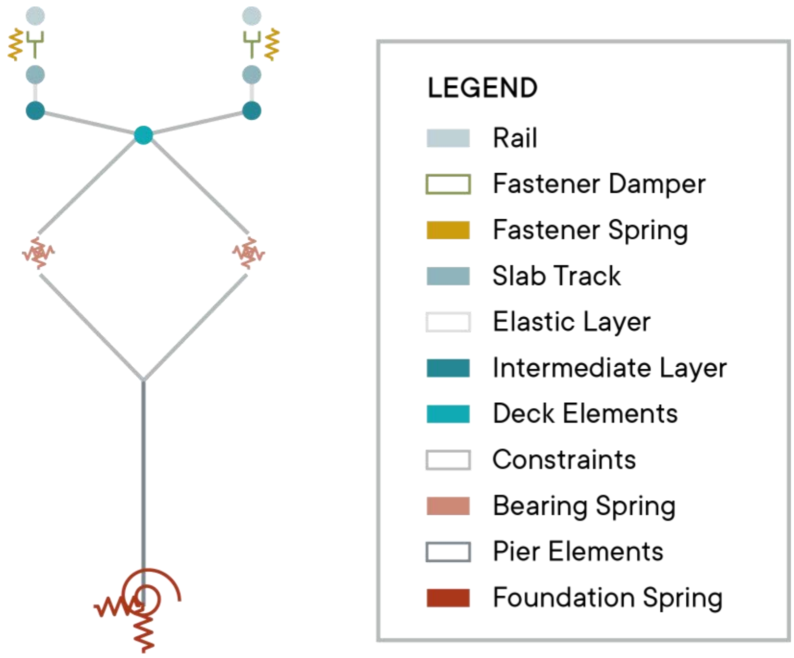


Figure 9: Schematic view of the FE model used for representing the bridge

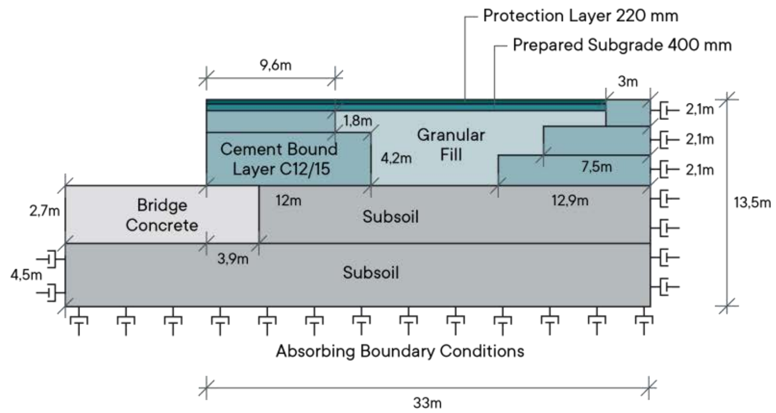
All properties used for the analysis were provided to Vienna Consulting Engineers ZT GmbH for the purpose of performing the dynamic analysis. It is worth noting that the impedance function of the foundation spring stiffness was not available. For this reason the analysis was performed only with the static stiffness accompanied by a sensitivity analysis

4.3 Modelling of earthworks (embankment)

A dense mesh of solid elements is used to model the transition zone (see Figure 11). PML's are used along the surrounding surface of the transition zone with the function of absorbing the waves being generated by the vehicle passing, and therefore, simulating an infinite elastic half-space. The mesh size is selected, such that a response of up to 30 Hz is captured.

Response in the higher frequencies is not considered necessary, since the first 5 dominant modes of the bridge are below 30 Hz (see criterion defined per Eurocode 1990).

A sketch of the transition zone containing geometric and physical properties is presented below.



	E [MPa]	γ	ρ [kg/m³]
Subsoil	200	0.3	1700
Embankment	300	0.3	1750
Granular Fill	300	0.3	1750
Prepared Subgrade 400 mm	350	0.25	1800
Protection Layer 220 mm	400	0.25	1900

Figure 10: Assumed geometric and material properties for the transition zone

The following figure shows the finite element model of the transition zone. The solid elements along the outer edge of the transition zone are the non-reflecting boundaries marked as dashpot damper on Figure 10.

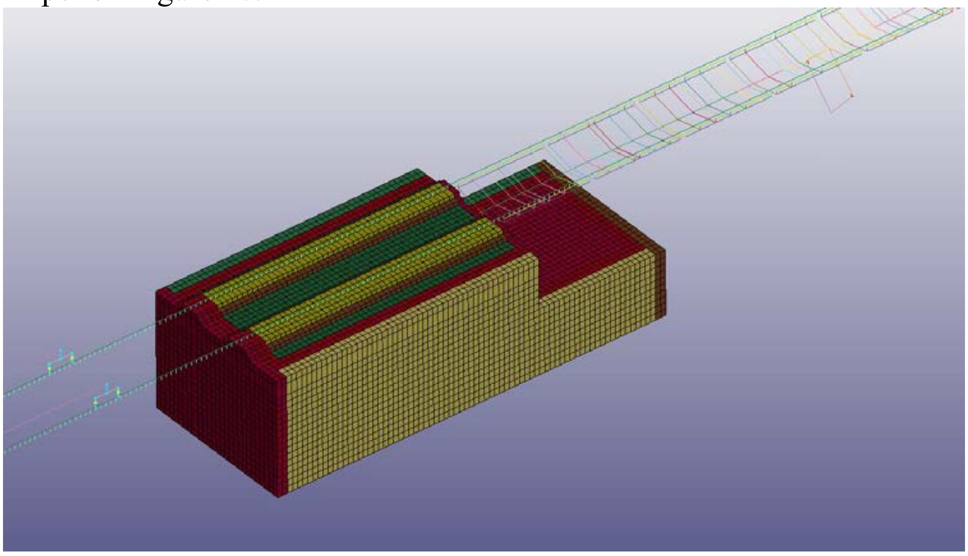


Figure 11: Finite element model of the transition zone

4.4 Model of the train

The train is modelled according to the multibody approach. This approach consists of representing the vehicle by a set of masses, springs, dampers, and constraints. This approach is widely accepted in research and professional engineering because of the good balance between accuracy and computational expenses.

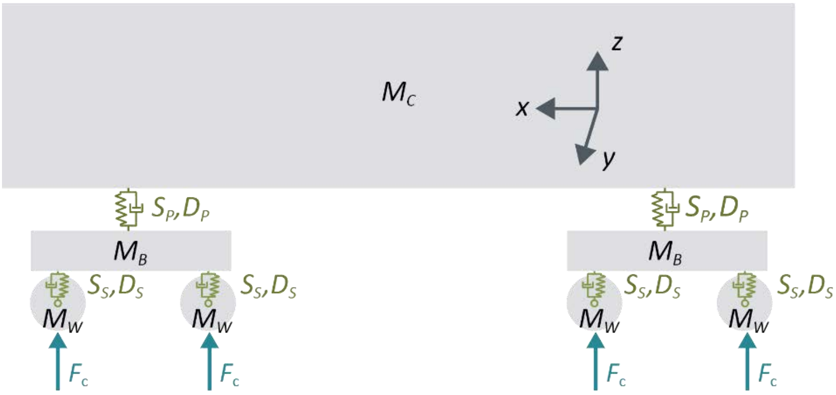


Figure 12 Multi-body representation of the train

The following variables are used:

M_C – mass of the car;
M_{CI} – inertia of the car at axis y
M_B – mass of a bogie;
M_{BI} - inertia of the bogie at axis y
M_W – mass of a wheel;
S_P – primary spring stiffness;
D_P – primary damper property;
S_S – secondary spring stiffness;
D_S – secondary damper property;
F_C – axle load;

Note: Only data relevant for the performed planar analyses is used. The moments of inertia at axes x and z are not of relevance since the performed analyses are two dimensional. The moments of inertia of the wheels have no effect on the results and therefore are not considered.

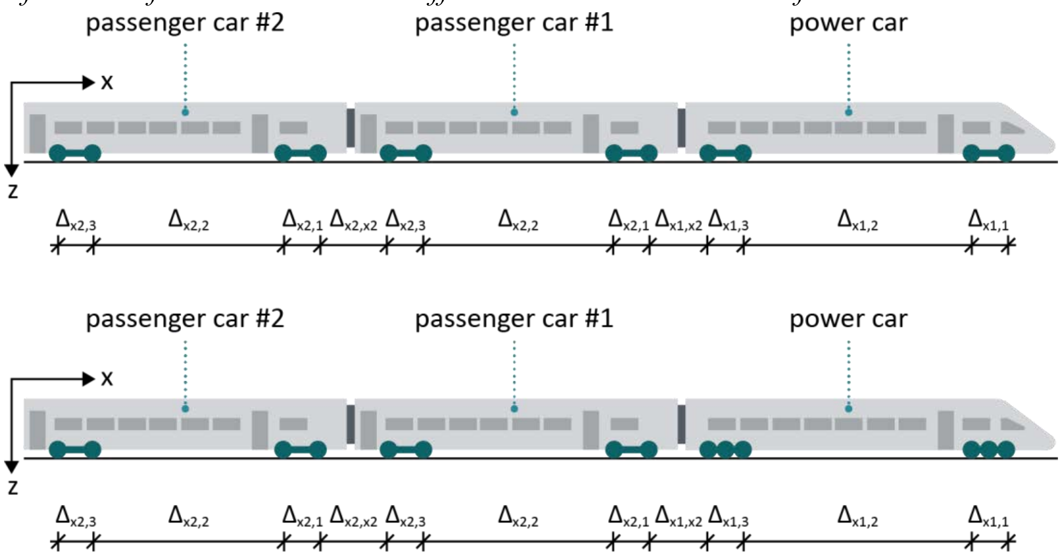


Figure 13: Sketch of the geometric parameters of the model

A total of one power car and three wagons are moving together over the bridge in the model:

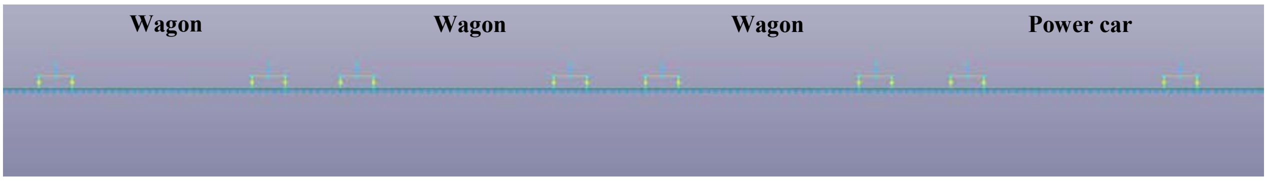


Figure 14: Multibody vehicle definition in LS-DYNA

The damping is defined according to Rayleigh model with coefficients corresponding to a damping ratio of 3% between for 1 and 50Hz.

4.5 Definition of rail irregularity

An irregularity profile generated from a power spectrum corresponding to a high-quality track is applied at the wheel rail contact. The power spectrum generation is according to [3]. Figure 14 depicts a single generation of a rail irregularity profile evaluated in the longitudinal direction of the rail. The ordinate shows the vertical deviation of the rail from the perfect rail geometry.

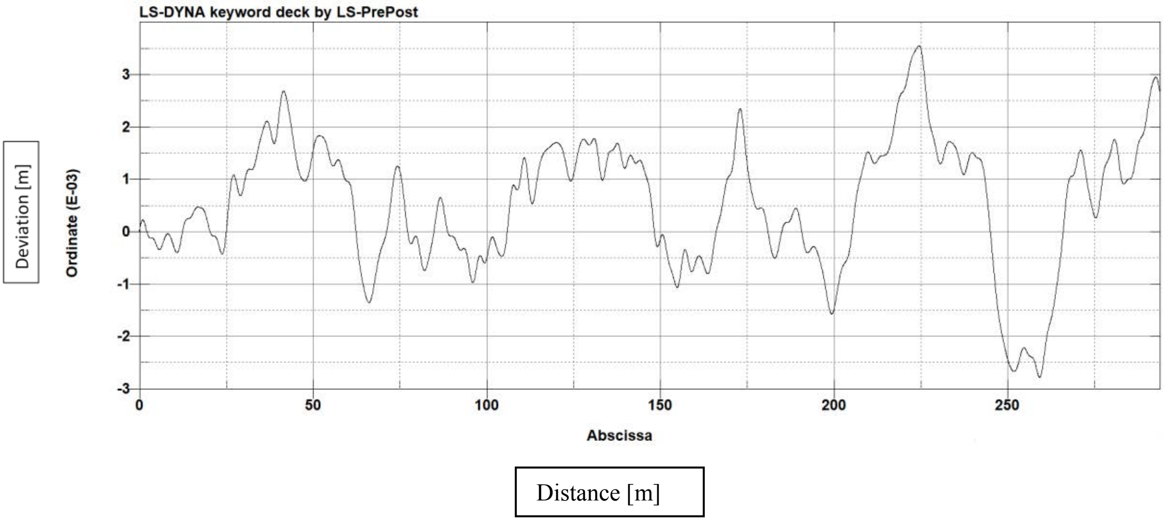


Figure 15: Single realization of a rail irregularity profile

4.6 Contact between wheel and rail

A penalty based contact is used to constrain the wheel loads to moving along the rails. For every time step a search for penetration between rail and wheel is performed. When a penetration is found a force proportional (VERTSTF) to the penetration depth (D) is applied to reverse the penetration.

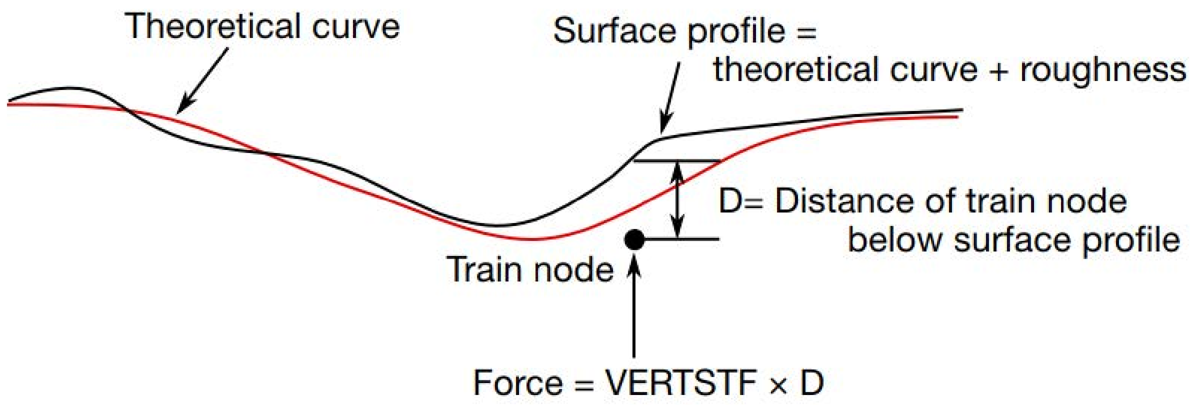


Figure 16: Contact definition according to the penalty method (extracted from LS-DYNA reference manual [4])

4.7 Results

The response is extracted for velocities between 50 km/h and 360 km/h. The dynamic amplification for a velocity of 50 km/h is negligible. Additionally no rail irregularities are introduced for the analysis with 50 km/h. Therefore, it can be assumed that the analysis with a velocity of 50 km/h has no meaningful dynamic amplification and can be used as a basis for calculating the amplification at 360 km/h and 400 km/h (in other words the dynamic response at 50 km/h is a good approximation of the static response). The following figures present the results for velocities of 50 km/h, 360 km/h and 400 km/h. Results at 360 km/h and 400 km/h are extracted for 3 different irregularity profiles for the purpose of estimating the sensitivity of the response to rail irregularity.

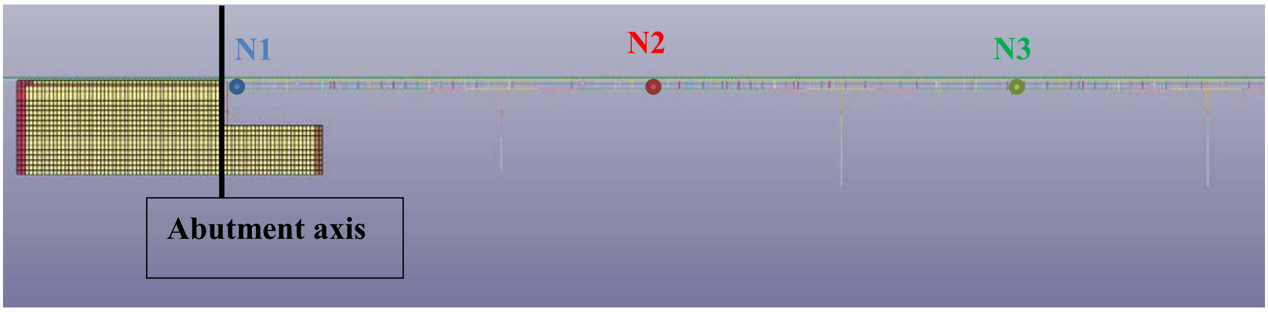


Figure 17: Result extraction points

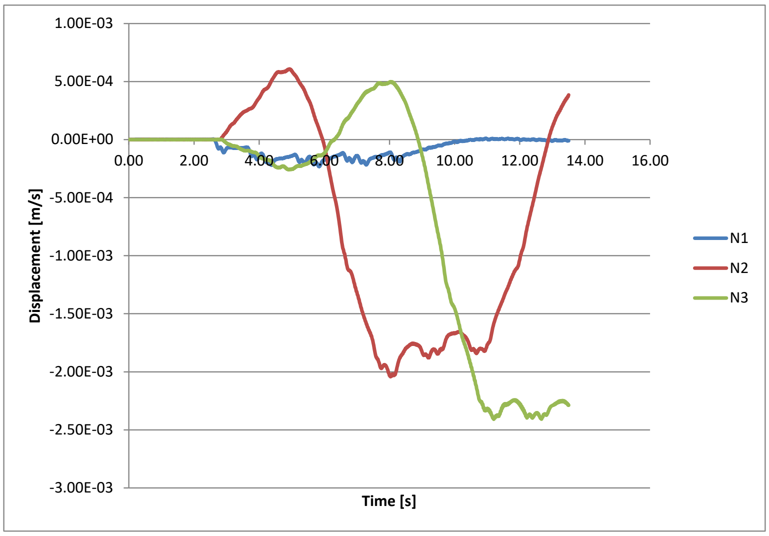


Figure 18: Vertical displacement at 50 km/h

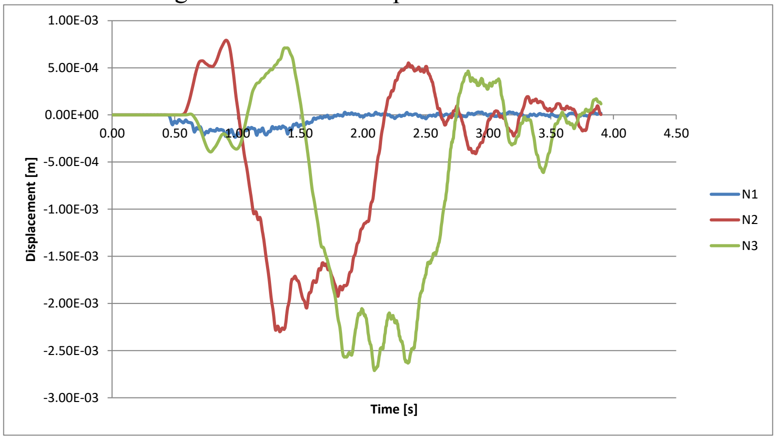


Figure 19: Vertical displacement of deck for 360 km/h

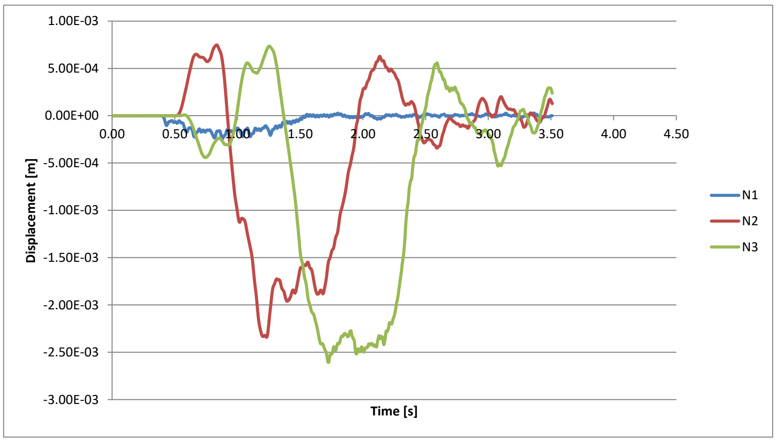


Figure 20: Vertical displacement of deck for 400 km/h

The dynamic amplification factor at the *i*-th Node α_i is the ratio of the peak vertical displacement from dynamic analysis $max(|u_{D,i}(t)|)$ and the peak vertical displacement from static analysis $max(|u_{S,i}(t)|)$:

$$\alpha_i = \frac{max(|u_{D,i}(t)|)}{max(|u_{S,i}(t)|)} \tag{5}$$

The following table presents the results for a set of velocities. It is observed that a plateau in the amplification occurs between the velocities 250 -360 km/h followed by a drop at 400 km/h and then a sharp increase. Performing the analyses with 450 and 500 km/h was necessary only for the purpose of identifying the general relationship between the amplification factor and velocity. In this way the expected general trend of increase in the amplification factor with the increase in the velocity was confirmed. This behaviour is not strict and depends on the relationship between the excitation frequency of the external loading for a given train speed and the natural frequencies of the bridge.

Velocity [km/h]	Displacement [mm]	Amplification factor
200	2.6	1.08
250	2.73	1.12
300	2.75	1.13
360	2.76	1.13
400	2.64	1.10
450	2.78	1.16
500	2.90	1.21

Table 1: Velocity vs. deck displacement and amplification facto.

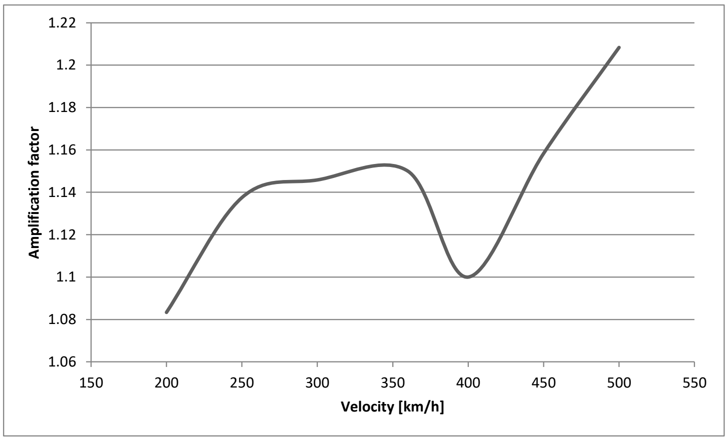


Figure 21 :Amplification factor vs. velocity

The maximum permitted peak design values of bridge deck acceleration calculated along the line of a track shall not exceed the recommended values given in A2 of EN 1990 (5 m/s^2). As can be concluded from Figure 22 this limit is kept.

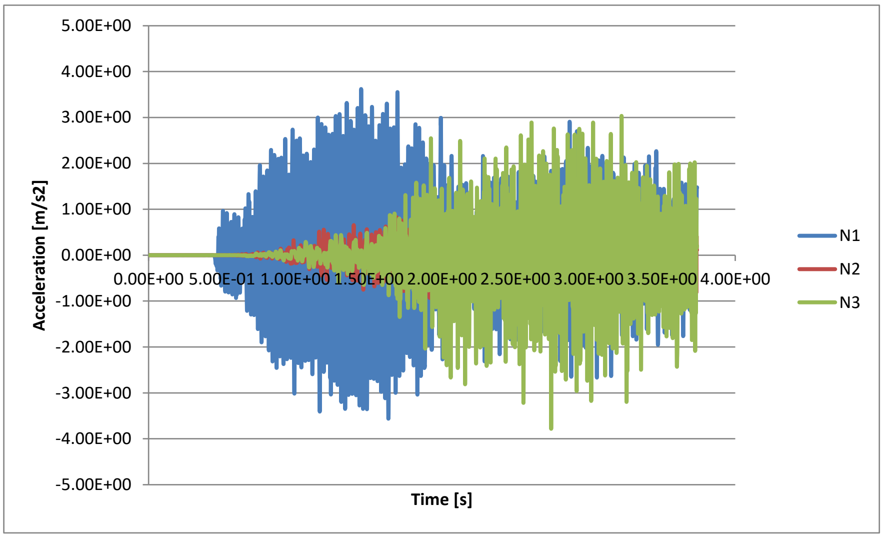


Figure 22: Peak accelerations at top of slab track obtained from the multi-body FEM analysis

From the above presented results it was concluded that no unusual, or unexpected dynamic responses should occur. The response parameters at the transition are within limits, indicating that the life-time expectation of the track will be satisfied. Finally the passenger comfort criterium according to [1] is also satisfied even for an operational velocity of 400 kph. No local resonant effects of the track slabs occur, which was considered a possibility due to the elastic polymer layer on which they are supported.

5 CONCLUSIONS

The above presented case-studies demonstrate the applicability of the PML method for problems usual for the civil engineering practice. With the highly efficient explicit time-integration in LS-DYNA the large numerical models described in the paper are solved with a relatively low computational cost. One present challenge is that the applied PLM procedure does not allow for a static analysis, which presents a limitation. To overcome this limitation the problem has to be decoupled into a static and dynamic part presenting additional challenges to the engineer when aiming to use a single model for the entire design. Nevertheless, in the experience of the authors, the PLM procedure as implemented in LS-DYNA can easily address most engineering problems related to the simulations of unbounded domains.

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