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MODELING THE INFLUENCE OF STRAIN-RATE IN SUPERELASTIC SHAPE MEMORY ALLOYS

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Abstract

Shape Memory Alloys (SMAs) are materials with appealing properties. They have the ability to recover stress-induced strains performing the superelastic effect, ideally resulting in insignificant residual strain (re-centering capability). A dependency of their behavior, exhibiting different hysteresis loops, has been reported on temperature and loading rate variations, maximum deformation, material composition and treatments. Presenting a wide applicability potential, SMAs have been intensively investigated in various sectors possessing a considerable place in structural engineering. Studies have been published developing constitutive models to simulate their behavior presenting some drawbacks: the inability to capture the rate-dependent nature, the large number of parameters, complexity and difficulty of implementation. In this context, a uniaxial rate-dependent constitutive model based on Graesser and Cozzarelli model for superelastic SMAs, aimed at capturing their essential hysteretic response has been developed. After the verification of the model prediction comparing it with experimental data derived from literature, a comparison with the rate-independent model of Auricchio and Sacco (1997) is attempted to indicate the effect of the rate-dependency. Results showed that the proposed model managed to successfully simulate the influence of strain rate using a simple equation rendering feasible the potential implementation of a robust solution algorithm easy to program in commercial software constituting a valid and effective computational tool for the analysis of superelastic SMAs.

Keywords: Shape Memory Alloys, Superelastic Effect, Rate-dependency, Hysteretic Behavior, Constitutive Modeling.

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1 INTRODUCTION

Shape Memory Alloys (SMAs) are smart materials possessing attractive properties. They have the ability to recover apparently permanent strains through the martensitic phase transformations being solid-solid diffusionless phase transformations [1] between a high-symmetric, usually cubic, austenitic phase and a low-symmetric martensitic phase, brought about by temperature change performing the shape memory effect or by unloading following stress application performing the superelastic effect, ideally resulting in insignificant residual strain (re-centering capability). They exhibit different types of hysteresis depending on which effect is demonstrated. Experimental evidence shows a dependency of their behavior on temperature and loading rate variations, maximum deformation, material composition and treatments. SMAs have demonstrated energy dissipation capabilities, large elastic strain capacity, hysteretic damping, good high and low-cycle fatigue resistance, recentering capabilities and excellent corrosion resistance [2].

SMAs have been intensively investigated over the past decades through experimental and analytical studies illustrating their wide implementation ability in various sectors. In structural engineering, superelastic (stress-induced) SMA applications possess a considerable place as through the energy dissipation capacity and the re-centering capability constitute a potential candidate for replacing conventional materials setting the basis for innovative applications, mitigating permanent residual deformations and allowing for achieving demanding serviceability and safety requirements. In this context, the research interest has focused on the development of SMA-based energy-dissipation devices to achieve vibration control [3-6] demonstrating the efficiency of the proposed schemes. Another interesting research effort regarded the use of SMAs as structural connecting elements proving the enhancement of the performance of steel connections [7, 8]. SMA applications in framing systems as diagonal braces exploiting the superelastic effect showed the reduction both of interstory drift ratios and residual displacements [9-10] achieving the prescribed performance objectives [11]. SMAs have been employed in vibration mitigation in bridges by conducting experiments on an SMA device to be used as restrainer cable of in-span hinges of concrete bridges [12], analytical studies on the effectiveness of superelestic NiTi SMA reinforcing bars [13] and on a novel concept for bridge columns with SMA bars placed inside a plastic hinge element of rubber [14], proving the effectiveness of SMAs in eliminating damage. The potential use of SMAs in developing advanced seismic protective systems for structure retrofitting has been investigated proposing the deployment of SMA-based devices to increase the in-plane seismic resistance of masonry walls by exploiting the superelastic effect [15-17]. In [18, 19] Cu-based SMAs were employed to address the issues encountered with the use of conventional steel reinforcing bars and their inability to restrain residual deformations in structures during and after intense earthquakes.

Considering the growing research interest in superelastic SMAs, studies have been published developing constitutive models to simulate their behavior. The reported models present some drawbacks as for instance the inability to capture the rate-dependent nature, the large number of parameters, complexity and difficulty of implementation. To address these issues, a uniaxial rate-dependent model based on Graesser and Cozzarelli model [20] for superelastic SMAs has been developed. After the verification of the model prediction comparing it with experimental data derived from literature, a comparison with the rate-independent model of Auricchio and Sacco [21] is attempted to indicate the effect of the rate-dependency.

2 RATE-DEPENDENCY OF SUPERELASTIC SMAS

Superelasticity is the recovery of large deformations during mechanical loading-unloading cycles of an SMA in the austenitic state through the phase transformation from austenite to martensite, exhibiting the flag-shaped loop depicted in Figure 1. As reported, the austenite is stable at high temperatures and low values of the stress, while the martensite is stable at low temperatures and high values of the stress [22]. The austenitic alloy displays an elastic behavior upon the application of stress level up to the transformation stress σ_s^{A-M}, after which the stress-induced martensitic transformation occurs [23]. During transformation the stress-strain curve follows a stress plateau until the complete transformation to martensite at stress σ_f^{M-A} with further straining causing its elastic loading. Upon unloading, since martensite is stable due to the presence of the applied stress, the reverse transformation takes place [20]. Due to defect generation mechanisms taking place during martensitic transformation, a residual strain, ε_r, can be induced in the material during deformation which will be carried over to the next stress cycle in case of incomplete recovery [23].

The hysteretic behavior of austenite SMAs depends on loading rate variations. The stress-induced phase transformations are associated with the release and absorption of latent heat, with the forward transformation increasing the temperature of the material (exothermic reaction) and with the reverse transformation consuming energy from the environment and reducing the temperature of the material (endothermic reaction) [24]. This heat alteration can affect the material temperature and consequently its response to mechanical stimuli. At low strain rates, the heat generated allows maintenance of a thermal balance as the exchange with the surroundings occurs sufficiently fast, which allows the SMA temperature to be identical with the ambient temperature and, as a result, such loading can be considered to be an isothermal process [25]. However, at high strain rates this thermal equilibrium is affected as the material cannot exchange heat with its surroundings and it experiences a gradual accumulation of heat, leading to an increase in the internal temperature due to the Clausius–Clapeyron law [24], working under adiabatic conditions [25]. This thermomechanical coupling is the cause of the strain rate dependence observed in SMAs [24]. Passing to frequencies of seismic interest, the hysteretic loop decreases in size and the loading and the unloading stresses increase, affecting the energy dissipation capacity of the material.

All the above were proved experimentally as this phenomenon engaged the special interest of the research community. Shaw and Kyriakides [26] conducted a series of experiments investigating the effects of latent heat and loading rate on the response of specimens at different temperatures. The uniaxial tensile tests on Ni-Ti wires at two different temperatures at isothermal (water) and non-isothermal (air) conditions indicated that the self-heating and cooling of the specimen that result from transformations increases the stress required for the forward transformation from austenite to martensite and reduces the stress required for the reverse transformation. The temperature evolution under the cyclic loading of different frequencies was investigated in [27] proving that the temperature rise becomes significant in the high-frequency range. Tests under various loading rates were also conducted in [28-29] indicating the self-heating of the specimens due to the loading rate increase.

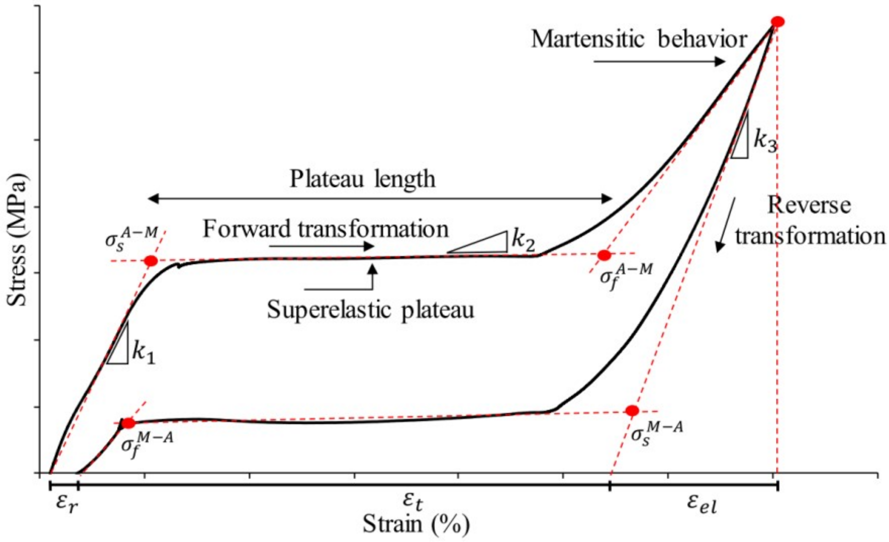

Figure 1: A typical superelastic stress-strain curve [23].

3 MODELING THE INFLUENCE OF STRAIN RATE

To simulate the superelastic behavior of SMAs various research efforts have been conducted. Auricchio et al. [1] formulated one- and three-dimensional thermomechanical constitutive models using an internal-variable inelastic framework able to describe solid-solid phase transformations considering an isothermal time-discrete formulation. In another study, [21] a superelastic beam model was presented based on the classical Euler-Bernoulli theory to allow the use of a reduced but significant set of kinematic parameters, not considering though the influence of strain rate. A few years later, Auricchio and his team [30] formulated a model for SMA behavior based on two scalar internal variables, the static martensite fraction and the dynamic martensite fraction to simulate rate phenomena. A limitation of the model is the increased number of parameters and the requirement of the advance knowledge of the maximum residual strain of the SMA. Another interesting effort in SMA modeling came from Graesser and Cozzarelli [20] who modified Özdemir's force-displacement relationship for a metallic damper [31] and specifically the equation describing the backstress, in order to capture the behavior of SMAs due to the shape memory as well as the superelastic effect, creating a rate-independent model.

To model the rate-dependent nature of superelastic SMAs, an extension of Graesser and Cozzarelli model [20] is presented. The expression of stress σ is identical to the one of Graesser and Cozzarelli model, given in Equation (1), while the backstress β given in Equation (2), involves the addition of one more term, namely stress σ_2, introduced to capture the rate-dependence, yielding Equation (3).

Aiming to determine the form of the stress equation, data from experiments conducted by DesRoches et al. [32] on 7.1 mm diameter bars under cyclic loading, which causes stable strain rate at frequencies of 0.025 Hz, 0.5 Hz and 1 Hz were used. The digitized stress-strain curves obtained from the experiment for these three frequencies are depicted in Figure 2, presented as full loops as only the continuous parts of the curves were reported. The formulation of Equation 3 is based on expressing the stress difference of the dynamic response of 0.5 Hz to the quasistatic response of 0.025 Hz as a rate-dependent function [33]. This is done by curve fitting on the stress difference data as shown in Figure 3, deriving Equation (4):

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$$\dot{\sigma} = \mathrm{E}\left[\dot{\varepsilon} - \dot{\varepsilon}\left(\frac{\sigma-\beta}{\mathrm{Y}}\right)^{\eta}\right] \tag{1}$$

$$\beta = \mathrm{E}\alpha\left\{\varepsilon^{in} + f_t\left|\varepsilon\right|^{c} erf\left(a\varepsilon\right)\left[u\left(-\varepsilon\dot{\varepsilon}\right)\right]\right\} \tag{2}$$

$$\beta_{tot} = \beta + \sigma_2 \tag{3}$$

$$\sigma_2 = \Delta\sigma_2 \cdot sign(\varepsilon)\left(\left|\varepsilon\right|\right)^{\left|\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}}\right|} \tag{4}$$

Where E is the Young's modulus of elasticity, $\dot{\varepsilon}$ is the strain rate, β is the backstress, Y is the "yield" stress and η is a constant controlling the sharpness of transition from elastic to plastic states, α is a constant controlling the slope of the stress-strain curve, parameters f_t, c and a are material constants and notations $erf(\ )$ and $u(\ )$ are used to represent the error function and the unit step function respectively. $\Delta\sigma_2$ is the stress difference at maximum strain between the experimental response at a quasi-static frequency and at a dynamic frequency, $\dot{\varepsilon}_0$ is the strain rate produced at a quasi-static frequency and $\dot{\varepsilon}$ is the strain rate induced by the frequency in the dynamic range.

In order to verify the effectiveness of the proposed equation capturing the strain rate, data from experiments conducted by Dolce and Cardone [34] on 1.84 mm diameter austenitic wire were used. The frequencies of loading of the experimental data available were 0.02, 0.2 and 2 Hz presented as full loops in Figure 4, as only the continuous parts of the curves were reported. The fitted curve on the stress difference data is depicted in Figure 5.

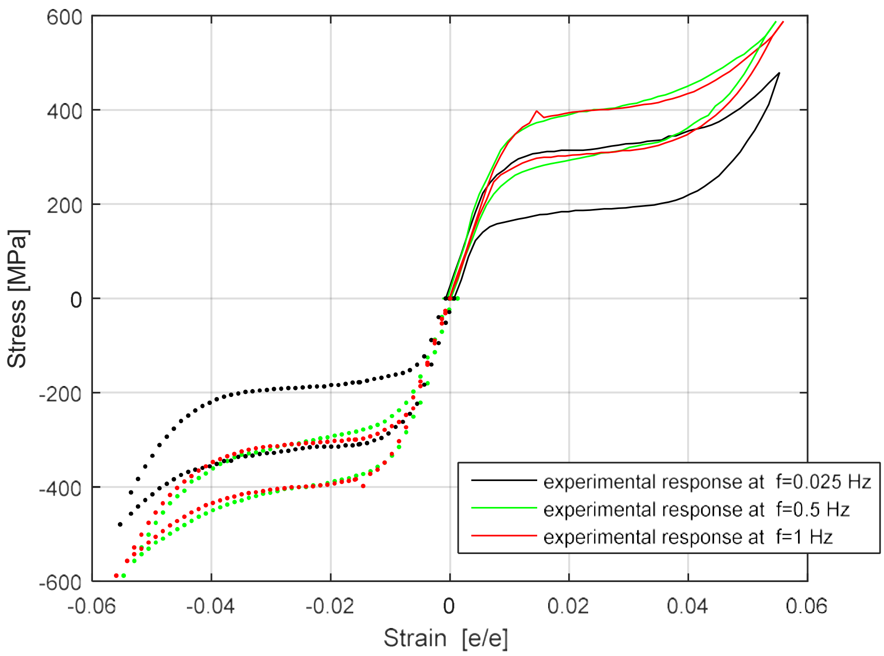

Figure 2: Comparison of the experimental responses at frequencies of 0.025, 0.5 and 1 Hz (data from [32]).

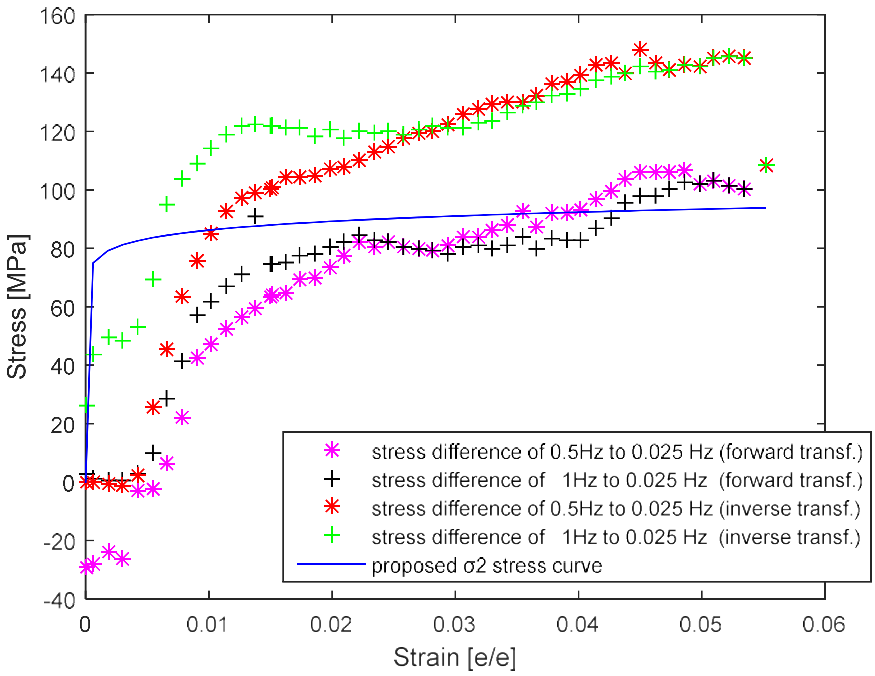

Figure 3: Comparison The fitted curve on the stress data from [32].

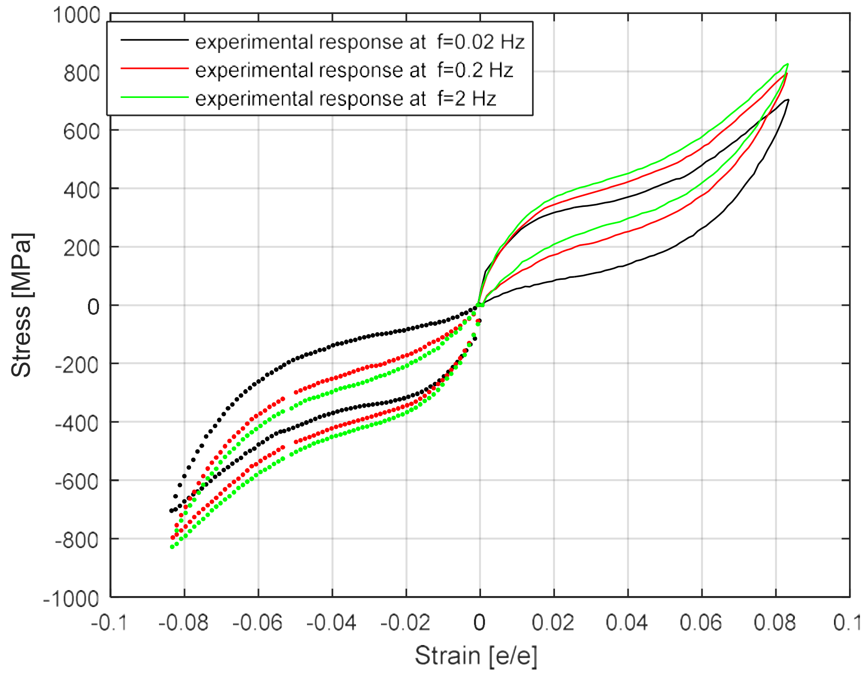

Figure 4: Comparison of the experimental responses of experiments conducted in [34] at frequencies of 0.02, 0.2 and 2 Hz.

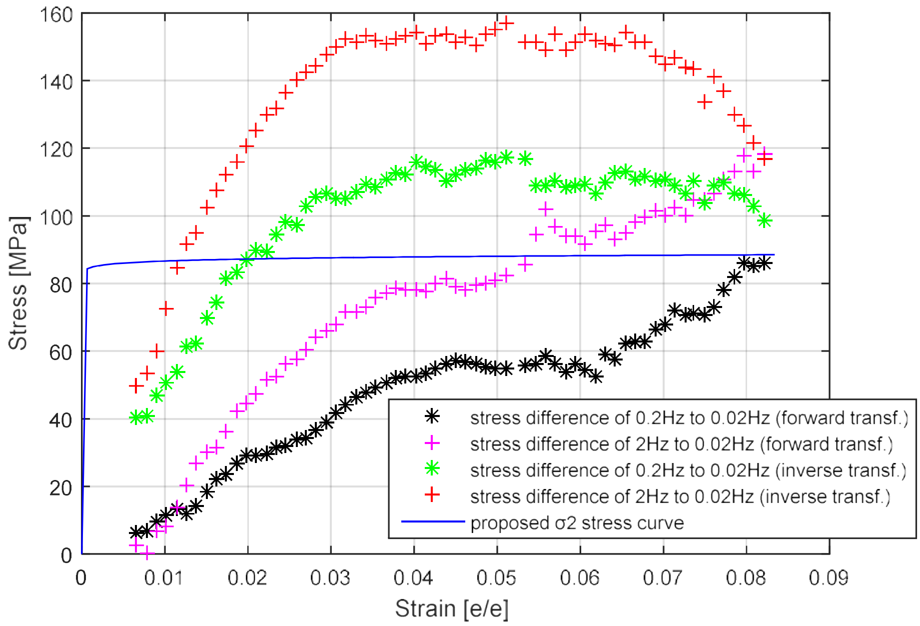

Figure 5: The fitted curve on the stress data from [34]

Comparison of the proposed model prediction to Graesser and Cozzarelli rate-independent model prediction and to the experimental responses from [32] at frequencies of 0.5 and 1 Hz (Figure 6), using the following model parameter values: $E = 35GPa$, $Y = 300MPa$, $\eta = 7$, $\alpha = 0.0488$, $f_t = 0.268$, $c = 0.002$ and $a = 165$, shows that the increase of the stresses attributed to heat exchanges especially when passing from quasi-static to dynamic conditions is well captured by the proposed model. The rate-independent Graesser and Cozzarelli model gives the same response prediction for the frequencies of 0.5 and 1 Hz. Comparison of the proposed model prediction to Graesser and Cozzarelli rate-independent model prediction and to the experimental responses from [34] at frequencies of 0.2 and 2 Hz (Figure 7) using the following model parameter values: $E = 30GPa$, $Y = 250MPa$, $\eta = 7$, $\alpha = 0.16$, $f_t = 0.05$, $c = 0.002$ and $a = 165$, denotes as well the stress alteration exhibited from the proposed model. Another characteristic of the SMA superelastic behavior the elastic loading of the fully transformed martensite at higher strains is also modeled in [33] adding another equation to backstress, an issue which is beyond the scope of this paper.

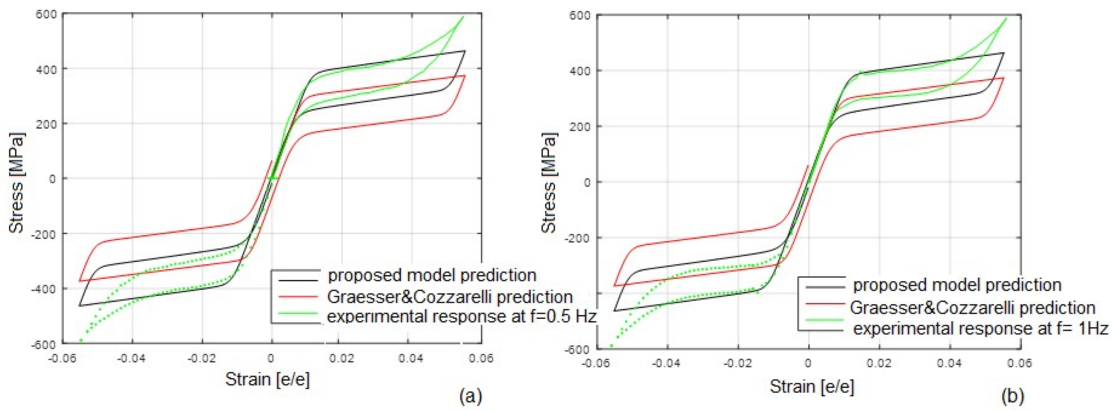

Figure 6: Comparison of the proposed model prediction to Graesser and Cozzarelli model and the experimental response (a) at a frequency of 0.5 Hz, (b) at a frequency of 1 Hz.

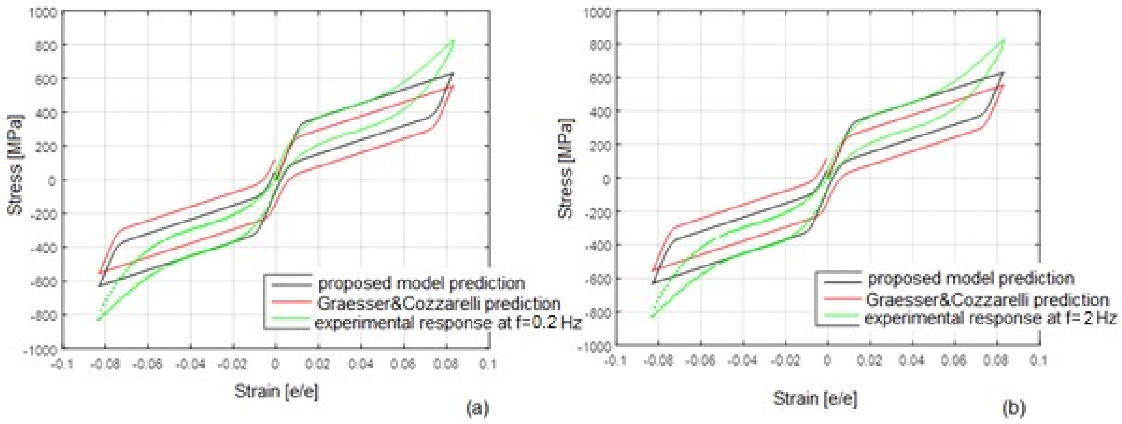

Figure 7: Comparison of the proposed model prediction to experimental response (a) at a frequency of 0.2 Hz, (b) at a frequency of 2 Hz.

To further evaluate the proposed model prediction, the performance of a single-storey Special Truss Moment Frame (STMF) with SMAs as X-diagonals incorporated in the pin-ended middle segment (Figure 8), conducting non-linear dynamic analysis in SeismoStruct software [35] was compared with the response of the same single-storey STMF simulated via a 4DOF model (u_1, ϕ_1, u_2, ϕ_2) consisting of two lumped masses (Figure 9), solved in MATLAB software [36] through the direct integration of the equations of motion. SeismoStruct embedded SMA model is rate-independent and uniaxial and follows the constitutive relationship proposed by Auricchio and Sacco [21] assuming a constant stiffness for both the fully austenitic and the fully martensitic behavior while the MATLAB model incorporated the proposed SMA model described herein. The two models' predictions are compared, comparing also the response of the left mass of the mechanical model with the response of node N1 of the SeismoStruct model incorporating 0.025 m and 0.035 m diameter SMA bars in each diagonal, under seven benchmark earthquakes [33]. Results for Northridge earthquake (NE) are presented herein (Figures 10 and 11). Response differences are justified, since the 4-DOF model is consisted of two rigid masses, while the model in SeismoStruct is a structure modeled with frame and truss elements, hence stiffer, analyzed with the finite element method. The comparison of the rate-independent SeismoStruct model with the proposed rate-dependent model indicates the stress increase. This was expected as the proposed model takes into consideration the influence of strain rate. Moreover, the loop derived from SeismoStruct model is of triangular shape that is not representative of the actual SMA loading-unloading loop while the loop generated form the proposed model resembles the "full" SMA curve. SeismoStruct analysis gives larger SMA deformations and this happened due to larger rotations derived than those of the 4-DOF model.

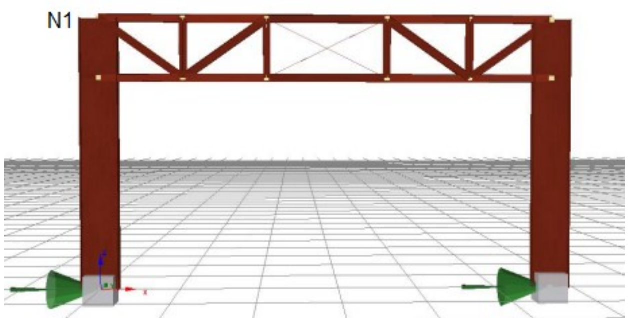

Figure 8: The SeismoStruct model of the STMF with X-diagonal SMA dampers in the special segment.

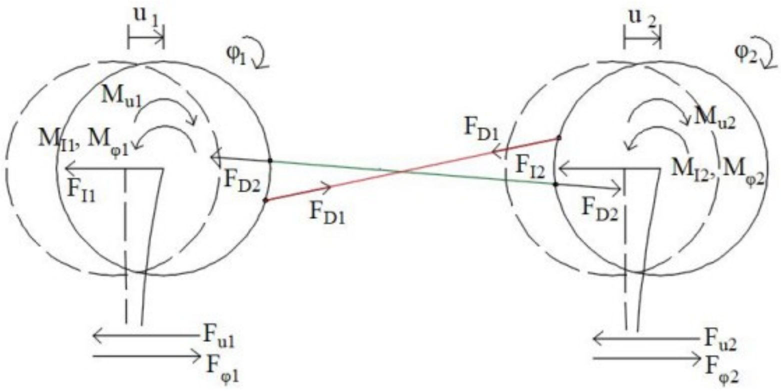

Figure 9: The free body diagram of the STMF with X-diagonal SMA dampers in the special segment.

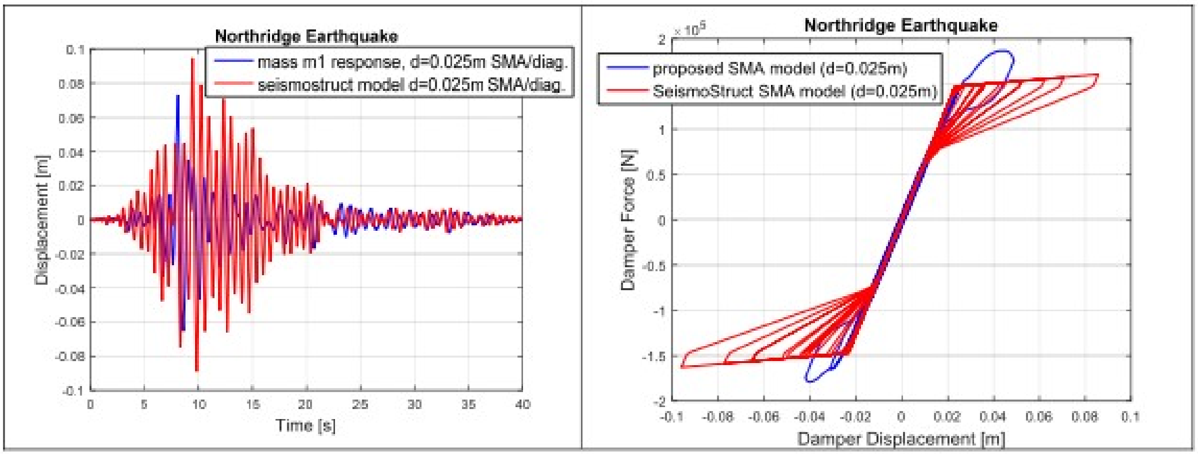

Figure 10: Comparison of frame response and damper force-deformation curve of the damper of the mechanical model and SeismoStruct (d=0.025 m) for Northridge Earthquake.

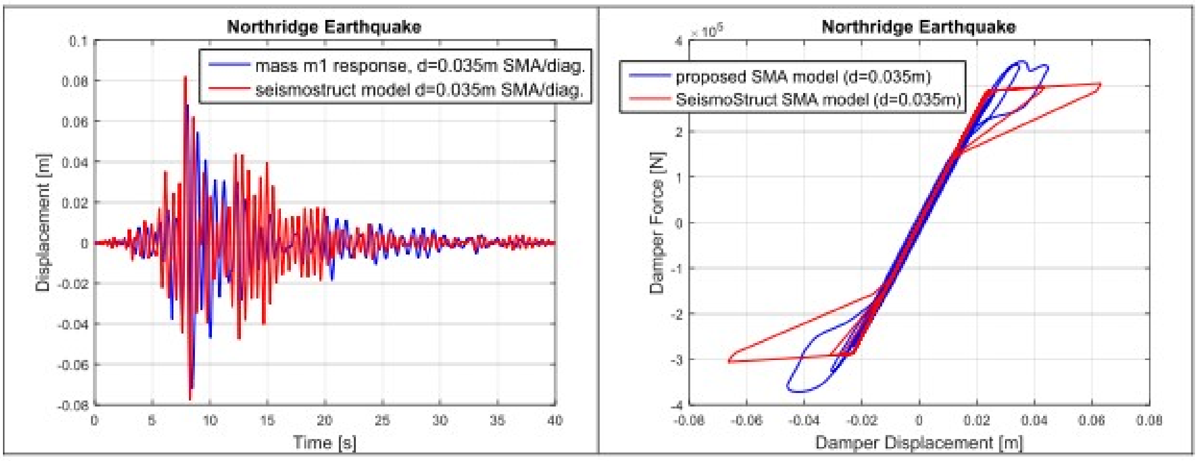

Figure 11: Comparison of frame response and damper force-deformation curve of the damper of the mechanical model and SeismoStruct (d=0.035 m) for Northridge Earthquake.

4 CONCLUSIONS

This work deals with the development of a constitutive model capturing the influence of strain rate in superelastic SMAs. The proposed model is an extension of Graesser and Cozzarelli rate-independent model that includes the addition of a rate-dependent function in the equation describing the backstress, formulated from experimental data found in the literature. Comparison of the model prediction with the aforementioned data as well as with Auricchio and Sacco [21] rate-independent model showed that the proposed model managed to successfully reproduce the complex superelastic behavior using a simple equation for considering the influence of strain rate, rendering feasible the potential implementation of a robust solution algorithm easy to program in commercial software. The model can be applied for either wires or bars while material parameters can be obtained conducting simple uniaxial tests, constituting a valid and effective computational tool for the analysis of superelastic SMAs that will enhance the advancement of knowledge regarding their systematic use in civil structures.

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