

# APPLICABILITY OF GYRO-LUMPED PARAMETER MODEL IN IDENTIFYING HORIZONTAL IMPEDANCE FUNCTIONS OF PILE GROUPS

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## Abstract

*In recent years, the effects of frequency dependency of impedance functions (IFs) of various soil-foundation systems upon nonlinear structural systems have been revealed. So-called "Gyro-Lumped Parameter Models (GLPMs)" have been playing a vital role on numerical computations for estimating the dynamic response of nonlinear structural systems with frequency dependent IFs. GLPMs consist of frequency independent mechanical elements such as springs, dampers, and gyromasses, which generate a reaction force that is proportional to the relative acceleration of two nodes where the element is placed. These models can help predict the seismic response of structures interacting with soil-foundation systems, mostly for design purpose. When it comes to real existing structures, however, there have never been a technology proposed to identify the frequency dependent IFs. A proper identification of IFs is indispensable in predicting damage to structures in enhancing their resilience against earthquakes. Current work proposes a new method for identifying the frequency dependent IFs using GLPMs based on transfer functions processed from recorded small-amplitude earthquake waves measured on and near the target structures. In this paper, a simple two degrees of freedom system consisting of superstructure and footing supported by a 2×4 pile group is targeted. In the identification procedure, the trust region method (TRM) is applied to fit the transfer functions of the system with variables of the system including the mechanical elements of GLPMs. In the TRM, various objective functions are dealt with and hypothetical unknown quantities such as the static stiffness of the soil-pile system, the damping of the superstructure, and the masses of the structural system are assumed to verify the performance of the identification. The results indicate that accurate mass identification is prerequisite to accomplish an appropriate identification of the frequency dependent IFs.*

**Keywords:** Impedance Functions, System Identification, Gyro-Lumped Parameter Models, Trust Region Method, Pile Foundations, Frequency Dependency

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## 1 INTRODUCTION

For decades, researchers struggled for finding a way to incorporate the frequency dependent characteristics of soil-foundation systems into ordinary time-stepping methods for estimating the dynamic response of structures. These days, several lumped parameter models (LPMs) have been contrived for representing a variety of frequency dependencies [e.g.,1-7], and they have been used for computing the dynamic response of structures even beyond the elastic region of structural members. The so-called ‘‘Gyro-Lumped Parameter Models (GLPMs)’’ have been recognized as a vital tool to compute the structural response when subjected to earthquake ground motions due to use of gyromass instead of using the ordinary mass in LPMs [e.g.,8-13]. Gyromass generates a reaction force that is proportional to the relative acceleration of two nodes where the element is placed. The advantage of the gyromass is that any inertia force due to the acceleration of ground motions never appears in it at all whereas inertia force occurs when the ordinary mass is used, which distorts the original frequency dependent characteristics of IFs. In addition, a smaller number of mechanical elements can emulate intricate fluctuations of IFs by using GLPMs than other LPMs.

GLPMs can help predict the seismic response of structures interacting with soil-foundation systems, mostly for design purpose. When it comes to real existing structures, although the frequency dependent IFs should be considered in predicting the response for achieving high accuracy, there have never been a technology proposed to identify the frequency dependent IFs. A proper identification of IFs is indispensable in predicting damage to structures in enhancing their resilience against earthquakes.

Current work proposes a new method for identifying the frequency dependent IFs using GLPMs based on transfer functions (TFs) processed from recorded small-amplitude earthquake waves measured on and near the target structures. In this paper, a simple two degrees of freedom system consisting of superstructure and footing supported by a 2×4 pile group is targeted. In the identification procedure, the trust region method (TRM) is applied to fit the TFs of the system with variables of the system including the mechanical elements of GLPMs. In the TRM, various objective functions are dealt with and hypothetical unknown quantities such as the static stiffness of the soil-pile system, the damping ratio of the superstructure, and the masses of the structural system are assumed to verify the performance of the identification.

## 2 METHODOLOGIES

In this study, a soil-pile-structure system is targeted to verify the performance of the proposed IF identification method. A superstructure is represented by a single degree of freedom system while a footing by a mass, where they are allowed to move only in the horizontal direction. The footing is supported by the published IFs of a 2×4 pile group embedded in a layered soil [9]. Properties of the considered system are as follows: the masses of the superstructure  $m_s$  and footing  $m_f$  are 450t and 350t, respectively; the stiffness  $k_s$  and damping ratio  $\zeta$  of the superstructure are  $7.1 \times 10^4$  kN/m and 0.05, respectively; and the static stiffness of the soil-pile system  $K$  is  $2.4 \times 10^5$  kN/m.

In this study, the target TFs of the superstructure and the footing with respect to input ground motions is estimated. In practice, this procedure is equivalent to the data process to obtain the TFs from recorded small-amplitude earthquake waves measured on the target structures, and on the ground surface near the structures as the input motion.

In the identification procedure, the frequency dependent IFs of the superstructure and footing model are replaced by a so-called Type-III GLPM [9] as shown in Figure 1. The GLPM consists of three core systems (a total of 15 mechanical elements). The properties of the elements are determined by applying the TRM to reproduce the TFs. The TRM is a well-known

method applicable to nonlinear least squares problems. This study uses the following three types of objective functions to dealt with in the TRM.

$$f = \delta_{\text{super-R}}^2 + \delta_{\text{super-I}}^2 + \delta_{\text{foot-R}}^2 + \delta_{\text{foot-I}}^2 \tag{1}$$

$$f = \delta_{\text{super-R}}^2 + \delta_{\text{super-I}}^2 \tag{2}$$

$$f = \delta_{\text{foot-R}}^2 + \delta_{\text{foot-I}}^2 \tag{3}$$

where  $\delta$  denotes the difference in the TFs estimated by the TRM from the target TFs. Here, subscripts “super” and “foot” indicate the TFs of the superstructure and the footing, respectively. In addition, “R” denotes the real part while “I” the imaginary part.

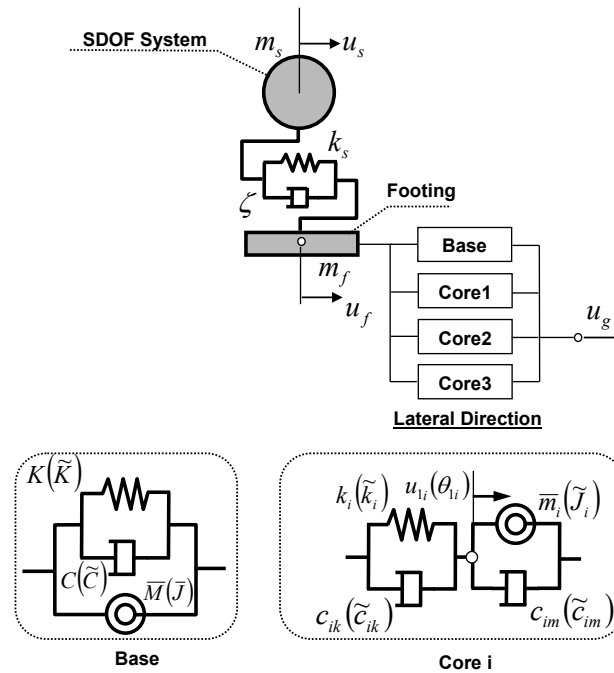


Figure 1: Structural system with Type-III GLPM

The objective functions are minimized by adjusting the properties of the elements through the algorithm of the TRM. The TRM needs the initial values of the variables, and this study assumes 0.01 for each value. The IFs expressed by the GLPM having the converged properties through the TRM are considered as the identified frequency dependent IFs.

### 3 VERIFICATION OF IF IDENTIFICATION METHOD

In this section, at first, the influence of the objective functions is investigated. Then the influence of parameters such as the static stiffness of the soil-pile system, the damping ratio of the superstructure, and the masses of the structural system, which may be unknown or vague in practice without detailed inspections, on the accuracy of the IF identification is studied.

#### 3.1 Objective functions

Figures 2-4 show the resultant TFs and the identified IFs with different objective functions. In case of using Eq.1, although the fitted TFs of both the superstructure and the footing show a good match with the target TFs, the identified IFs show a discrepancy above 2Hz, especially around the local maxima of both the real and imaginary parts. In case of using Eq.2, a larger

discrepancy appears in the IFs than those in case of Eq.1. On the other hand, a good match can be achieved in case of using Eq.3. These results imply that the targeting TFs of footing is effective to achieve the IF identification with appreciable accuracy. Applying the TFs of the superstructure to the objective function may dilute the characteristics of IFs due to its large amplitude of TFs associated with the resonance of the superstructure. In the following parts, therefore, Eq.3 is applied to the TRM.

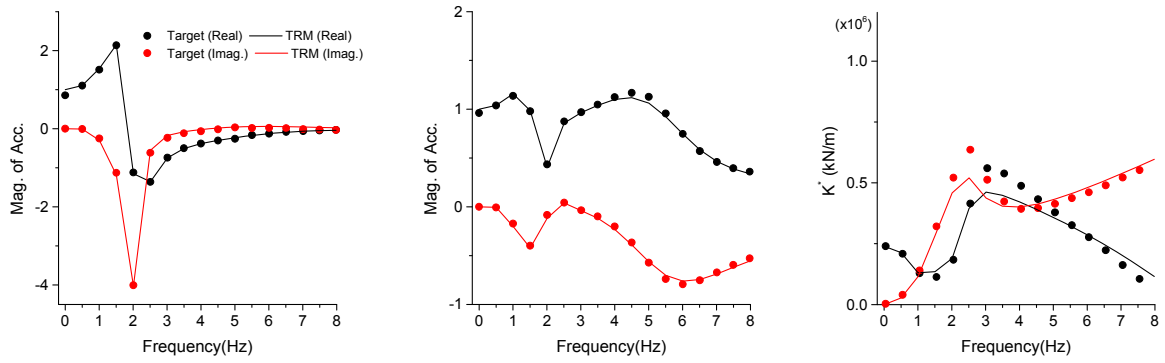


Figure 2: Transfer functions (left: superstructure, center: footing) and impedance functions (right) obtained from TRM: Equation 1 is used for objective function

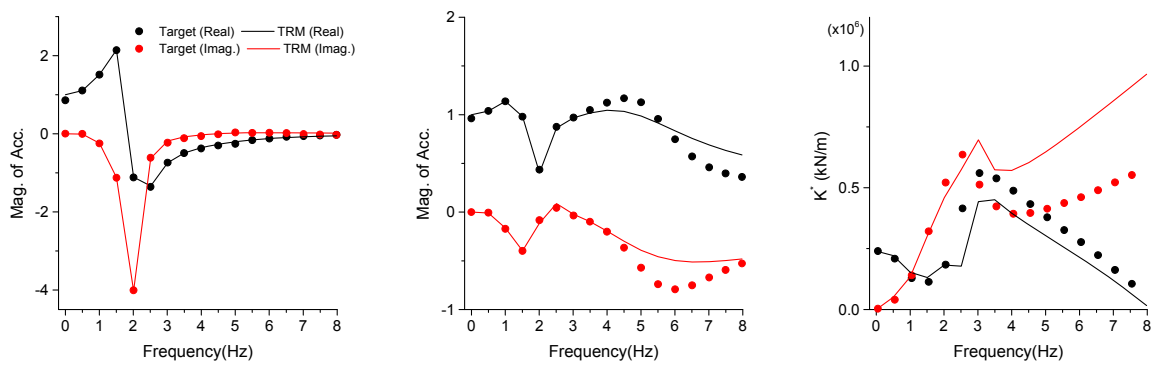


Figure 3: Transfer functions and impedance functions obtained from TRM: Equation 2

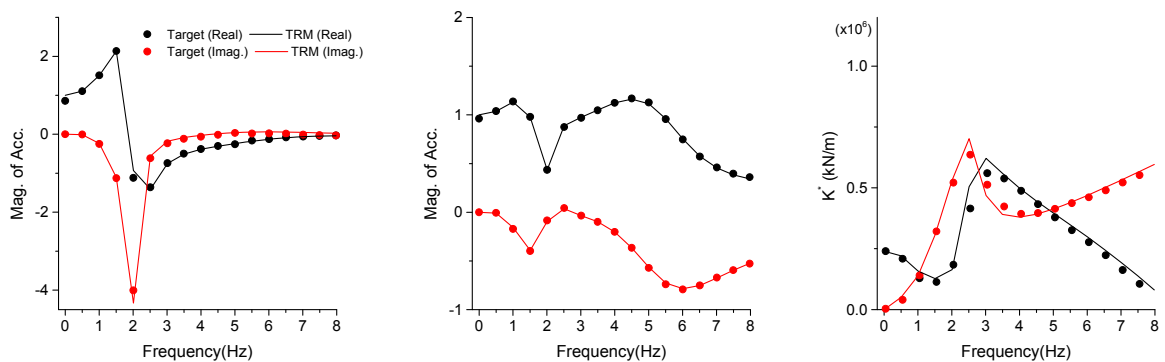


Figure 4: Transfer functions and impedance functions obtained from TRM: Equation 3

### 3.2 Static stiffness of soil-pile system

The static stiffness of the soil-pile system is the property of one of the mechanical elements in the GLPM. In practice, a couple of non-destructive technologies have been proposed to identify the static stiffness whereas the accuracy of them is still insufficient. Therefore, it is interesting to see how the proposed method applies to identify the static stiffness.

Figure 5 shows the results when the static stiffness is provided as a variable of the TRM. The TFs show a large discrepancy while the identification of the IFs is failed. Instead of setting the initial stiffness as a variable, the initial stiffness is set as a fixed value with a certain discrepancy by multiplying by 0.6. Figure 6 shows that an appropriate match with the target TFs is attained while the identified IFs are quite well reproduced. In details of the IFs, a discrepancy in lower frequency range, whose amounts are almost proportional to  $\alpha$ , tends to appear. Therefore, it is presumed that not the exact but an approximate value (at least the same order) could be acceptable to identify the IFs with appreciable accuracy in the middle and high frequency range whereas the IFs in lower frequency range may show a certain discrepancy.

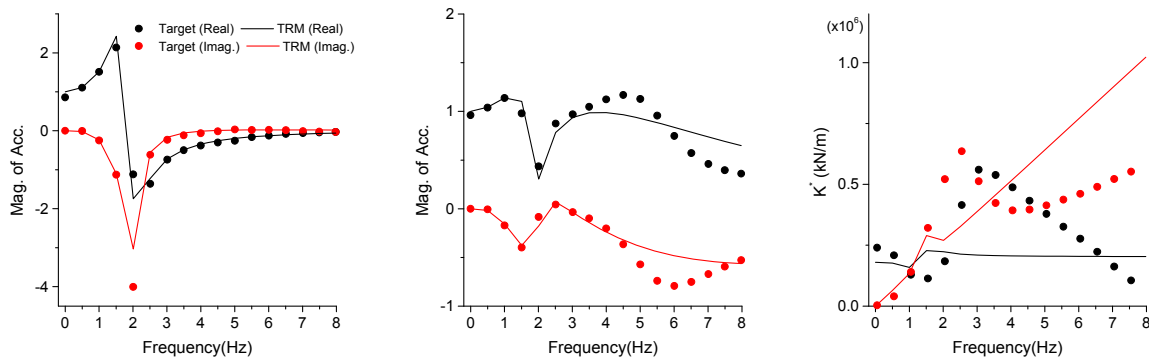


Figure 5: Transfer functions and impedance functions obtained from TRM: Static stiffness of soil-pile system as variable in TRM

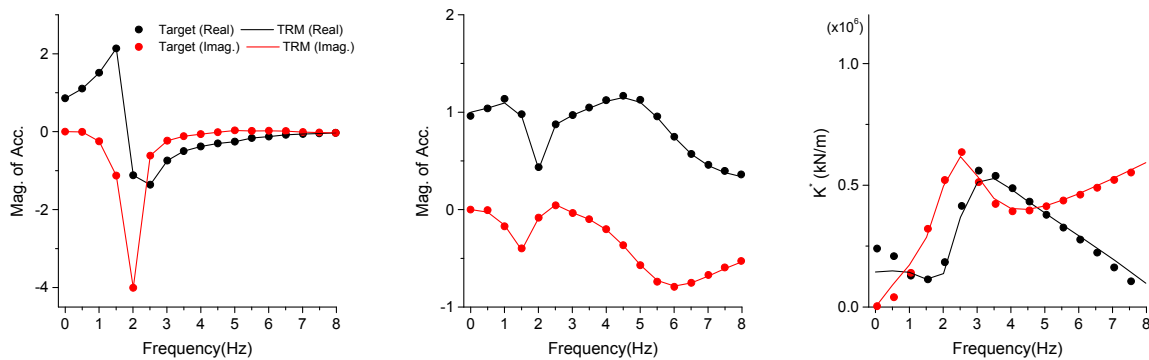


Figure 6: Transfer functions and impedance functions obtained from TRM: Static stiffness of soil-pile system containing discrepancy (multiplying by 0.6) treated as constant in TRM

### 3.3 Damping ratio of superstructure

In practice, the damping ratio of superstructures can be estimated from the TFs obtained from recorded data measured on the system. Generally, the estimated damping ratio has certain errors. Therefore, the influence of errors on the identified IFs is proved by multiplying the

exact damping ratio by 0.6. In the following identification, this damping ratio is applied to the TRM as an initial value. Therefore, the damping ratio is treated as a variable in the TRM.

Figure 7 shows that the imaginary part of the TFs of the superstructure at the first predominant frequency (around 2Hz) increases. It is found that the identified value of the damping ratio is almost the same as the initial value. Therefore, the decrease in the damping ratio attributes to the increase in the TFs. On the other hand, the TFs of the footing show a good match with the target TFs as Eq.3 is used for the objective function of the TRM, which minimizes the discrepancy in the TFs of the footing. The IFs are identified in a fairly good manner. In details, the imaginary part of the identified IFs tends to increase rapidly at the predominant frequency. This is because the imaginary part deals with the damping effect, which compensates the decrease in the damping ratio of the superstructure to attain the accurate fitting of the TFs of the footing. Therefore, errors in the damping ratio of superstructures tend to distort the IFs locally around the predominant frequency of superstructures.

A possible alternate option is that the damping ratio with an error (multiplying by 0.6) is applied as the initial value to the TRM whereas Eq. 1 is selected as the objective function to adjust the damping ratio appropriately by considering both the differences in the TFs of the superstructure and the footing.

Figure 8 shows that, although a good match in the TFs of the superstructure is attained, the TFs of the footing cannot sufficiently reproduce the target TFs, indicating inadequacy of identified IFs.

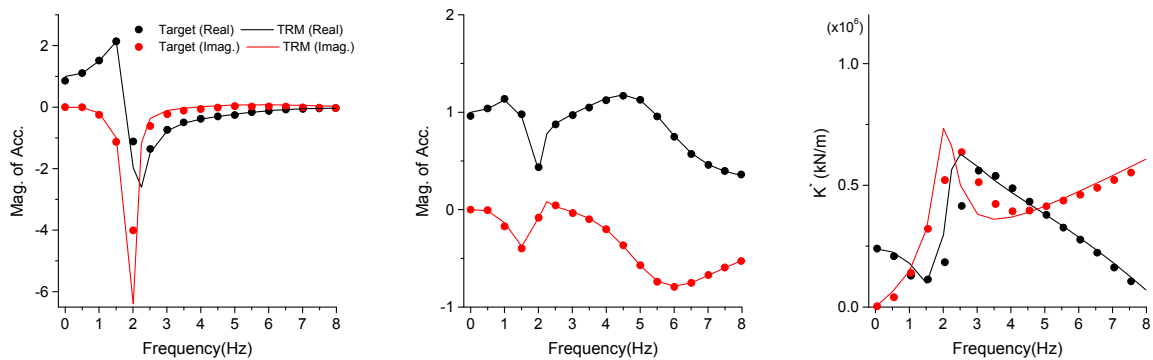


Figure 7: Transfer functions and impedance functions obtained from TRM: Damping ratio of superstructure containing discrepancy (multiplying by 0.6) treated as initial value in TRM

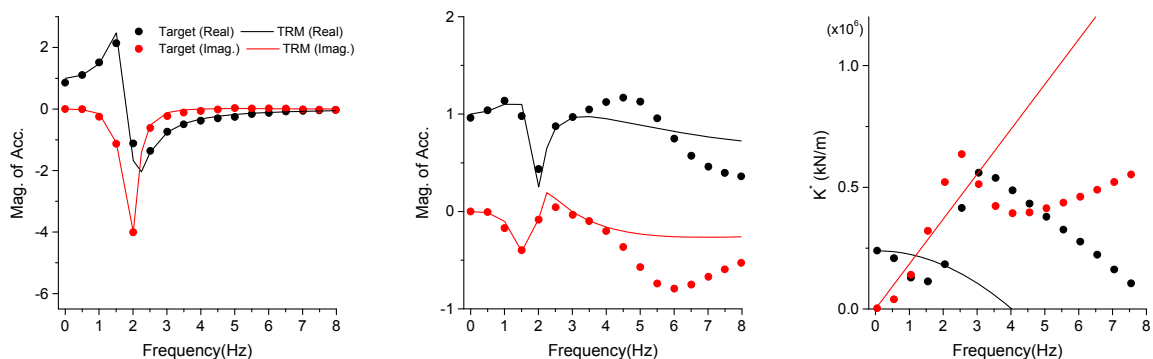


Figure 8: Transfer functions and impedance functions obtained from TRM: Damping ratio of superstructure containing discrepancy (multiplying by 0.6) treated as initial value in TRM using Equation 1

### 3.4 Masses of superstructure and footing

In practice, the estimation of the masses of structural systems is quite approximate. Non-structural elements may change the weight of each floor, which may affect the accuracy of the identification of IFs. In this part, the masses of the superstructure and the footing are assumed as the variables of the TRM. Here, a certain discrepancy is provided to them, and they are applied as their initial values in the TRM. In the following identification, the initial value of the mass of the footing is given as the product of  $m_f$  by  $\alpha_f$ ; the ratio of the mass of the superstructure to the footing is expressed as  $\beta$  multiplied by  $\gamma$ .

Figure 9 shows the resultant TFs and IFs in case of  $\alpha_f=1$  and  $\gamma=1$ . The converged values of  $\alpha_f$  and  $\gamma$  through the TRM are 1.01 and 1.19, respectively. The figure indicates a good performance of the identification. Figure 10 shows the results in case of  $\alpha_f=0.9$  and  $\gamma=1$ . In contrast, although the resultant TFs shows a good match with the target TFs, a significant difference appears in the identified IFs. The converged values of  $\alpha_f$  and  $\gamma$  are 1.88 and 1.16, respectively. This is an important issue when the identification of the IFs is performed for structural systems whose masses are unknown. Figure 9 implies that the masses should not be a variable in the TRM as even a small difference in the initial value of the masses will converge to far different IFs. The reason for the marked difference in the identification can be simply explained from the following simultaneous equations of motion for the system.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1/\beta \end{bmatrix} \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_f \end{Bmatrix} + \begin{bmatrix} 2\zeta\omega_s & -2\zeta\omega_s \\ -2\zeta\omega_s & 2\zeta\omega_s \end{bmatrix} \begin{Bmatrix} \dot{u}_s \\ \dot{u}_f \end{Bmatrix} + \begin{bmatrix} \omega_s^2 & -\omega_s^2 \\ -\omega_s^2 & \omega_s^2 + g(p_i)\omega_f^2 \end{bmatrix} \begin{Bmatrix} u_s \\ u_f \end{Bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1/\beta \end{bmatrix} \begin{Bmatrix} \ddot{u}_g \\ \ddot{u}_g \end{Bmatrix} \quad (4)$$

where  $u_s$  and  $u_f$  are the response displacements of the superstructure and the footing, respectively;  $u_g$  is the input motion;  $\omega_s = \sqrt{k_s/m_s}$ ;  $\omega_f = \sqrt{K/m_f}$ ; and  $g(p_i)$  is the frequency dependent impedance function normalized by the static stiffness, consisting of GLPM parameters  $p_i$ .

Equation 4 indicates that  $g(p_i)\omega_f^2$  is the product of the two terms: and the same response occurs in different structural systems when the product is consistent with each other. There are infinite numbers of combinations of  $g(p_i)$  and  $\omega_f^2$  to achieve a consistent value of  $g(p_i)\omega_f^2$ . Accordingly, the masses of the system should not be the variables of the TRM.

Figure 11 shows the influence of discrepancy in the mass of the footing where  $\alpha_f=0.8$  is assumed. Herein, the masses are assumed to be constants in the TRM. The TFs show a good agreement with the target TFs whereas the identified IFs tend to deviate from the target IFs.

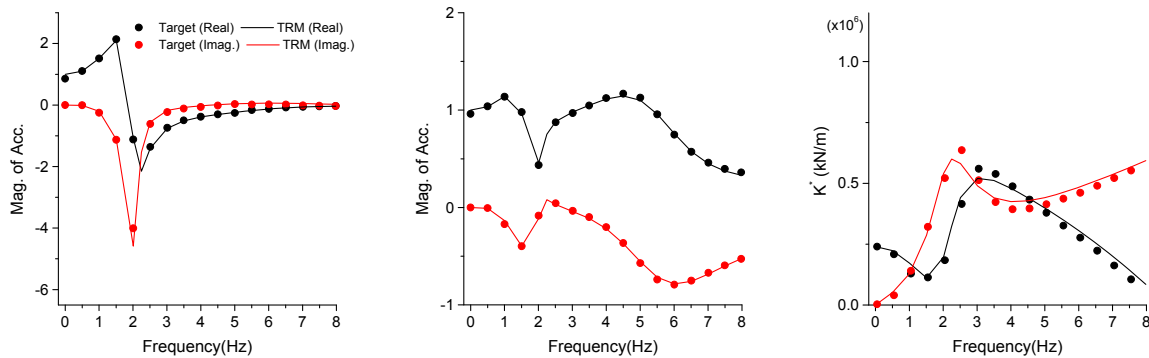


Figure 9: Transfer functions and impedance functions obtained from TRM: Masses of superstructure and footing treated as variables in TRM ( $\alpha_f=1$  and  $\gamma=1$ )

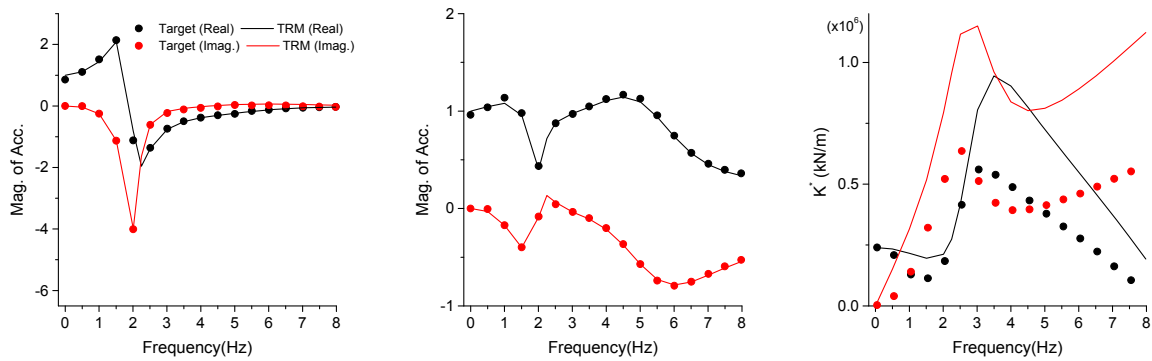


Figure 10: Transfer functions and impedance functions obtained from TRM: Masses of superstructure and footing treated as variables in TRM ( $\alpha_f=0.9$  and  $\gamma=1$ )

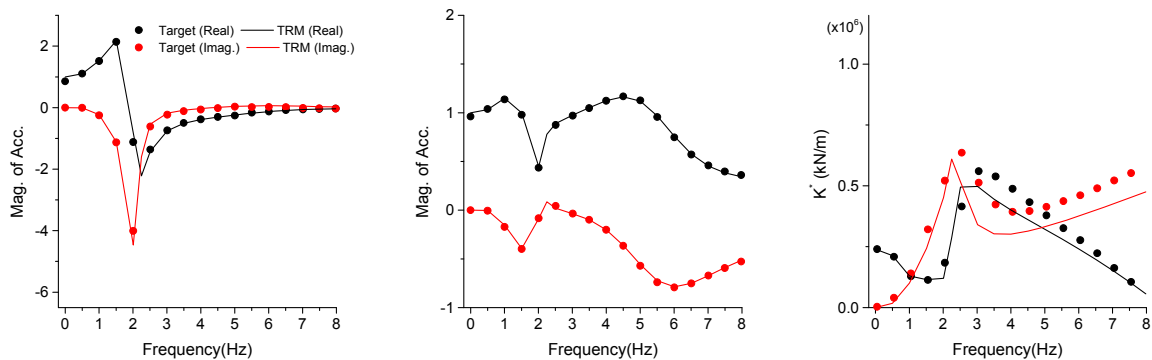


Figure 11: Transfer functions and impedance functions obtained from TRM: Masses of superstructure and footing containing discrepancy (multiplying by 0.8) treated as constants in TRM

#### 4 CONCLUSIONS

- This study proposes an identification method for frequency dependent impedance functions (IFs) of soil-foundation-structure systems. In this method, so-called “Gyro-Lumped Parameter Models (GLPMs)” are utilized to represent the IFs. The trust region method (TRM) is applied for finding variables comprising the mechanical elements of GLPMs where objective functions regarding the transfer functions (TFs) are minimized through the TRM procedure. In this paper, a simple two degrees of freedom system consisting of a superstructure and a footing supported by a  $2 \times 4$  pile group is targeted.
- In the TRM, objective functions in terms of the TFs of the footing is found to be effective to achieve the IF identification with appreciable accuracy. Applying the TFs of the superstructure to the objective function may dilute the characteristics of IFs, thus leading to undermining the accuracy of the IF identification.
- The static stiffness of soil-foundation systems should not be set as a variable in the TRM. The TFs show a large discrepancy while the identification of the IFs is inadequate. A certain discrepancy in the static stiffness may show a difference in lower frequency range. In practice, an approximate value of the static stiffness having the same order with the exact value could be acceptable to identify the IFs with appreciable accuracy in the middle and high frequency range.



- The damping ratio of superstructures tend to distort the IFs locally around the predominant frequency of superstructures.
- When the masses of superstructure and footing are unknown, and they are dealt with as variables in the TRM, the resultant TFs shows a good match with the target TFs whereas a significant difference may appear in the identified IFs. In addition, the masses are assumed to be constants with some discrepancies from the exact values in the TRM, although the TFs show a good agreement with the target TFs, the identified IFs tend to deviate from the target IFs.

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