

## **AN INTERFACE FOR CHECKING DYNAMIC COMPATIBILITY OF ROLLING STOCK WITH EXISTING RAILWAY BRIDGES**

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### **Abstract**

*Since the application of the Directive (EU) 2016/797 of the European Parliament and of the Council [1] on the interoperability of the rail system within the European Union and the 4th railway package, for any new rolling stock, its Route Compatibility before being placed into service could be carried out by the railway undertakings (RU). However, the dynamic compatibility of a train with existing rail bridges is always a technically complicated subject. In order to ensure a high level of traffic safety and facilitate the studies carried out by the RUs, SNCF offers a simplified, reliable and efficient interface to the compatibility check between rolling stock and existing SNCF Réseau bridges. This article aims to explain the analytical approach to build this interface and a quick overview of its application. By using the sum of a geometric progression to describe the deflection at mid-span of a beam with 2 articulated supports under the repetitive passage of repeated loads, the maximum acceleration at the resonance expressed by two parameters  $R_{dB}$  and  $R_{z,n}$  depending on the parameters of the rolling stock. If the bridges respect their frequency domain and have standard damping values, the maximum acceleration of a bridge only depends on the axle load, the wheelbase of the bogies, the distances between the groups of bogies and the number of axles. All of these load and geometric criteria will define the frequency domain of the rolling stock for a required speed.*

**Keywords:** Existing bridges, railway, dynamic behavior, spectral approach

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## 1 INTRODUCTION

The circulation of new rolling stock on the European rail network is only authorized after numerous checks have been carried out, in particular the compatibility checks between the Rolling Stock and the rail bridges. Previously these checks were made, except at high speeds, by using these quasi-static analyses: the effects of the vehicle's speed on the rail bridges were taken into account by means of an enhancement factor calibrated from experimental tests. This study is carried out on railway bridges in the 1960s by the Office for Research and Experimentation (ORE) et published in UIC leaflet 776-1R,1979 [2] by International Union of Railway (UIC). These quasi-static checks regulated by UIC 700 [3] and then by European standard EN15528 [4] are rather simple and allow to guarantee the safety of freight train traffic in Europe at speeds up to 120 km/h or even 100 km/h for heavy freight at 22,5 Tonnes per axle.

Nowadays, with fourth railway package for the open access for Rail passenger Services in Europe, the European directives impose through the Technical Specifications for Interoperability (TSIs) on RU to fulfill additional dynamic checks. This aims to guarantee the safety of traffic whose speed exceeds 100 km/h. These additional dynamic checks are required because of the high evolution of new passenger train's aggressivity at European level regarding new architectures, loads and higher design speeds than before:

- The length of the cars and the composition of the trains have diversified to adapt to the needs of passengers, particularly those with reduced mobility.
- The axle loads of passenger trains have increased from 17T to 20T or even 21.5T, in particular with the use of double decker trains to transport more passengers
- The design speed of passenger trains has increased from 160km/h to 200km/h to effectively compete with other modes of transport

These additional dynamic checks are essential for the safety of passenger traffic because they aim to exclude the risk of the bridge's resonance when the train passes over it. The resonance phenomenon occurs when the frequency of passage of the train's bogies on the rail bridge coincides perfectly with the bridge's natural frequency:

- The vertical vibrations at mid-span of the rail bridge deck are then amplified each time the bogie passes.
- The accelerations transmitted to the ballasted railway track, when they exceed a certain threshold, cause a loss of the ballast's cohesion which can cause a loss of rail-wheel contact or even a buckling of the track in hot weather.

Even if the probability of this risk is low but the severity is very high because the risk of the track buckling can cause the derailment of the train and the next one. It is absolutely imperative to exclude this risk in the compatibility checks of rolling stock with existing railway bridges before being placed into service. However, two problems arise for railway companies, slow down the open access competition trains in Europe:

- The number of existing railway bridges on a route is very high and the input data to perform the calculations are not easily accessible (there is no digitized database of rail bridge archives in Europe).
- There is no simplified method to check the dynamic compatibility between a train and a bridge. Dynamic calculations therefore require sufficient resources with the right skills for complex calculations.

Thereby SNCF RESEAU has developed a simplified spectral method to facilitate and secure dynamic compatibility studies between new passenger trains and the 15000 railway bridges on conventional lines in which the passenger train's speed exceeding 100 km/h.

## 2 THEORETICAL ANALYSIS

### 2.1 Fundamental analysis of a single load

In this study, the bridges are considered as a beam with 2 simply articulated supports. We define by  $y(x, t)$  the dynamic deflection of an isostatic beam of span  $L$  (m) subjected to a moving point load of intensity noted  $P$ . The position of the moving load varies according to its speed ( $v$  in m/s) and time ( $t$  in seconds).

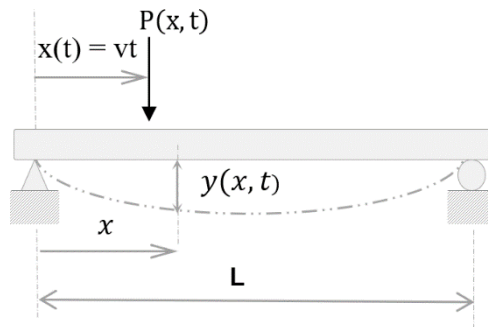


Figure 1: Deflection of an isostatic beam of span  $L$  under a mobile load  $P$  at  $x(t)$

Parameter	Value
Length	$L$ [m]
Modulus of elasticity	$E$ [N.m-2]
Moment of inertia	$I$ [m4]
Linear mass	$m$ [kg.m-1]
Damping rate	$\zeta$ (%)

Table 1: Deck model as Bernouilli-Euler simply supported beam

For a moderate load  $P$ , the transverse displacement remains small compared to the transverse dimensions of the beam. And the equilibrium between the forces of inertia, the internal forces and the external force  $P(x, t)$  applied to the beam allow to write this following partial differential equation

$$P(x, t) = m \frac{\partial^2 y(x, t)}{\partial t^2} + EI \frac{\partial^4 y(x, t)}{\partial t^4} \quad (1)$$

In order to simplify the resolution of this equation, the dynamic deflection  $y(x, t)$  of the beam under a moving point load  $P(x, t)$  can be expressed as a function of its deflection at mid-span  $U(t)$  and of its first eigenmode called  $\phi_1(x)$ :

$$y(x, t) = U(t)\phi_1(x) \quad (2)$$

Under a point load, the first eigenmode mode represents 99% of their global dynamic response with  $\phi_1(x) = \sin(\pi x/L)$ . Thus, the deflection of the beam can be expressed with very good precision as a function of its deflection at mid-span and of the first mode. Due to the independence of the two functions, one function of time  $U(t)$  and the other function of the

abscissa of the beam  $\phi_1(x)$ , a simplified equivalent model with one degree of freedom can be determined [5][6]. The equilibrium function can be written as follow, with  $M^*$  and  $K^*$  respectively the overall mass and stiffness of the equivalent 1 DOF

$$P(x, t) = m \frac{\partial^2 y(x, t)}{\partial t^2} + EI \frac{\partial^4 y(x, t)}{\partial t^4} \equiv M^* \ddot{U}(t) + K^* U(t) = P^*(t) \quad (3)$$

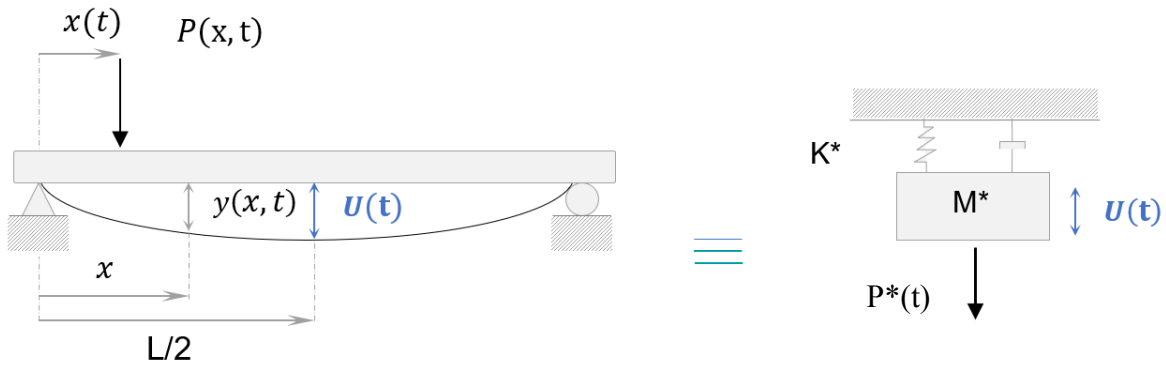


Figure 2: Transformation of a Bernoulli-Euler beam into 1 DOF oscillator

The parameters of this simplified model are determined by equalizing the kinetic and elastic potential energies of the two models. The equivalent load  $P^*(t)$  is calculated to produce the same work identical to the load  $P(x, t)$ :

$$K^* = EI \frac{\pi^4}{L^4} \int_0^L \sin^2(\pi x/L) dx = EI \frac{\pi^4 EI}{L^3} = \frac{48,7EI}{L^3} \approx \frac{48,7EI}{L^3} \quad (4)$$

$$M^* = m \int_0^L \phi_1^2(\pi x/L) dx = m \int_0^L \sin^2(\pi x/L) dx = mL/2 \quad (5)$$

$$P^*(t) = \int_0^L P(x, t) \phi_1(x) dx = \int_0^L P \delta(x - vt) \phi_1(x) dx = P \phi_1(vt) = P \sin\left(\frac{\pi}{L} vt\right) \quad (6)$$

The equivalent force depends on the parameters of the moving load (intensity and speed) but also on the span  $L$  of the beam. By introducing the pulsations of the beam  $\omega$  and the mobile load  $\tilde{\omega}$ , the differential equation of the equivalent simplified model is written:

$$\ddot{U}(t) + \omega^2 U(t) = \frac{P}{K^*} \omega^2 \sin(\tilde{\omega} t) \quad (7)$$

With

- $\omega = 2\pi F = \sqrt{\frac{K^*}{M^*}}$ , the pulsation of the first natural frequency beam  $F$  in Hz
- $\tilde{\omega} = \frac{\pi}{L} v$ , the pulsation of the moving load on the beam of span  $L$  and speed  $v$

By introducing the parameter  $\beta$  as the ratio between the two pulsations, the resolution of this equation helps to determine the maximum acceleration at mid-span of the beam in free oscillation under the action of a mobile load of intensity  $P$  according to few parameter [7]

$$\ddot{U} = \frac{P}{mL} f(\beta) \quad (8)$$

Where

$$f(\beta) = \frac{4\beta}{(1-\beta^2)} \left| \cos\left(\frac{\pi}{2\beta}\right) \right| \text{ and } \beta = (v/2FL) \quad (9)$$

## 2.2 Influence of two-axle bogie

Actually, railway bridges are mostly solicited by passenger train with two-axes bogies. The resolution can be carried out with two moving loads spaced by a distance noted  $d_B$ . The analog resolution with two loads differs simply in the initial conditions of the second load which are imposed by the first load. The acceleration in this case can be expressed as a function of the acceleration obtained under a single load weighted by an enhancement factor called bogie coefficient  $R_{dB}$

$$R_{dB} = 2 \left| \cos\left(\frac{\pi d_B}{2\beta L}\right) \right| \quad (10)$$

When the distance  $d_B$  is equal to  $2\beta L$ , the coefficient  $R_{dB}$  reaches 2. This phenomenon occurs while the frequency of passage of the second load ( $v/d_B$ ) coincides perfectly with the natural frequency of the beam ( $F$ ). From the equation (9)

$$F = \frac{v}{2\beta L} = \frac{v}{d_B} \quad (11)$$

Thus, for a beam of length  $L$ , we can define for each train's speed  $v$  a critical distance between the moving loads worth  $d_{critical} = 2\beta L = v/F$

## 2.3 Influence of repetitive loads - Coefficient $R_{\zeta,n}$ depending on the number of axle loads

When the frequency at which the axles or a group of axles pass over the bridge coincides with the natural frequency of the bridge, the dynamic free oscillation response of the second load joins to that of the first load and so on for the  $n$  repetitive loads. Thus, without damping, the maximum acceleration obtained for  $n$  repetitive loads will be multiply  $n$  times the response under the first load determined above. Taking into account the damping of the bridge, we can determine the additional coefficient  $R_{\zeta,n}$  which will be  $n$  without damping decreases toward 1 if the damping is infinite [7]. At resonance while  $F = v/d_{critical}$ , with  $d_{critical}$  the distance between the repetitive axles or group of axles, the coefficient of repetitive loads can be defined as

$$R_{\zeta,n} = e^{-2\pi\zeta^0} + e^{-2\pi\zeta^1} + e^{-2\pi\zeta^2} + \dots e^{-2\pi\zeta^{i-2}} + e^{-2\pi\zeta^{n-1}} = \frac{(1 - e^{-2\pi\zeta^n})}{(1 - e^{-2\pi\zeta})} \quad (12)$$

The figure 3 illustrates the evolution of this coefficient under the number of train axles and the bridge's damping.

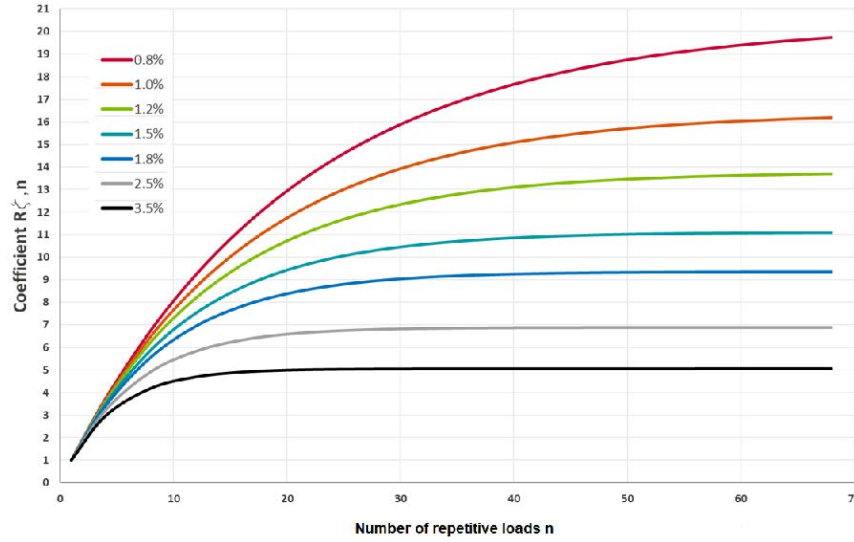


Figure 3: Variations of  $R_{\zeta,n}$  as a function of the number of axles for different bridge's damping

## 2.4 Particular case of harmonic as double of load frequency

While the bridge's natural frequency is double that of the moving loads,  $F=2v/d_{critical}$ . The deck oscillates twice between two consecutive loads. The influence of damping can be defined by the coefficient  $R_{2\zeta,\nu}$  as an analogue of the coefficient  $R_{\zeta,\nu}$  at resonance (equation (12)).

$$R_{2\zeta,n} = \frac{(1 - e^{-4\pi\zeta^n})}{(1 - e^{-4\pi\zeta})} \text{ while } F = 2v/d_{critical} \quad (13)$$

## 3 LAW OF BRIDGE'S NATURAL FREQUENCY AND ASSOCIATED SPEEDS

By analyzing the acceleration as a function of the parameter  $\beta$ , it is worth highlighting that when the pulsations of the beam and that of the single moving load are equal, the parameter  $\beta$  is equal to 1. A single moving load can therefore enter into resonance with the beam. However, thanks to the magnitude of the natural frequencies of all rail bridges ( $55/L$  at a minimum with  $L$  the length of the beam), it would be necessary for this single load to run at a speed greater than 400 km/h to cause this resonance.

The graph in the Figure 4 illustrates the impact of the natural frequency law of the bridge on the function  $f(\beta)$  which controls the acceleration response according to equation (8). We can note that the modification of the natural frequency law of the bridge only has the effect of translating the curves. The higher the F.L value, the more the acceleration peak will be shifted to a higher speed. We thus note that the acceleration increases quickly and exceeds the intermediate peaks for a speed:

- $v_{80/L} \approx 220 \text{ km/h } (= 61 \text{ m/s})$
- $v_{65/L} \approx 180 \text{ km/h } (= 50 \text{ m/s})$
- $v_{55/L} \approx 150 \text{ km/h } (= 42 \text{ m/s})$

It's worth to highlight that for these bridge's frequencies and at these speeds, the critical distance  $d_{critical}$  is similar and determined by :

$$d_{critical} = \frac{v}{F} = \frac{61}{80/L} \approx \frac{50}{65/L} \approx \frac{42}{55/L} = 0,76L \quad (14)$$

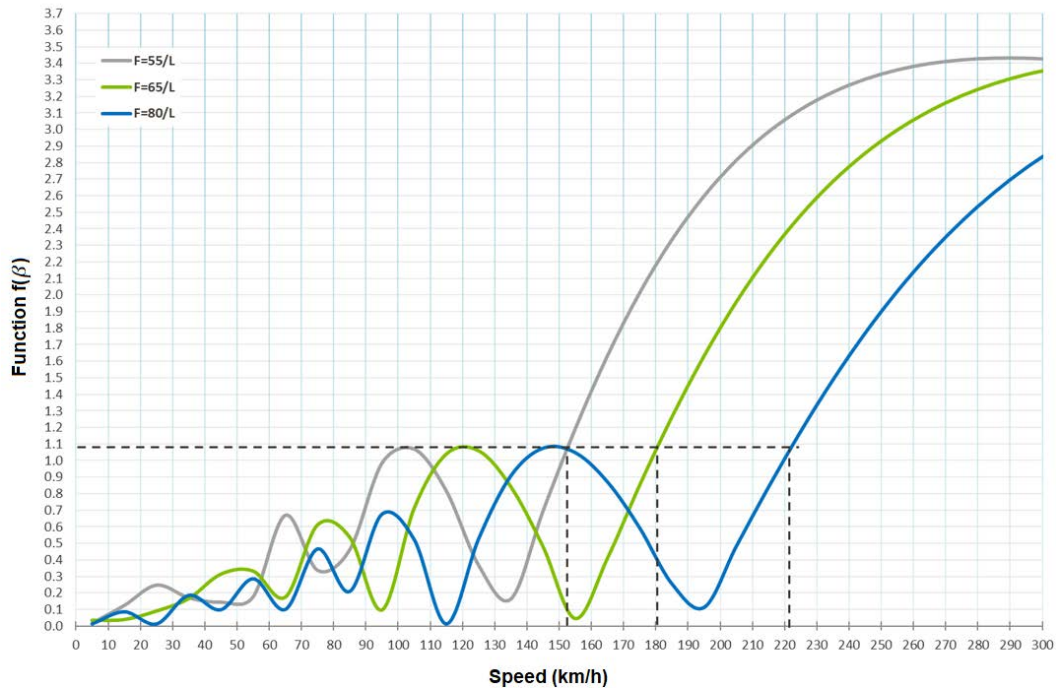


Figure 4: Acceleration of beam in function of the speed for different 1<sup>st</sup> frequency

The offset of this “limit” speed from which the acceleration under a single load increasing greatly is proportional to the frequency ratio because they occur at the same value of  $d_{critical}$ . It is possible to deduce for any frequency law the value of the limit speed from which the accelerations under a single load increase critically. It is also noted that the maximum of the function  $f(\beta)$  is approximately 1.07 when the speed of the mobile load remains lower than the limit speed determined for each natural frequency law.

In another hand, the intermediate peak is reached when  $\beta$  reaches  $1/4$  corresponding to the maximum of the  $\cos(\pi/2\beta)$  function (equation (10)). For any frequency law, we can thus determine the speed of the intermediate peak and the value of the associated critical distance between 2 loads:

$$\beta = \frac{v}{2LF} = 1/4 \rightarrow v \simeq \frac{LF}{2} \rightarrow d_{critical} = L/2 \quad (14)$$

We thus find the critical speed for the intermediate peak for the 3 laws studied, for example with  $F = 80/L$

$$v \simeq \frac{LF}{2} = \frac{L \times 80/L}{2} = 40 \text{ (m/s}^2\text{)} \quad (15)$$

- $v_{80/L} = 40 \text{ m/s} \approx 145 \text{ km/h}$
- $v_{65/L} = 32,5 \text{ m/s} \approx 120 \text{ km/h}$
- $v_{55/L} = 27,5 \text{ m/s} \approx 100 \text{ km/h}$

The minimum law of  $55/L$  corresponds to a static design of the bridges. The  $80/L$  law corresponds to a design for speeds up to 200km/h.



## 4 VALIDATION AND APPLICATIONS IN COMPATIBILITY CHECKS

### 4.1 Comparison with numerical modeling

To illustrate this spectral approach, take the example of the 20m span bridge. The figure 5 shows the variation of the acceleration in  $\text{m/s}^2$  at mid-span of a bridge according to the speed of a moving load of 200kN. The graphs are given in free and forced oscillation (when the load is on the bridge) according to the analytical method and by using of a numerical simulation software. This bridge respects the natural frequency law  $F=80/L$  and has the linear mass of 18370kg/ml and a damping of 1,8%.

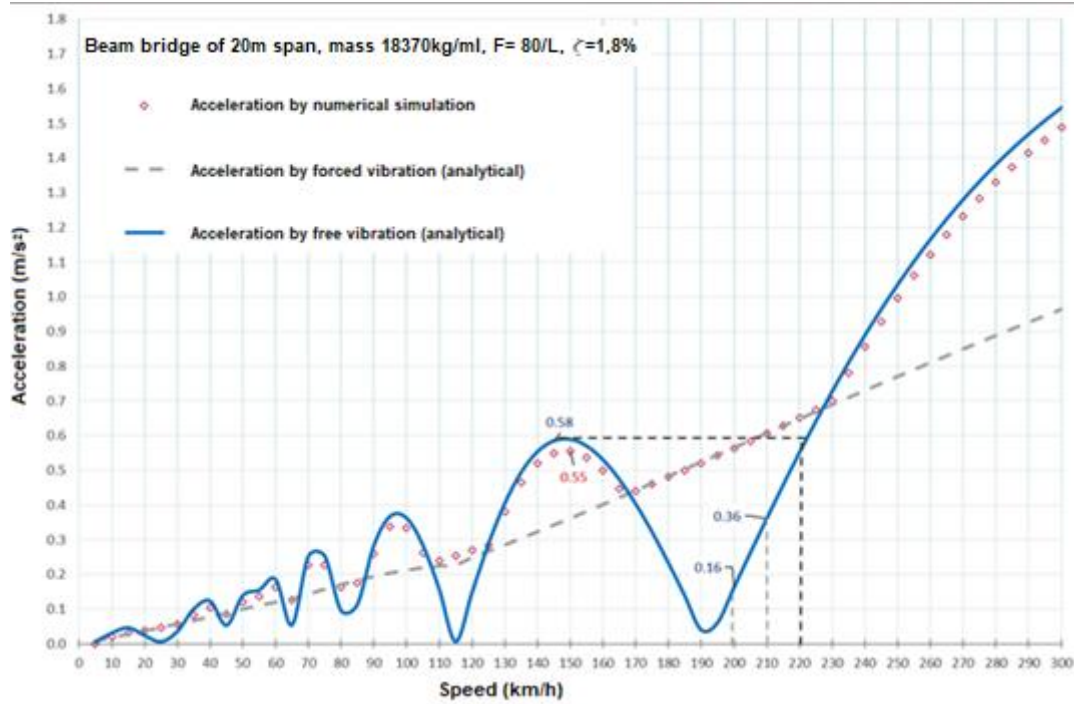


Figure 5: Comparison of acceleration in function of speed of bridge

The analysis 5 allows us to identify the speed range at risk which is indeed above 220km/h because the accelerations increase sharply over this speed. In the same way, the speed limits for the other frequency laws have also been adjusted.

The analytical solution without damping is very close to numerical simulations carried out with a damping of 1.8%. The influence of damping on the first load crossed the bridge is low, and this can be taken into account by a reducing factor  $e^{-\zeta\pi/2\beta}$  (further demonstrations in [7]). With this value of damping, the coefficient  $f(\beta) \times e^{-\zeta\pi/2\beta}$  is approximately equal to 1. By using of the analytical formula, the maximum value of the acceleration for a speed not exceeding 220 km/h (speed limit for a frequency law  $F=80/L$ ) can be found.

$$\ddot{U}_{max}(v < 220\text{km/h}) = -200.10^3 / (18370 \times 20) \times 1.07 = -0.58 \text{ with } \zeta=0 \quad (16)$$

$$\ddot{U}_{max}(v < 220\text{km/h}) = -200.10^3 / (18370 \times 20) \times 1.0 = -0.54 \text{ with } \zeta=1,8\% \quad (17)$$

The value of the acceleration obtained with the numerical simulation is 0.55  $\text{m/s}^2$ , for an analytical value of 0.54  $\text{m/s}^2$ , meaning an approximately of 2%. The spectral method has a suitable estimation of acceleration and more accurate to deal with a high number of bridges of the network.



## 4.2 Repetitive axle loads

For the same bridge of 20m span, we can determine the maximum response for 200kN repetitive loads. For a damping of 1,8%, at resonance, the coefficient  $R_{\zeta,n}$  is limited to 9,2 (see Figure 3). The maximum acceleration obtained is therefore  $0,54 \times 9,2 \approx 5 \text{ m/s}^2$ . This value of acceleration at 220km/h appears while distance between 2 consecutive axles of the rolling stock reaches  $0,76L$  (equation 14), so 15,2m. This distance can be found among the rolling stock which are authorized to operate in French network up to 200 km/h [8]. This peak is too high, the traffic speed for bridges with a natural frequency of 80/L will be reduced to 210km/h or 200km/h with a safety margin. Thus for a speed of 200km/h the acceleration will be limited to  $1.5 \text{ m/s}^2$  ( $= 9,2 \times 0,16 \text{ m/s}^2$ ) or at 210km/h to  $3,3 \text{ m/s}^2$  ( $= 9,2 \times 0,36 \text{ m/s}^2$ ) (see Figure 5).

Focus now into the intermediate acceleration peak at  $\beta=1/4$  occurs at the speed around 145km/h ( $F=80/L$ ) which exited by the critical distance  $d_{critical} = L/2 = 10\text{m}$  (see equation 14). However, this distance between axles 10m doesn't correspond to any train authorized to run up to 145km/h. The rolling stock having this distance is actually limited to 100km/h on the network. Therefore, for this intermediate peak, the calculation must be carried out for a critical distance of 20m to cover the case in which the bridge's frequency is double of load frequency (see section 2.4)

$$F = \frac{2v}{d_{critical}} \rightarrow d_{critical} = L \quad (18)$$

The acceleration obtained is  $0,54 \text{ m/s}^2 \times 4,9 \approx 2,65 \text{ m/s}^2$  because of maximum value of  $R_{2\zeta,\nu} = \frac{(1-e^{-4\pi\zeta^n})}{(1-e^{-4\pi\zeta})}$  is 4,9.

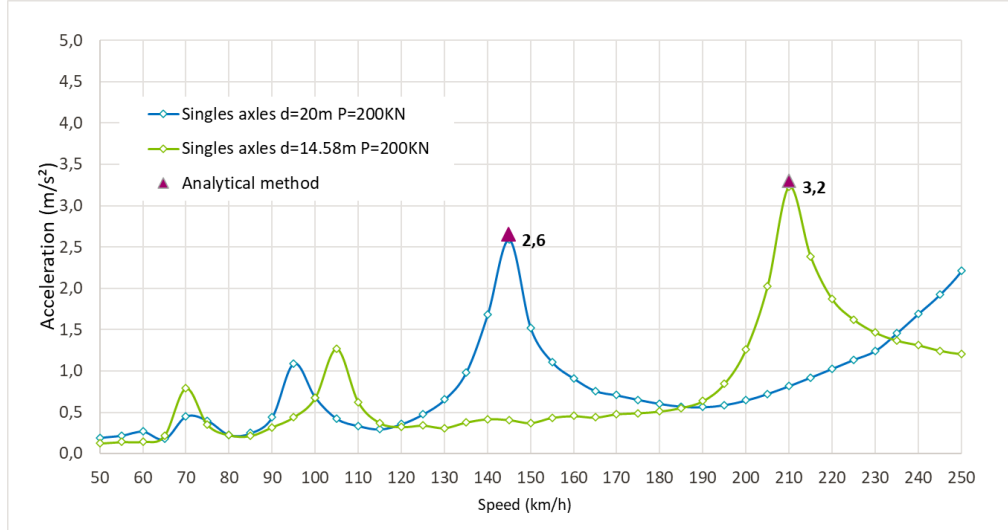


Figure 6: Validation of analytical method

The figure 6 illustrates the comparison between the analytical method and the results of the numerical calculations for the speeds of 210km/h and 145km/h (intermediate peak). We note that the results obtained with the analytical method are very similar with less than 3% deviation. However, the gain in time and simplicity is higher for the analytical approach. Because using of numerical method, for each speed, a vehicle must be modeled in order to evolve the distance between the axles in order to determine the resonance.

## 5 CONCLUSIONS

This spectral method has allowed to identify the main parameters that lead the dynamic response of bridges under repetitive loads. Speed limits and critical distances between repetitive axles have been defined for each railway bridge according to its natural frequency law. By using this approach to assess the existing bridges of the SNCF network for which the main characteristics as type, span and slenderness are known, numerous parametric studies are performed to figure out the sensible bridges. Among all bridges, 12000 bridges (80% of the national network) were validated without further dynamic calculations [8]. 3000 bridges were identified as having a dynamic risk under to the new rolling stock. For these bridges, traffic speed must be lower or equal to the limit speed corresponding to their natural frequency law. These 3000 bridges were consequently notified in the SNCF RESEAU infrastructure register (RINF) and will require specific controls by the railway undertakings.

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