

LARGE-SCALE VULNERABILITY ANALYSIS OF HISTORICAL MASONRY BUILDINGS

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Abstract

Masonry structures represent a large amount of the built heritage all around the world. Many of those buildings have significant historical and cultural value but they are characterized by high levels of vulnerability to multiple hazards. Although the vulnerability of masonry buildings to earthquakes has been largely investigated in previous studies, there are still major knowledge gaps on vulnerability to other hazards such as soil settlements and environmental degradation.

In this study, a new methodology for multi-hazard vulnerability assessment of historic masonry buildings is proposed. Aiming at supporting nation-wide disaster risk prioritization, a large-scale probabilistic approach is delineated. The Italian built heritage was considered for the implementation of the methodology. A huge data set on historical masonry buildings was collected and processed to define realistic building archetypes, with special focus on residential buildings built before 1920. Statistical data on some properties of the building archetypes are compared to empirical relationships available in past treatises and rules of thumb.

A multi-scale structural modeling procedure was used, looking to both global behaviour via equivalent frame modeling of the structure and local behaviour of individual elements and sub-systems. Furthermore, the most influential structural parameters were identified through a sensitivity analysis. Analysis results show the different impact of such parameters depending on the type of hazard under consideration.

Keywords: Historical masonry buildings, analytical vulnerability assessment, methodology, archetypes, system-scale structural models, sub-system-scale structural models.

1 INTRODUCTION

Italy is home to numerous historical masonry buildings that testify to the country's rich and varied architectural heritage. [1,2] Approximately 61% of the buildings on our territory are made of masonry [3,4]. These buildings, which date back to different historical periods, were built with traditional materials such as stone, brick and terracotta and were designed to last for many generations. Until the large-scale introduction of reinforced concrete, stone had always been the material most used by man, proving to be structurally reliable over the years. Despite the passage of time and difficult climatic conditions, many of these buildings are still standing and represent a valuable cultural heritage for the country.

The study of these structures and their structural behavior should not be considered a purely historical or artistic aspect, but a fundamental condition for the development of seismic risk prevention and reduction strategies.

From the point of view of seismic risk mitigation, an in-depth vulnerability analysis is necessary for existing buildings.[5] This must be carried out for entire territorial areas with different levels of detail depending on the quality and quantity of information available for the buildings. The complexity of this type of analysis is due to the multiplicity of factors that affect the vulnerability of historic buildings such as: the variability of structural forms, the different quality and type of materials used, and the various structural modifications carried out over the years.

The following study concerns the structural behavior of historical masonry buildings on the national territory. This work was carried out by integrating the study on the design criteria of existing masonry with the data obtained from the "Cartis" database [6,7], which was developed in the DPC-ReLUIS 2014-2016 project. In particular, the aim of the following work was to identify a methodology to assess the vulnerability of historical masonry buildings on a large scale.

We started from the characteristics of the existing masonry structures when certain features vary (e.g., type of masonry, age of construction, number of floors, etc.) and determined a building archetype that could represent a family of masonry buildings present on the national territory.

This archetype was subsequently modelled with the aid of MATLAB programming software using the equivalent frame modelling criterion (EFM) [8] with vertical and horizontal elements simulating pier and spandrel panels, respectively [2,9], which in addition to being suggested by various normative codes, is the most widespread method for the analysis of masonry structures to date.

Furthermore, in order to take into account, the influence of arches and/or vaults on structural performance, ARCO software [10] was used. Therein, the masonry arch is assessed based on the static theorem of limit analysis, allowing the computation of both vertical actions and thrust forces that the arch transfer to its supporting masonry walls and/or columns.

Subsequently, to evaluate a level of safety that could be extended to the plurality of the historic masonry buildings, an analysis was firstly carried out on the model of the archetype by means of gravitational loads, then a non-linear static analysis under displacement control (pushover).

In conclusion, a sensitivity analysis was conducted on the most influential parameters during the analysis and in the model in order to evaluate the results and arrive at possible conclusions and improvements of these structures.

2 METHODOLOGY

The conservation of historic masonry buildings is a complex and challenging task that requires a comprehensive approach to understanding their vulnerability. This work deals with the development and the implementation of a new methodology in order to assess the large-scale vulnerability of historic masonry buildings, based on statistical data from a database (e.g., Cartis). This methodology uses a combination of statistical data and design information from the period, through a multilevel modelling and through a identification of the most influential parameters to assess the current state of the buildings and identify their vulnerabilities, as illustrated in the flowchart in Fig. 1.

This methodology is able to provide a more accurate and reliable assessment of building vulnerability through the use of data from a comprehensive database; this lead to effective and targeted conservation interventions so as to ensure the preservation of historic masonry buildings for future generations.

2.1 Identification of historical building archetype

CARTIS is a database that provides mechanical and geometrical characteristics of masonry buildings in Italy [5]. It is a valuable tool for engineers and researchers who are studying the vulnerability of these historic structures.

The data collected in the CARTIS database are based on a comprehensive survey of masonry buildings in Italy, which includes information about the building's age, construction materials, size, and layout, as well as its structural features such as walls, arches, and vaults. This data is then used to create statistical models that can be used to assess the vulnerability of these buildings in the case of natural disasters such as earthquake.

The data in the CARTIS database can also be used to inform the design of new masonry buildings and to support the development of new building codes and standards. By providing a comprehensive overview of the mechanical and geometrical characteristics of masonry buildings in Italy, CARTIS helps to ensure that these structures are safe and resilient in the face of future hazards.

The evaluation strategy adopted in this study was applied to an archetype masonry building representative of the majority of masonry buildings fabricated in Italy before 1920, i.e., prior to the use of reinforced concrete [13].

The building archetype was generated from information extracted from the Cartis database. First of all, the information was filtered by age of construction in order to filter and take into account only pre-1920 buildings. Then the masonry types were investigated, and the respective geometric-structural characteristics were investigated in order to completely define the building. The relevant properties considered for the construction of the case study were:

- Number of storeys
- Presence and type of vaults
- Type of floors
- Average floor area
- Average wall thickness
- Average spacing between walls
- Average storey height
- Average ground storey height.

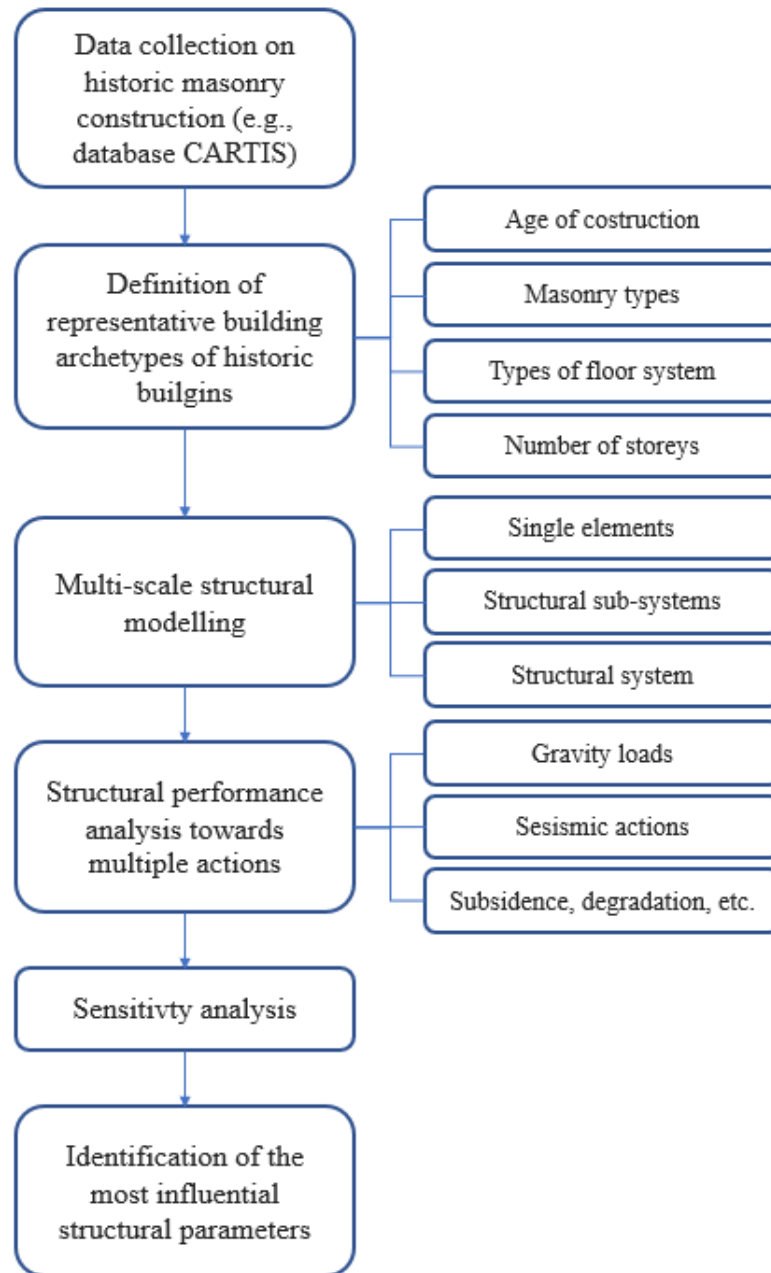


Figure 1. Flowchart methodology.

A statistical analysis was carried out on the extrapolated data, leading to the conclusion that the most recurring masonry types in the country for pre-1920 masonry buildings are: (i) irregular rough stone masonry (A2) covering the 53% of cases, (ii) regular solid brick masonry (C2) which is found in 26% and (iii) regular square stone masonry (C1) which is found in 10% of cases, as shown in Fig. 2. Based on this data, the A2 masonry type was chosen.

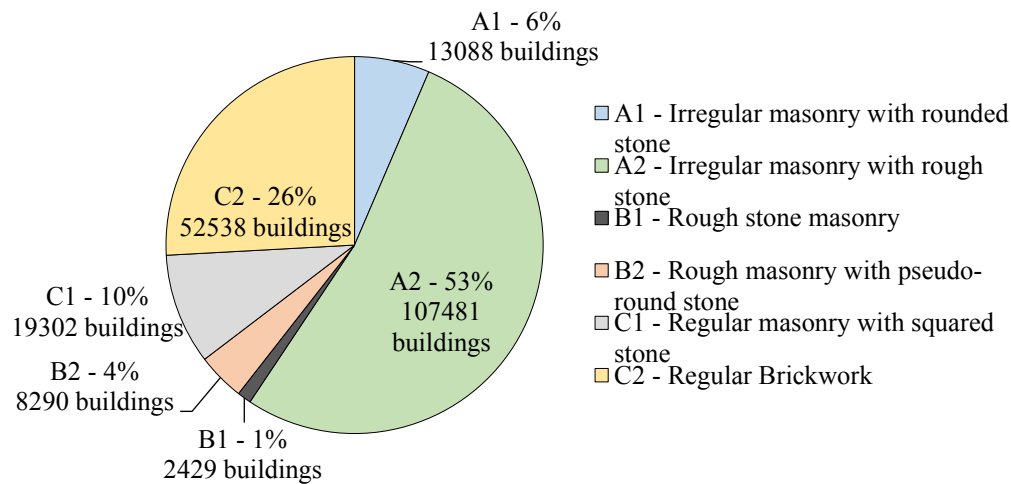


Figure 2. Types of masonry in Cartis database.

Subsequently, as already discussed, the other geometric data were extracted to generate the building and an Italian pre-1860 building archetype was identified. Its geometric characteristics are shown in Table 1. Concerning the mechanical characteristics, the “A2: irregular masonry with rough stone” under study was regarded as “rough stone masonry with parameters of uneven thickness” according to the Italian building code, i.e., NTC 2018 [11]. The mean values of mechanical properties assumed were set to the average of those provided by the Italian building code commentary, i.e., Circular n° 7/2019 [12], shown in Table 2.

Age of construction	<1860
Type of masonry	Irregular rough stone masonry (A2)
Number of storeys	3
Presence of vaults	Yes
Type of vaults	Barrel vaults at 1 st floor
Type of floors	Timber floors at 2 nd and 3 rd floors
Average floor area	150 m ²
Average wall thickness	0.75 m
Average wall spacing	5 m
Average height of storeys	3.50 m and 4 m at the ground floor

Table 1. Properties of the building archetype.

f	τ_0	f_{v0}	E	G	w
[MPa]	[MPa]	[MPa]	[MPa]	[MPa]	[kN/m ³]
-	min - max	-	min - max	min - max	-
2	0.035 - 0.051	-	1020 - 1440	340 - 480	20

Table 2. Mechanical properties of A2 masonry type.

Figure 3 shows the ground floor plan, typical plan and orthogonal vertical sections AA' and BB' of the selected building archetype.

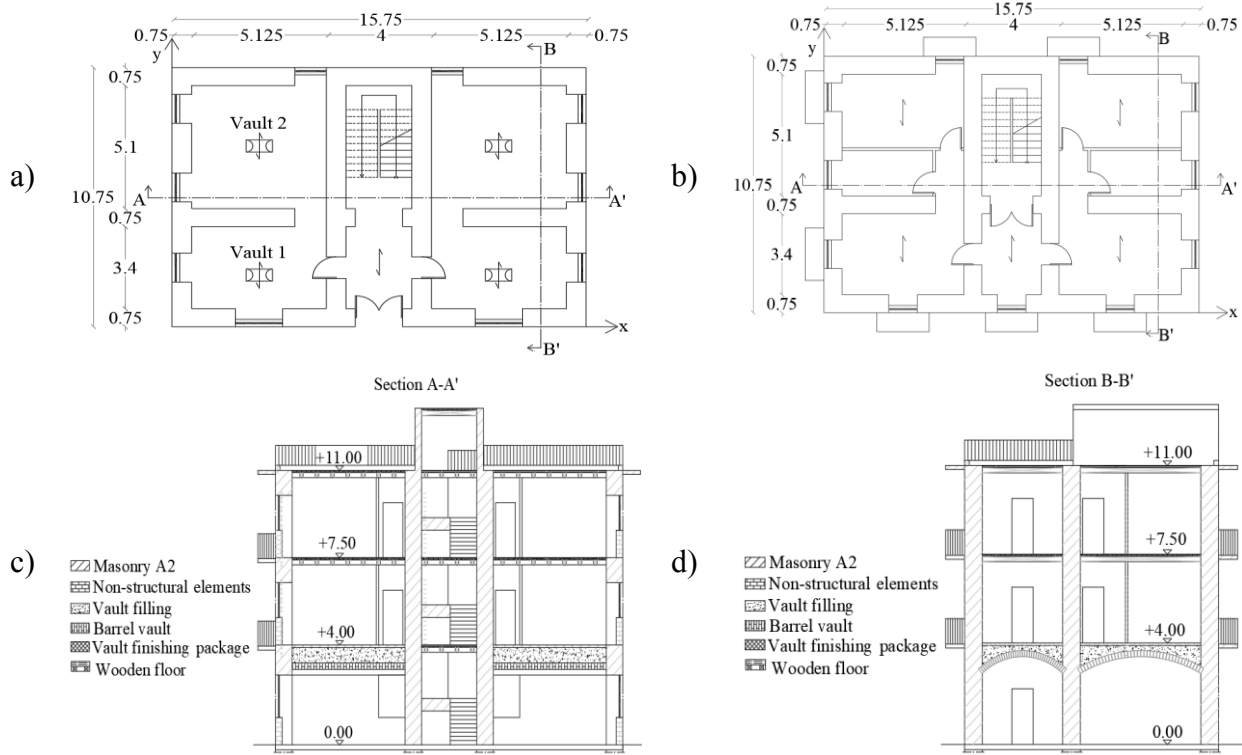


Figure 3. Archetype building: (a) ground floor plan; (b) typical floor plan; (c) section AA'; (d) section BB'.

2.2 Structural modelling of historical building archetype

After the geometrical definition of the building archetype and after the modeling of the uncertainties related to the materials, it was generated through the use of an automatic procedure implemented in MATLAB. The building was generated according to the hypothesis of a rectangular plan and with an equivalent frame model. The last one was characterized by macro-elements assimilated to beam elements connected by rigid offsets suitably sized to consider the greater or lesser deformability of the nodes, (Figure 5).

Specifically, the effective height of the piers panels was calculated using the criterion proposed by Lagomarsino et al. [13], for both external and internal piers panels.

The floor systems were modeled through equivalent diagonal trusses with equivalent diagonal roofs with appropriate stiffness depending on the timber floor and/or vault.

The vault was first resorted to an equivalent plate modeling [14] and then the equivalent plate was modeled with diagonal connecting rods [15] with a suitable equivalent stiffness. The vault was then studied through ARCO software [10] by assimilating it to an arch and obtaining the reactions to the kidneys that could be implemented in the MATLAB program. The arch schematization of the barrel vault 2 and the values of the reactions for the vaults 1 and 2 (Figure 3a) are respectively shown in Figure 4 and Table 3.

Vault	H [kN]	V [kN]
1	114.7	61.8
2	52.1	40.8

Table 3. Constraint reactions of barrel vaults.

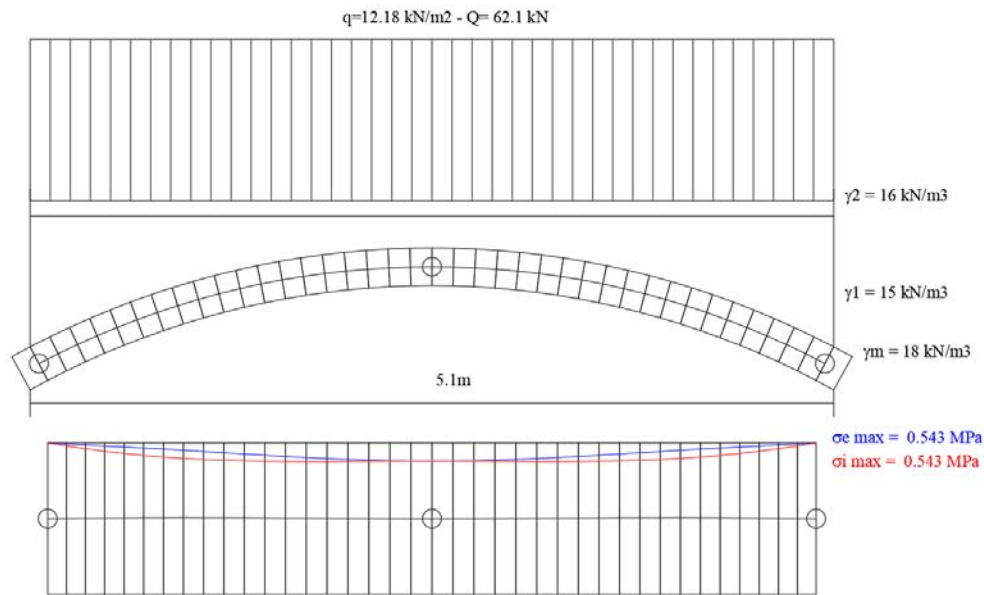


Figure 4. Arch schematization of the barrel vault 2 through the ARCO software

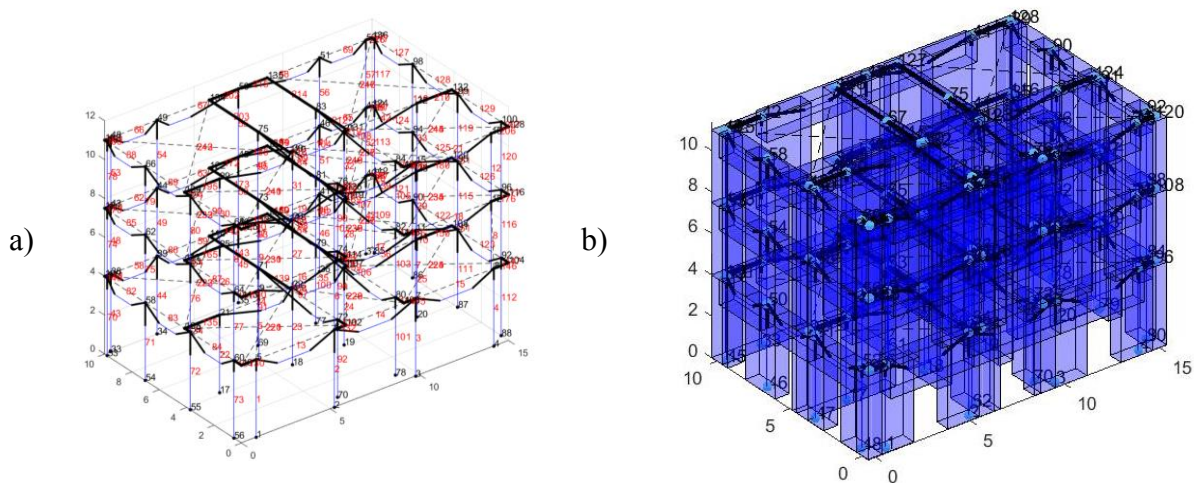


Figure 5. Structural system of archetype building: (a) equivalent frame view; (b) solid view.

3 MULTI-SCALE ANALYSIS OF BUILDING ARCHETYPE

Two kinds of analyses were carried out: a gravitational analysis and a non-linear static analysis.

The gravitational analysis was performed by analysing individual walls. The checks were found to be satisfied, and the ratio of the acting normal stress to the ultimate normal stress was assessed. More specifically, the $\zeta_{v,i}$ was evaluated, i.e. the ratio between the maximum value of the vertical load capacity by the i -th pier panel of the wall under investigation and the value of the acting vertical overload.

Figure 6 shows the results of this coefficient. The calculations for the three most loaded walls (those in the x direction) are shown. The value of this coefficient for the walls in the y direction is always between 3 and 16. To better understand the numbering of piers panels,

wall 1 was shown as an example in Figure 7. The numbering has been formulated as (i,j,k) with i equals the piers, j the spandrels and k the wall under consideration.

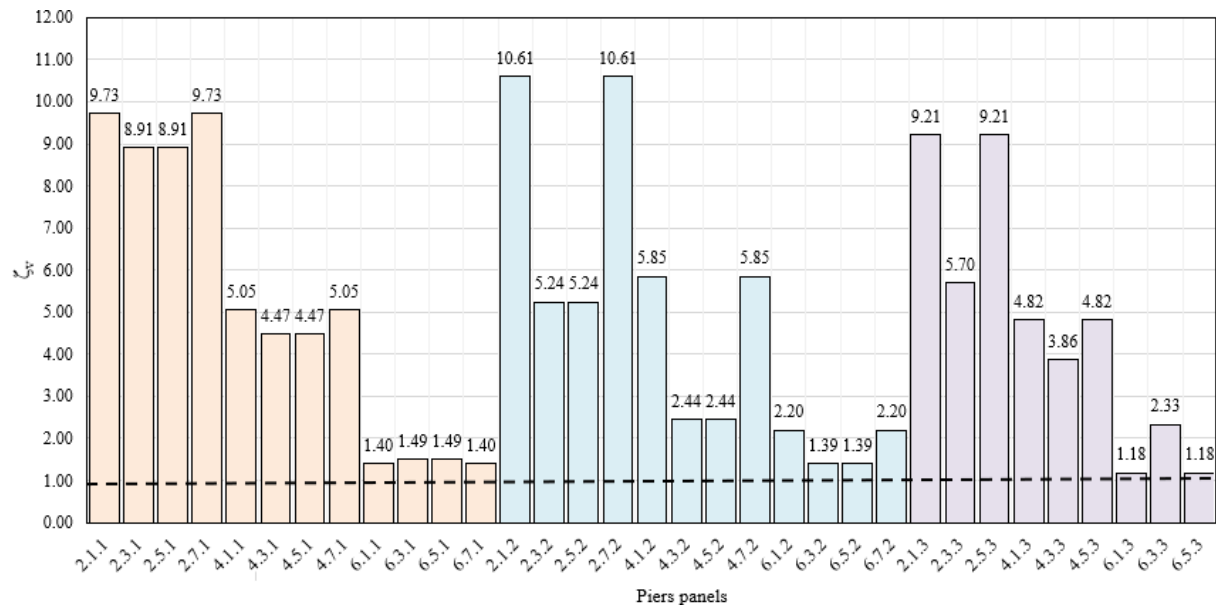


Figure 6. Demand-to-capacity ratio of pier panels under gravity loads.

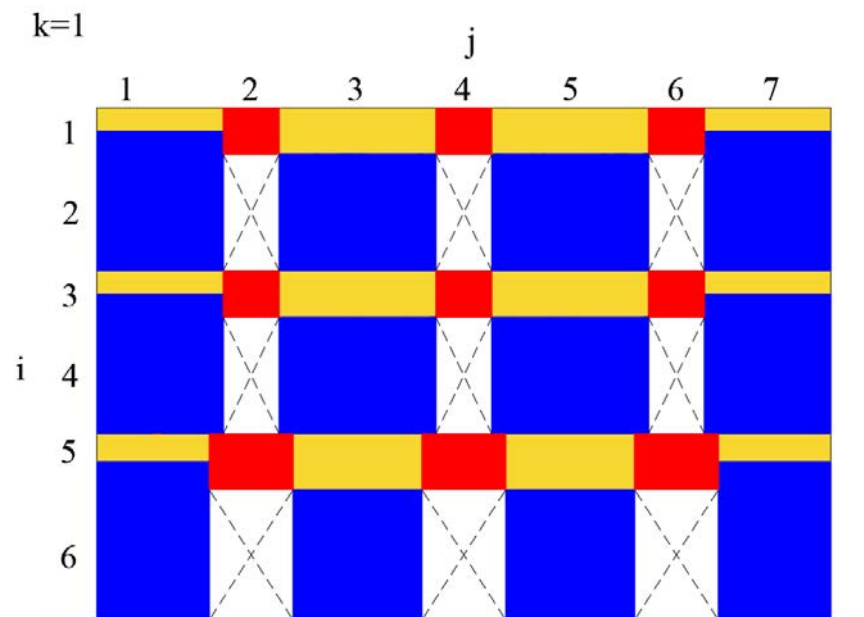


Figure 7. Macro-element modelling of wall 1.

The pier panels 6.1.1, 6.3.1, 6.5.1, 6.7.1, 6.3.2, 6.5.2, 6.1.3, and 6.5.3 has a lower slenderness ratio compared to the other panels. This was due to the unloading of the horizontal thrust caused by the presence of the barrel vault.

One solution for those pier panels was to limit the use of the i -th portion to the maximum load bearing capacity, or to introduce chains to reduce the thrust acting on the pier panels.

Subsequently, a non-linear static analysis was performed on the building archetype considering the average values of the mechanical properties, Figure 8.

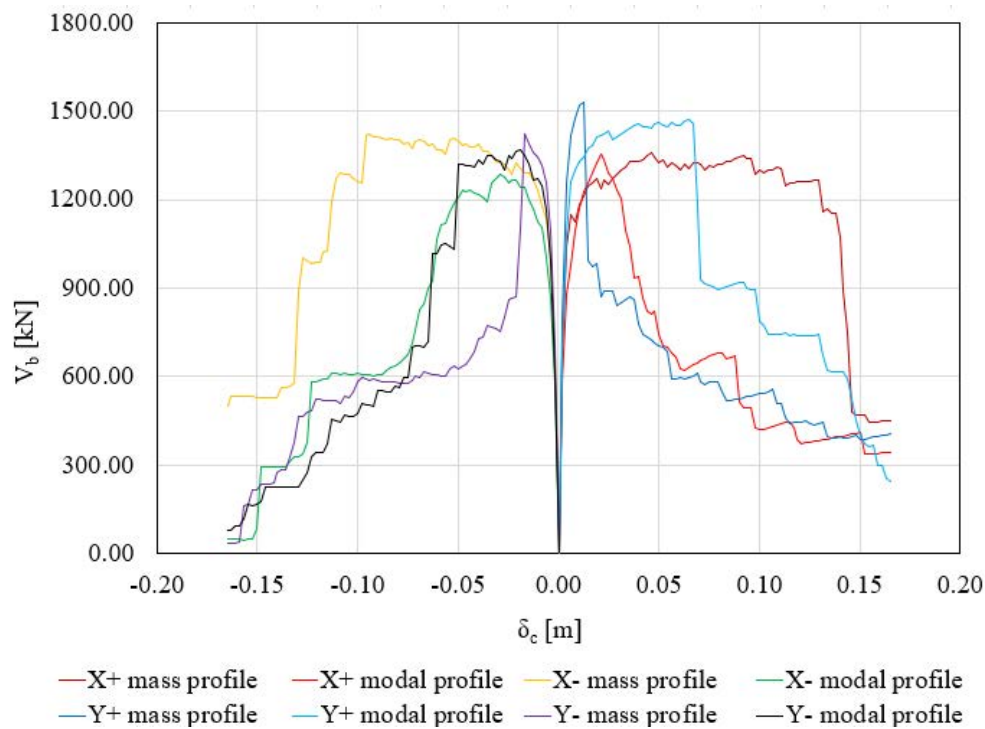


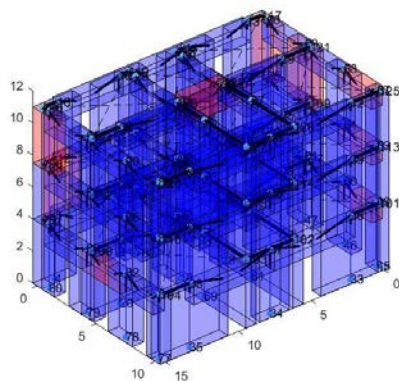
Figure 8. Pushover capacity curves of case-study building.

The results of the analysis outline that the building archetype had a greater shear resistance at the base along the y-direction than the x-direction; the latter one offered a greater ability to redistribute stresses because it is also the direction with greater wall length and less thin piers panels. The shear failure mechanism governed the analyses along the y-direction.

It was observed that once the maximum bearable shear was reached at the base, more than one panel reached the shear failure mechanism, in addition to shear crises in the other panels, this led to a drastic reduction of the load-bearing capacity of the wall.

Figure 9 shows the building during the analysis along the y-direction with a profile of perhaps proportional to the masses (blue curve in Figure 8) during the step at maximum shear and immediately afterwards. Specifically, the blue panels are panels that have not suffered damage, the red panels are panels that have suffered a buckling failure and the green panels are panels that have suffered a shear failure.

a)



b)

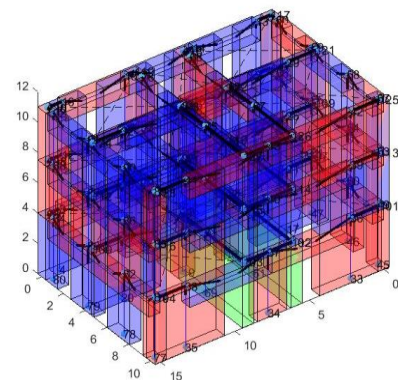


Figure 9. Identification of failure modes during seismic analysis: (a) maximum base shear (b) last step.

In addition to the numerous spandrel and piers panels in failure due to pressure deflection, it was found that in the step following the maximum shear at the base, the simultaneous failure of two piers panels due to shear is evident. These two panels, central and very large, are the panels carrying the weight of the central staircase. To this aim the curve showed a rapid reduction in bearable shear (change of -43%).

On all pushover capacity curves, five damage states were identified as:

- DS1 at 70% of the maximum base shear ($V_{b,max}$) on the increasing branch of the capacity curves.
- DS2 at $V_{b,max}$.
- DS3 at 20% degradation of $V_{b,max}$.
- DS4 at 50% degradation of $V_{b,max}$.
- DS5 when the ultimate displacement is reached.

The damage states considered are depicted in Figure 10 where they are identified on a significant global pushover curve of a building.

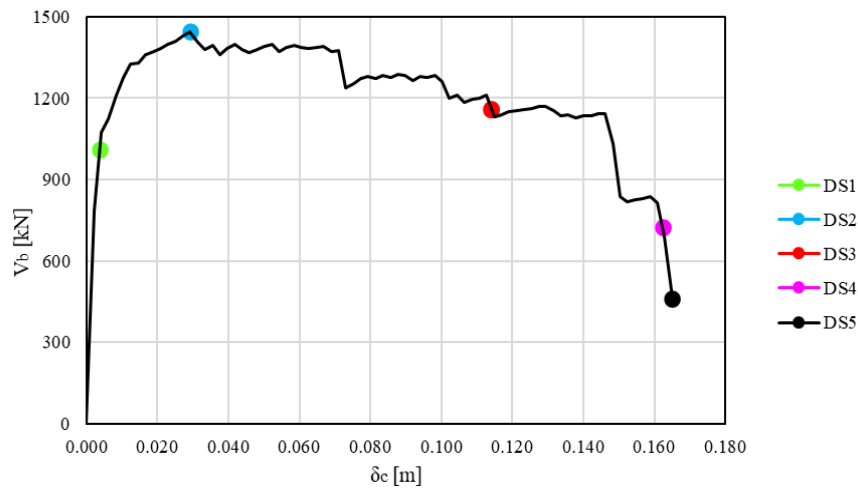


Figure 10. Definition of damage states on pushover curve.

Table 4 and 5 show the difference between the two distributions of forces in term of damage states (DSs).

Damage state	Force profile	V_b [kN]	ΔV_b [%]	δ_c [m]	$\Delta \delta_c$ [%]
DS1	Mass	950.32	0	0.00364	+56%
DS1	Modal	949.13		0.00566	
DS2	Mass	1357.6	0	0.04595	-55%
DS2	Modal	1355.90		0.02089	
DS3	Mass	1086.08	0	0.13953	-76%
DS3	Modal	1084.72		0.03383	
DS4	Mass	678.8	0	0.14465	-61%
DS4	Modal	677.95		0.05647	
DS5	Mass	449.7	+24%	0.16500	0
DS5	Modal	343.40		0.16500	

Table 4. Difference in displacement and shear for different DS for the two force profiles in the X direction.

Damage state	Force profile	V_b [kN]	ΔV_b [%]	δ_c [m]	$\Delta \delta_c$ [%]
DS1	Mass	1069.81	-4%	0.00332	+17%
DS1	Modal	1030.05		0.00388	
DS2	Mass	1528.3	-4%	0.01253	+417%
DS2	Modal	1471.50		0.06475	
DS3	Mass	1222.64	-4%	0.01373	-402%
DS3	Modal	1177.20		0.06889	
DS4	Mass	764.15	-4%	0.04051	-220%
DS4	Modal	735.75		0.12967	
DS5	Mass	406	-40%	0.16500	0
DS5	Modal	245.00		0.16500	

Table 5. Difference in displacement and shear for different DS for the two force profiles in the Y direction.

4 SENSITIVITY OF SEISMIC CAPACITY TO MECHANICAL PARAMETERS

After performing the non-linear static analyses considering the average characteristics in Table 2, a sensitivity analysis was carried out by varying the mechanical parameters. Specifically, the shear strength τ_0 and the elastic moduli E and G , varying them simultaneously in their range considering the minimum and maximum values.

Figure 11 shows the capacity curves for the two directions (X and Y) and the two force profiles (proportional to masses and proportional to the first mode of vibration).

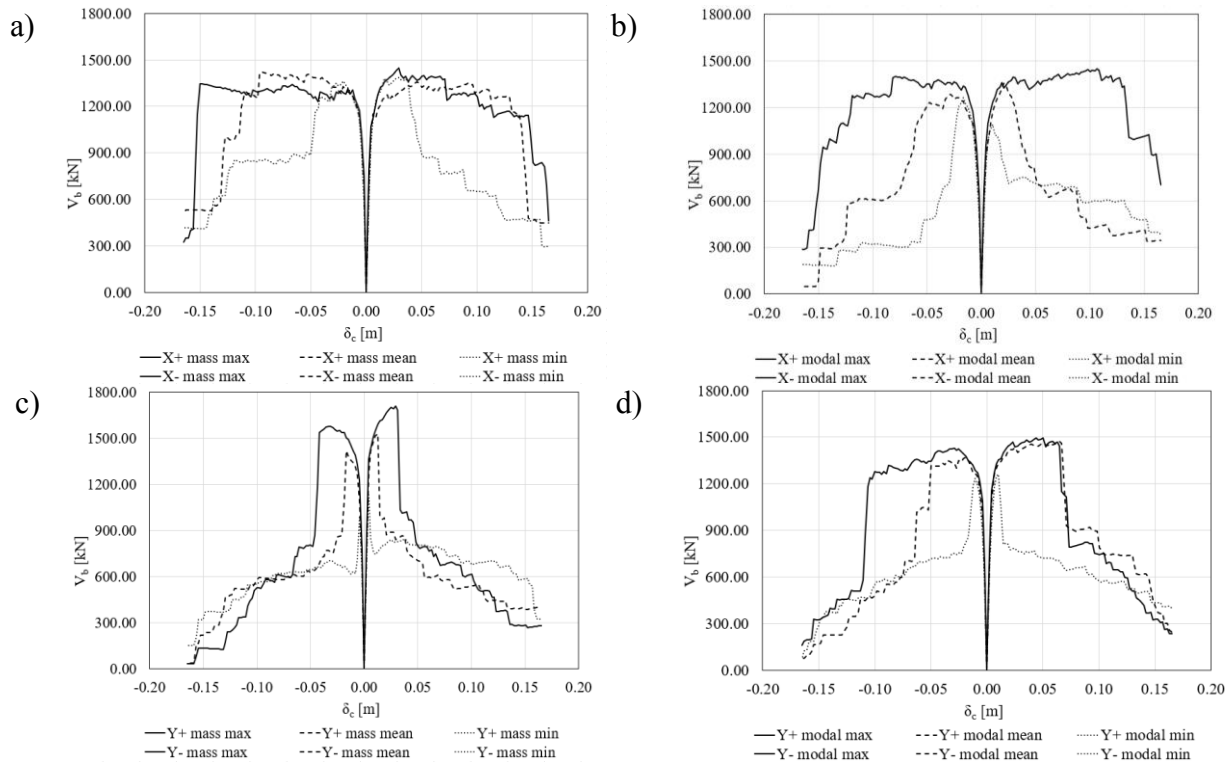


Figure 11. Pushover curves of case-study building: (a) X direction and mass profile; (b) X direction and modal profile; (c) Y direction and mass profile; (d) Y direction and modal profile.

The results outline that the variation of the mechanical parameters changed the behaviour of the building. Considering the minimum parameters in Table 2, the curves, for both directions and for both force profiles, show a sudden drop in capacity immediately after the maximum shear is reached. This is due to the reduction of the shear resistance to the minimum values.

The building, as shown in a similar condition before, had several panels experiencing a shear failure at the same time.

Tables 6 and 7 show the percentage changes in base shear, displacement and stiffness for different damage states. As regard the maximum mechanical properties the following conclusions can be drawn: (i) the change in shear with respect to the average properties is always an increase of approximately 10%, (ii) the change in displacement is always an increase in the X direction but always a decrease in the Y direction due to the numerous shear crises in the piers panels and (iii) in terms of stiffness, there is an increase in both directions but this is a consequence of the increase in the elastic modulus E.

On the other hand, in the case of the minimum mechanical properties, it was found a negative change in base shear except for the X direction with a force profile proportional to the masses; in terms of displacement there was almost always a decrease as well as in terms of stiffness. These results led to the conclusion that the structure had a better behaviour when the maximum mechanical characteristics were considered.

		X direction											
DS		Mass profile						Modal profile					
		V _b	ΔV _b	δ _c	Δδ _c	k	Δk	V _b	ΔV _b	δ _c	Δδ _c	k	Δk
		[kN]	[%]	[m]	[%]	[kN/m]	[%]	[kN]	[%]	[m]	[%]	[kN/m]	[%]
DS1	max	1011.6	6%	0.004	3%	270669	3%	1016.1	7%	0.005	-12%	201485	17%
	mean	950.3		0.004		261358		949.1		0.006		167620	
	mean	950.3	2%	0.004	20%	261358	-15%	949.1	-19%	0.006	-32%	167620	19%
	min	970.5		0.004		222326		764.1		0.004		199770	
DS2	max	1445.1	6%	0.029	-57%	49421	40%	1451.6	7%	0.107	80%	13628	-
	mean	1357.6		0.046		29546		1355.9		0.021		64919	376%
	mean	1357.6	2%	0.046	-36%	29546	60%	1355.9	-19%	0.021	-50%	64919	61%
	min	1386.4		0.029		47414		1091.5		0.010		104519	
DS3	max	1156.1	6%	0.114	-22%	10124	23%	1161.3	7%	0.134	75%	8654	-
	mean	1086.1		0.140		7784		1084.7		0.034		32061	270%
	mean	1086.1	2%	0.140	-69%	7784	226%	1084.7	-19%	0.034	-45%	32061	46%
	min	1109.1		0.044		25374		873.2		0.019		46888	
DS4	max	722.6	6%	0.162	11%	4451	-5%	725.8	7%	0.165	66%	4412	-
	mean	678.8		0.145		4693		678.0		0.056		12005	172%
	mean	678.8	2%	0.145	-38%	4693	65%	678.0	-19%	0.056	139%	12005	-66%
	min	693.2		0.089		7748		545.8		0.135		4039	
DS5	max	462.7	3%	0.165	0%	2804	3%	700.5	51%	0.165	0%	4245	51%
	mean	449.7		0.165		2725		343.4		0.165		2081	
	mean	449.7	-34%	0.165	0%	2725	-34%	343.4	12%	0.165	0%	2081	12%
	min	295.6		0.165		1792		383.6		0.165		2325	

Table 6. Displacement and shear difference for different DS for the two force profiles in the X-direction considering maximum, mean and minimum mechanical properties.

		Y direction											
DS		Mass profile						Modal profile					
		V _b [kN]	ΔV _b [%]	δ _c [m]	Δδ _c [%]	k [kN/m]	Δk [%]	V _b [kN]	ΔV _b [%]	δ _c [m]	Δδ _c [%]	k [kN/m]	Δk
DS1	max	1196.7		0.003		347941		1047.3		0.004		290217	
	mean	1069.8	11%	0.003	3%	321933	7%	1030.1	2%	0.004	-7%	265685	8%
	mean	1069.8		0.003		321933		1030.1		0.004		265685	
	min	792.1	-26%	0.002	-27%	324780	1%	883.2	-14%	0.004	-3%	234460	-12%
DS2	max	1709.5		0.029		58463		1496.2		0.050		29848	
	mean	1528.3	11%	0.013	57%	121955	-109%	1471.5	2%	0.065	-29%	22727	24%
	mean	1528.3		0.013		121955		1471.5		0.065		22727	
	min	1131.6	-26%	0.004	-67%	270898	122%	1261.7	-14%	0.010	-84%	120817	432%
DS3	max	1367.6		0.032		42274		1197.0		0.067		17996	
	mean	1222.6	11%	0.014	58%	89077	-111%	1177.2	2%	0.069	-4%	17088	5%
	mean	1222.6		0.014		89077		1177.2		0.069		17088	
	min	905.3	-26%	0.006	-58%	155510	75%	1009.4	-14%	0.013	-82%	79748	367%
DS4	max	854.8		0.048		17978		748.1		0.097		7677	
	mean	764.2	11%	0.041	15%	18865	-5%	735.8	2%	0.130	-33%	5674	26%
	mean	764.2		0.041		18865		735.8		0.130		5674	
	min	565.8	-26%	0.154	279%	3681	-80%	630.9	-14%	0.089	-31%	7069	25%
DS5	max	281.3		0.165		1705		234.9		0.165		1424	
	mean	406.0	-44%	0.165	0%	2461	-44%	245.0	-4%	0.165	0%	1485	-4%
	mean	406.0		0.165		2461		245.0		0.165		1485	
	min	315.1	-22%	0.165	0%	1910	-22%	399.7	63%	0.165	0%	2422	63%

Table 7. Displacement and shear difference for different DS for the two force profiles in the X-direction considering maximum, mean and minimum mechanical properties.

5 CONCLUSIONS

This study focused on the formulation of a methodology that, starting from data collection of data, allows large-scale probabilistic vulnerability analysis of historical masonry buildings.

The first part of the study was aimed at delineating a URM masonry building archetype that can be representative of typical historical buildings located in Italy. Geometric data sets on such buildings were collected from CARTIS database, deriving statistics for probabilistic simulation at territorial scale. Such an information was then implemented in MATLAB software to generate structural models via equivalent frame modelling. Seismic performance of the selected building archetype was evaluated through nonlinear static analysis, using the mean values of the mechanical properties provided by the Italian Building Code Commentary, i.e., Circular n°7/2019 [12]. As expected, the highest levels of earthquake resistance were found in the longitudinal direction of the building plan, where load-bearing walls have the greatest number of piers and sectional area.

A sensitivity analysis was also conducted under varying mechanical properties, evaluating variations in nonlinear response of the structure and corresponding capacity curves.

This showed that the archetype has a higher capacity and greater stiffness considering the mechanical properties with their maximum values. The redistributive capacity improves in the building panels and higher base shear are achieved.

Future developments of this study will involve consideration of other masonry types (e.g., clay brick masonry that is the second most common category in Italy) and building archetypes, taking into account the influence of the age of construction and number of floor levels.

Another future development will be to use this methodology by considering additional types of actions, such as subsidence and masonry degradation.

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