

## NUMERICAL INVESTIGATION ON THE SEISMIC RESPONSE OF A MASONRY CROSS VAULT BASED ON THE DISCRETE MACRO- ELEMENT METHOD

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### Abstract

*This paper focuses on the application of an innovative numerical approach to assessing the nonlinear dynamic response of an unstrengthened masonry cross vault subjected to shaking table testing performed within a blind prediction context. The numerical approach corresponds to the Discrete Macro-Element Method (DMEM), which consists of the assemblage of 2D panels and allows the assessment of the main in-plane and out-of-plane mechanisms of masonry structures according to a simplified mechanical scheme. A sensitivity analysis was carried out to determine the influence of the linear and nonlinear properties on the seismic response of the unstrengthened masonry cross vault. The calibration of the numerical model allowed reasonable agreement between experimental and numerical dynamic properties (natural frequencies and mode shapes). In addition, the numerical model provided acceptable results in terms of history of displacement and collapse mechanisms due to the application of nonlinear dynamic analyses. The results presented in this paper demonstrate the applicability of a simplified numerical method for assessing the seismic response of complex structures.*

**Keywords:** HiStrA Software, Fiber Calibration, Sensitivity Analysis, Model Updating, Non-linear Dynamic Analysis.

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## 1 INTRODUCTION

Masonry vaults constitute structural elements that play a relevant role in the seismic performance of existing historical buildings. These elements influence the global performance by increasing the lateral stiffness and affecting the load distribution during a seismic event. Nonetheless, these elements are characterized by a brittle response due to the constituent material's nature, which reduces the building capacity. Due to their complex geometrical characteristics, the heterogeneous material composition, or the presence of backfill, the structural and seismic assessment of these elements constitutes a complicated task. The failure of these structural typologies is mainly associated with the in-plane shear distortions or the lack of lateral instability due to relative displacements [1, 2]

To better understand the behavior of these elements, experimental and numerical research has been carried out considering different loading and boundary conditions. For instance, laboratory testing focused on assessing the response of these elements due to vertical loadings such as imposed differential displacements or concentrated loads [3-7] and shaking table tests [8-10] have been performed in the last decades. On the other hand, the numerical investigations associated with masonry cross vaults involved applying nonlinear analyses based on different approaches, such as the Finite Element Method [11-15] or the Discrete Element Method [16-18]. Nonetheless, it is worth noting that experimental and numerical approaches present relevant limitations due to the affordability of laboratory testing or the significantly high computational demand of nonlinear simulations.

This paper presents the seismic response assessment of an unstrengthened masonry cross vault using an innovative and simplified modeling approach known as the Discrete Macro-Element Method (DMEM). In addition, this investigation focused on determining the accuracy of this modeling approach to simulate the complex response of a masonry cross vault subjected to shaking table tests. This method was initially conceived for assessing the in-plane response of masonry structures [19]. Furthermore, the method was enhanced to simulate unreinforced masonry structures' combined in-plane and out-of-plane behavior [17], which was implemented in the HiStrA software [20]. The presented study is part of a competition involving several research teams on the blind prediction of the shaking table response of a masonry cross vault [21, 22]. Precisely, the presented analyses refer to the post-diction phase. The numerical model was calibrated to reproduce the modal properties from dynamic identification tests, obtaining acceptable results in natural frequencies and mode shapes. The seismic assessment of the unstrengthened cross vault was numerically investigated by nonlinear dynamic analyses. A reasonable agreement was obtained when comparing histories of displacements and collapse mechanisms. These results demonstrated that the proposed modeling approach represents a suitable tool for assessing the seismic response of masonry cross vaults.

## 2 DISCRETE MACRO-ELEMENT METHOD

The Discrete Macro-Element Method (DMEM) was initially developed to assess the in-plane response of masonry structures [19]. The mechanical scheme of the DMEM consists of 2D panels composed of a hinged quadrilateral composed of rigid edges. Adjacent panels are connected by zero-thickness interface elements whose calibration procedure follows a straightforward fiber approach. Based on its initial formulation, the DMEM can simulate in-plane failure mechanisms of masonry panels such as flexural, shear-sliding, and shear-diagonal. Additional mechanisms were subsequently implemented for assessing the out-of-plane response of unreinforced masonry walls [23] and curved structural components such as arches [24]. To this aim, the panels are composed of four rigid plates and plane interface elements (see Figure 1a). The kinematics of these panels is described by seven degrees of free-

dom (DOF): six rule the rigid body motion, and one governs the in-plane shear deformability. Simplified constitutive laws rule the simulation of the different collapse mechanisms. The bi-flexural response is described by a Takeda constitutive law in which the post-peak response in tension and compression can be ruled by exponential, linear, or parabolic softening curves with different unloading conditions. On the other hand, the shear mechanisms can be ruled by a Mohr-Coulomb yielding criterion in which the initial stiffness can define the unloading cycles. Examples of the constitutive laws available in the DMEM are depicted in Figure 1b.

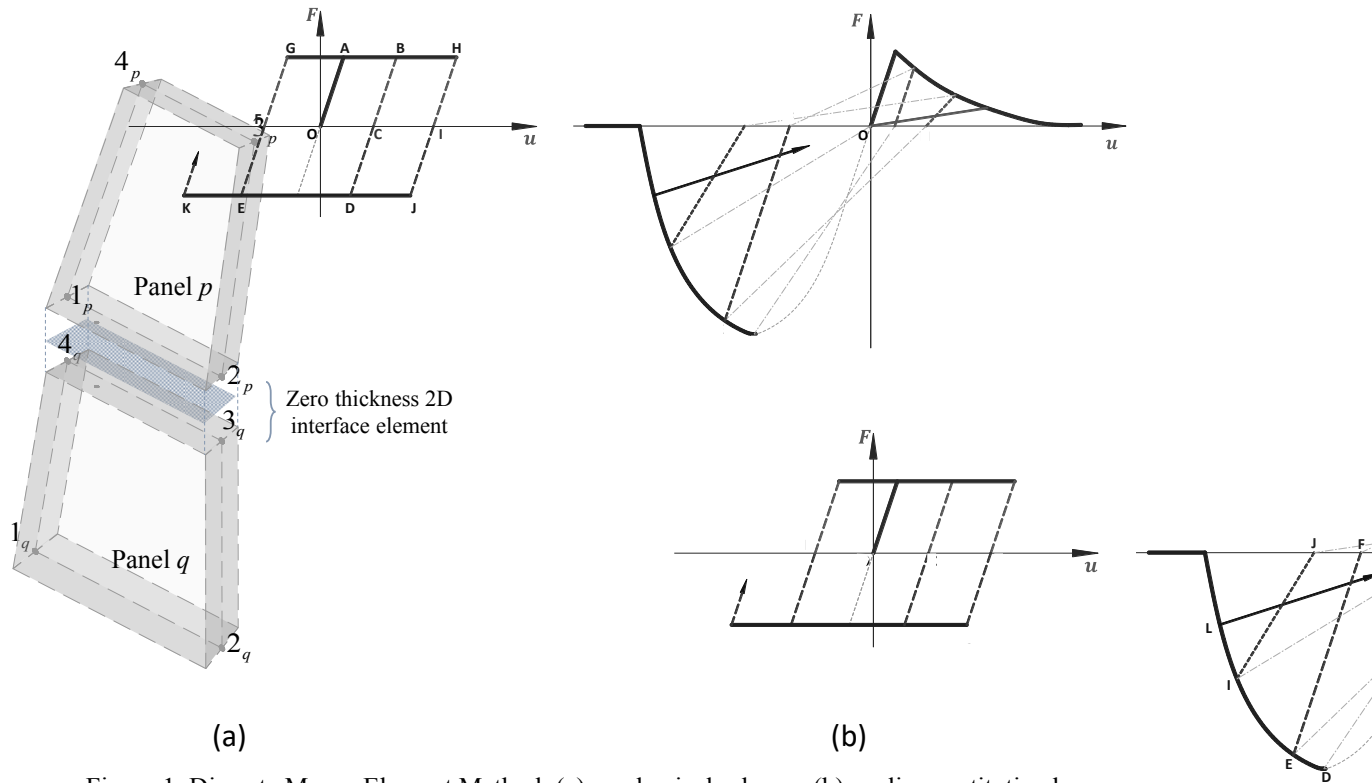


Figure 1. Discrete Macro-Element Method: (a) mechanical scheme, (b) cyclic constitutive laws.

### 3 UNSTRENGTHENED MASONRY CROSS VAULT

An unstrengthened masonry cross vault was built at the National Laboratory of Civil Engineering (LNEC) in Lisbon in December 2020 within the scope of the **SERA Project – Seismic Response of Masonry Cross Vaults: Shaking table tests and numerical validations** [21, 22]. The brick units used to construct the cross vault presented average dimensions of 4.5 cm x 12.0 cm x 23 cm, whereas a thickness of approximately 1 cm characterized the mortar joints. The units were arranged so that the bed joints were orthogonal to the four edges of the cross vault (see Figure 2a). The cross vault was characterized by a rectangular shape in which the lateral edges presented a length of approximately 3.0 m. On each corner of the vault, an infill compound made of brick masonry was placed to introduce additional weight. However, it is worth noting that the infill did not present any structural contribution to the cross vault. In addition, the corners of the vaults are supported by two steel masses and two masonry piers that simulate the presence of walls and columns. Finally, steel plates and rods were introduced to the specimen to restrain the rotations of the elements and to enhance and simulate the actual conditions of cross vaults. The 3D geometrical model of the full-scale masonry cross vault is depicted in Figure 2b.

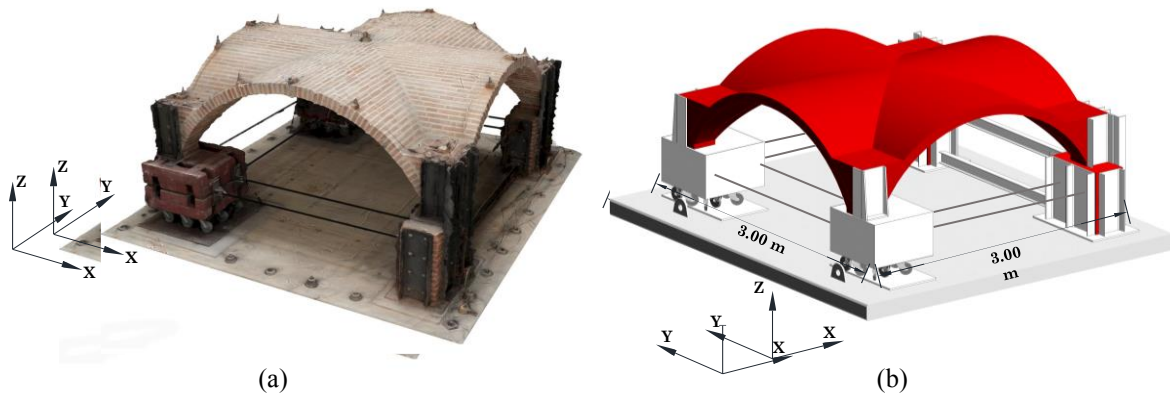


Figure 2. Unstrengthened masonry cross vault: (a) plan and (b) isometric views [21].

The boundary conditions were defined to guarantee the occurrence of an in-plane shear mechanism when assessing the seismic response of the unstrengthened masonry cross vault [21]. As illustrated in Figure 3, two sets of boundary conditions along two edges of the structure were considered. The first focused on the masonry piers, which restrained their displacements and rotations (fixed edge). The second was characterized by the introduction of wheels below the steel masses, which enabled the displacements along X, and Y directions (moveable edge) and the rotation around the vertical axis [21].

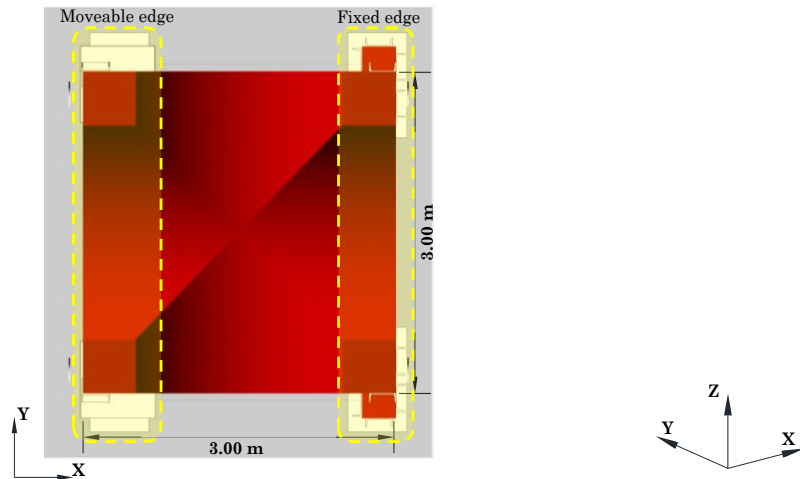


Figure 3. Boundary conditions of the unstrengthened masonry cross vault [21].

The instrumentation setup implemented for the unstrengthened masonry cross vault consisted of piezoelectric accelerometers (Acc), optical cameras (OC), and video cameras (VC), as illustrated in Figure 4. The piezoelectric accelerometers were placed in seventeen points of the masonry cross vault, measuring 29 directions. It is worth noting that additional accelerometers were placed close to one fixed pier and on the shaking table. Displacements were measured at four points of the structure using optical cameras. Measured point OC1 was at the center of the masonry cross vault, whereas OC2 and OC4 were on top of the masonry infill. The remaining measured point (OC3) was placed on the middle span of the moveable edge. In addition, four video cameras were located at each edge and one on top of the masonry vault. It is worth noting that the axial forces of the steel rods were measured during the experimental campaign using six load cells [21].

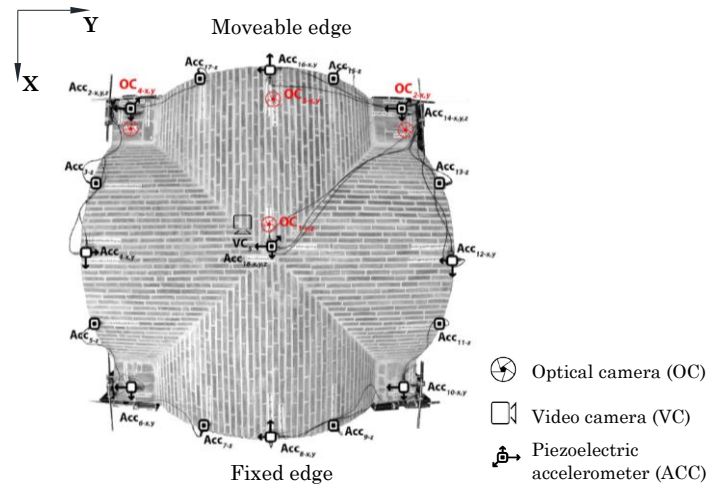


Figure 4. Instrumentation setup for the dynamic identification and shaking table tests [21].

## 4 LABORATORY TESTING

An experimental campaign involving the application of laboratory tests was carried out at the LNEC. These tests aimed at characterizing the mechanical properties of the constituent materials, identifying the dynamic properties, and assessing the seismic response of the unstrengthened masonry cross vault. The dynamic properties of the structure were identified under different damage conditions due to the application of shaking table tests with a certain seismic intensity. The application sequence of dynamic identification and shaking table tests is summarized in Table 1. It is worth noting that further details regarding the mechanical characterization can be found in [21].

Shaking table tests	Dynamic identification tests
2. Considering 25% of seismic input	1. Under no damage condition
4. Considering 50% of seismic input	3. After 25% of seismic input
6. Considering 75% of seismic input	5. After 50% of seismic input
	7. After 75% of seismic input

Table 1. Testing sequences for the dynamic identification and shaking table tests.

### 4.1 Mechanical characterization

The experimental campaign consisted of three-point bending and uniaxial compression tests for mortar [25] and masonry units [26]. Based on these tests, the Young's modulus of mortar presented a mean value of 370 MPa and compressive and flexural strengths with average values of 1.68 MPa and 0.66 MPa. The units were characterized by average Young's modulus and compressive strength of 6200 MPa and 25.0 MPa, respectively. The mechanical characterization of the composite material involved the application of triplet [27], uniaxial compression [28], and diagonal compression tests [29]. The triplet tests (see Figure 5a) were carried out on twelve samples of three fired brick units with total approximate dimensions of 17.3 cm x 22.3 cm x 11.8 cm. Four specimens with dimensions of 61.9 cm x 46.8 cm x 11.8 cm (see Figure 5b) were subjected to uniaxial compression tests. Finally, the diagonal compression tests were applied to two masonry specimens (see Figure 5c).

Based on the application of these tests, it was possible to determine the main mechanical properties of the fired brick masonry material. From the triplet tests, it was possible to obtain mean cohesion and friction coefficient values of 0.031 MPa and 0.785, respectively. In the case of the uniaxial compression tests, the mean value of Young's modulus was 2223 MPa,

whereas the mean compressive strength was 9.1 MPa. Finally, the diagonal compression tests allowed the estimation of a mean tensile strength of 0.31 MPa and a shear modulus of 762 MPa. A summary of the mean values, together with their corresponding coefficient of variation (COV), is reported in Table 2.

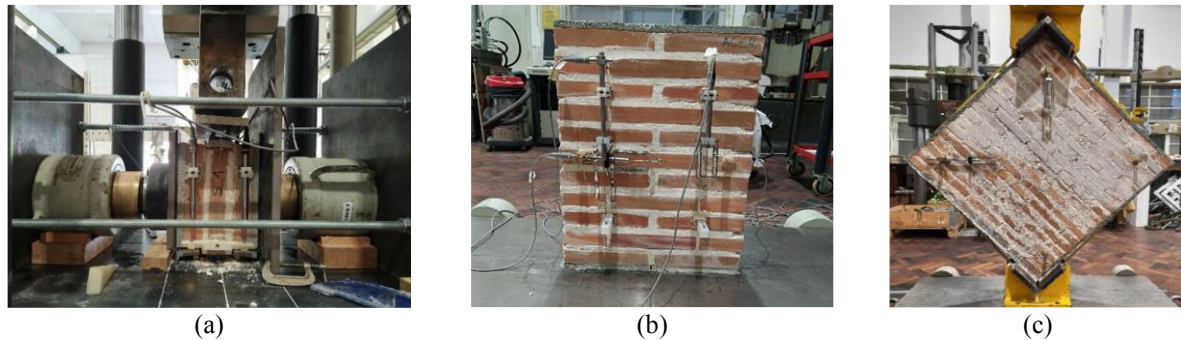


Figure 5. Mechanical characterization of the masonry material: (a) triplet, (b) uniaxial compression, and (c) diagonal compression tests [21].

	Triplet tests		Uniaxial compression tests		Diagonal compression tests	
	Cohesion	Friction coefficient	Young's modulus	Compressive strength	Shear modulus	Tensile strength
Mean value	0.031 MPa	0.785	2223 MPa	9.1 MPa	762 MPa	0.31 MPa
COV	-	-	14.0%	2.0%	68.0%	14.3%

Table 2. Results of the mechanical characterization of fired brick masonry material [21].

## 4.2 Dynamic identification

Dynamic identification tests were conducted to estimate the structure's modal properties, namely mode shapes and vibration modes. The dynamic identification tests were carried out by applying an artificial accelerogram and measuring the corresponding response using piezoelectric acceleration transducers with a sensitivity of 100 mV/g and a measurement range of  $\pm 50$  g pk. The duration of the dynamic identification tests was approximately 160 seconds with a sampling frequency of 200 Hz. The acceleration data's signal processing was carried out using the ARTeMIS software [30] following a Frequency Domain Decomposition (FDD) method. A detailed description of the results of the dynamic identification tests is reported in [22].

The identification of the dynamic properties of the structures was first carried out considering an undamaged condition. As illustrated in Figure 6a, the first vibration mode is associated with a pure shear vibration mode with a natural frequency of 6.15 Hz. The second mode, with a natural frequency of 11.62 Hz, was characterized by a horizontal displacement of the movable edge (see Figure 6b). Finally, the third one is associated with a vertical mode in which the maximum displacement is focused on the center of the vault (see Figure 6c), presenting a natural frequency of 19.39 Hz.

The natural frequencies of the unstrengthened masonry cross vault were compared to those identified after applying each shaking table test. It was possible to observe no relevant variation between the results from the undamaged condition and after the shaking table test with 25% of seismic input. This may indicate that the structure did not experience significant or permanent damage. On the other hand, the natural frequencies identified after the shaking table tests with 50% and 75% of seismic input were characterized by lower frequencies which may be related to the stiffness degradation of the material and the propagation of cracks, especially for the first and second vibration modes. For the dynamic identification after 50% of



seismic input, the reduction in natural frequencies of these modes presented absolute values of 4.23% and 7.14%, respectively. This reduction increased after the shaking test with 75% of seismic input with absolute values of 9.43 for the first mode and 13.08 for the second mode. It is worth noting that it was not possible to identify the third vibration mode after applying the shaking table test with 75% of seismic input. A summary of the natural frequencies throughout the different damage conditions is presented in Table 3.

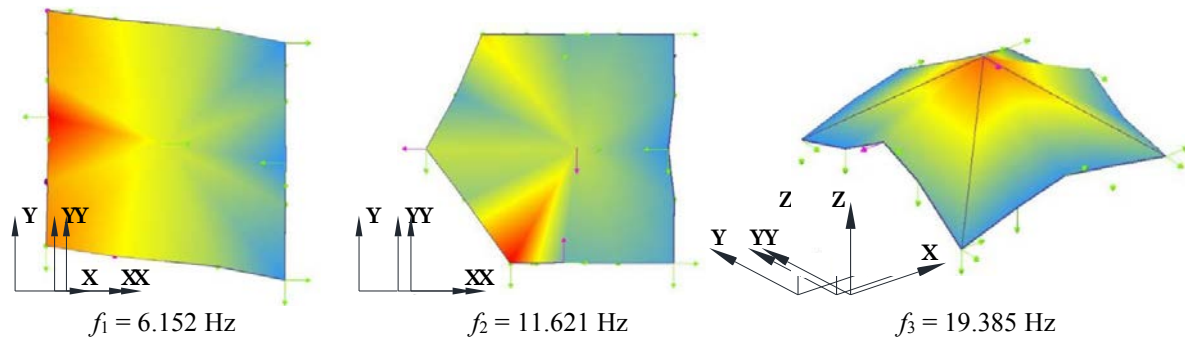


Figure 6. Identified vibration modes of the unstrengthened masonry cross vault (undamaged condition) [22].

	Mode 1	Mode 2	Mode 3
Undamaged	6.15 Hz	11.62 Hz	19.39 Hz
After shaking table tests with 25% of seismic input	6.15 Hz	11.38 Hz	19.38 Hz
After shaking table tests with 50% of seismic input	5.89 Hz	10.79 Hz	19.30 Hz
After shaking table tests with 75% of seismic input	5.57 Hz	10.10 Hz	-

Table 3. Natural frequencies identified at different damage conditions [22].

### 4.3 Shaking table tests

The application of shaking table tests allowed assessing the response of the unstrengthened masonry cross vault under dynamic loading with different intensities. The seismic input considered for these tests corresponded to the record of the 2009 L'Aquila earthquake. The record was subjected to a baseline correction, a bandpass filter ranging between 0.05 Hz and 20 Hz, and a Butterworth filter with a sixth pole order. The one-component input (see Figure 7) was applied along the Y–Y direction to assess the structure's in-plane shear seismic response. In addition, three sequential shaking table tests were carried out with increasing steps of 25% amplitude increment.

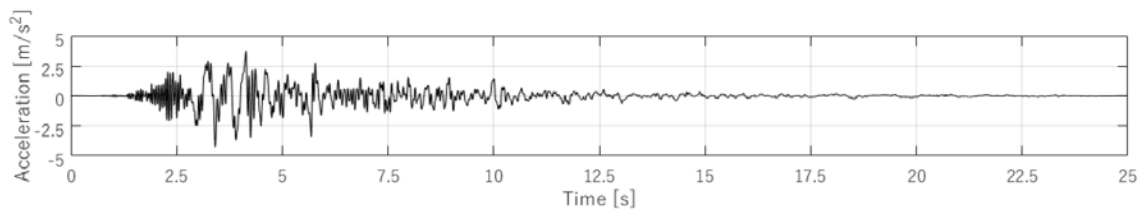


Figure 7. Filtered history of acceleration considered for the shaking table tests [21].

As reported in [22], the collapse mechanism of the unstrengthened masonry cross vault after applying the shaking table test with 75% of the seismic input was described as a crack propagation along the main diagonals of the structure due to a shear mechanism. Moreover, it was possible to observe that the structure also suffered damage in terms of plastic hinges at the masonry webs. Additional cracks were identified in the connection between the masonry

web and the infill. This detachment was mainly focused on the fixed edge. A top view of the collapse mechanism of the unstrengthened masonry cross vault is illustrated in Figure 8.

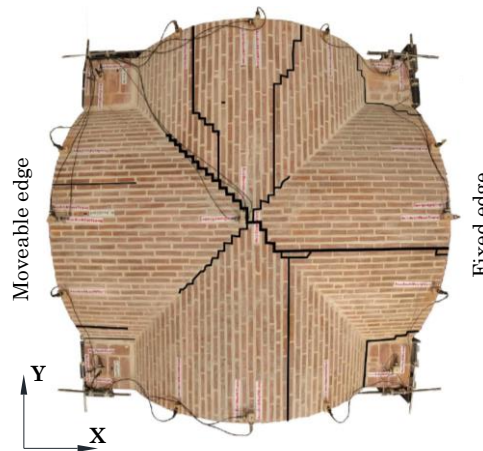


Figure 8. Collapse mechanism of unstrengthened masonry cross vault [22].

## 5 NUMERICAL SIMULATIONS

The numerical assessment of the unstrengthened masonry cross vault's seismic response was performed using the DMEM implemented in the HiStrA software [20]. The numerical model was composed of 2D quadrilateral and triangular elements for the idealization of the masonry vault, infill, binder, and steel components. In addition, trusses were used to model the steel rods and steel profiles that connect the masonry piers and the steel masses (see Figure 9). Two sets of boundary conditions were defined to simulate the actual laboratory characteristics. The first consisted of a fixed restraint of the piers, and the second one allowed the displacements in the X–X and Y–Y directions and the rotation around the Z axis. Based on the number of computational elements and the boundary conditions, the DME model of the masonry cross vault was described by 4822 DOFs.

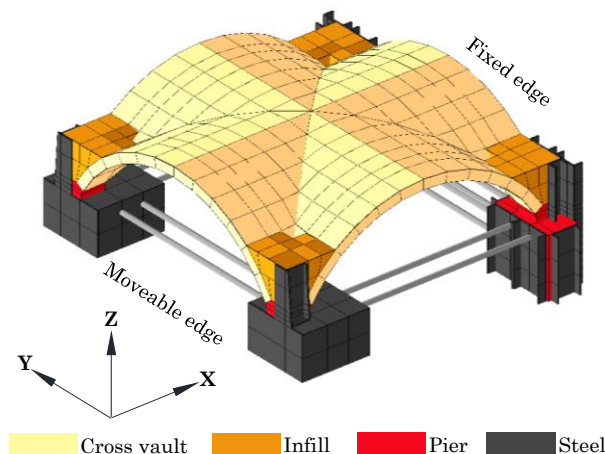


Figure 9. DME model of the masonry cross vault

The initial linear elastic mechanical properties defined for the numerical models were taken primarily from the mechanical characterization. Regarding the fired brick masonry material, the Young's and shear moduli presented values of 2,223 MPa and 762 MPa, respectively, and a specific weight of 22.55 kN/m<sup>3</sup>. In the case of the structural steel material, typical val-



ues of Young's modulus and Poisson ratio were considered in this investigation. A summary of the mechanical properties is reported in Table 4.

	Young's modulus $E$ [MPa]	Poisson ratio $\nu$ [-]	Shear modulus $G$ [MPa]	Specific weight $\gamma$ [kN/m <sup>3</sup> ]
Fired brick masonry	2,223	-	762	22.55
Structural steel	210,000	0.25	-	78.50

Table 4. Linear elastic mechanical properties of fired brick masonry and structural steel materials.

## 5.1 Model updating

The numerical simulations initially consisted of updating the DME model following a sensitivity analysis and comparing experimental and numerical modal parameters. The sensitivity analysis focused on assessing the influence of the Young's modulus of the fired brick masonry material in the natural frequencies and mode shapes of the unreinforced cross vault. The accuracy of the mode shapes was evaluated using the Modal Assurance Criterion (MAC), which is determined as a function of experimental and numerical mode shapes. The MAC value ranges between when there is no resemblance and 1 when there is a perfect resemblance between experimental and numerical results [31].

The comparison of the dynamic properties focused on the first and third experimental modes since it was not possible to identify the second one with the DME model. In contrast, the estimation of the MAC value considered the mode shapes associated with the numerical model's first and fourth vibration modes. A first comparison between experimental and numerical results was carried out considering the initial value of the fired brick masonry material (2223 MPa). It was observed that there was a significant difference in natural frequencies, presenting absolute differences of 19.6% and 22.0% for the first and third experimental modes, respectively. In the case of mode shapes, there was a reasonable agreement regarding the first mode presenting a MAC equal to 0.878. Unfortunately, the MAC associated with the third experimental mode presented a value of 0.404. It was noted that these differences were associated with the rigidity of the numerical model. In this sense, the initial value of the Young's modulus of the masonry material was subjected to five reduction factors (RF = 0.9, 0.8, 0.7, 0.6, and 0.5) aiming at assessing its influence on the dynamic properties of the unstrengthened cross vault. It was observed that the difference in terms of natural frequency reduced, and the MAC ratio of the third experimental mode increased with a lower RF. In addition, it was evidenced that when using RFs of 0.6 and 0.5, the absolute differences in natural frequencies and the MAC presented acceptable results. The sensitivity analysis results are reported in Table 5.

		Initial	RF = 0.9	RF = 0.8	RF = 0.7	RF = 0.6	RF = 0.5
1 <sup>st</sup> num mode / 1 <sup>st</sup> exp mode	$f_{\text{num}}$ [Hz]	7.35	7.05	6.74	6.38	5.99	5.55
	error [%]	19.6	14.6	9.7	3.8	2.6	9.7
	MAC [-]	0.878	0.879	0.881	0.883	0.885	0.887
4 <sup>th</sup> num mode / 3 <sup>rd</sup> exp mode	$f_{\text{num}}$ [Hz]	23.65	22.88	22.04	21.09	20.02	18.79
	error [%]	22.0	18.0	13.7	8.8	3.3	3.1
	MAC [-]	0.404	0.458	0.533	0.609	0.685	0.751

Table 5. Frequency and mode shape comparison between experimental and numerical results.

Based on this sensitivity analysis, it was decided to use an average RF equal to 0.55 as the optimum value for the Young's modulus of the fired brick masonry. The mode shapes of the numerical model considering a RF of 0.55 are illustrated in Figure 10. The first mode presented a natural frequency of 5.78 Hz (with an absolute difference of 6% and a MAC ratio of

0.89), and it involved a pure shear behavior of the cross vault (see Figure 10a). The remaining dynamic properties were associated with the fourth vibration mode, whose response was mainly governed by the vertical component of the displacements, presenting a natural frequency of 19.34 Hz (difference of 0.2%) and a MAC ratio of 0.72.

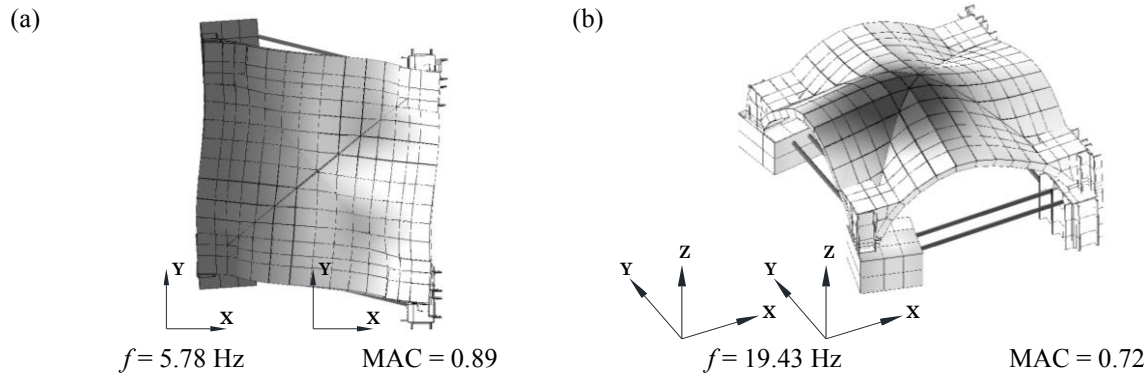


Figure 10. DME dynamic properties: (a) first and (b) fourth vibration modes

## 5.2 Nonlinear dynamic analysis

The unstrengthened cross vault's seismic response was numerically investigated through nonlinear dynamic analyses. The physical nonlinearity was focused on the mechanical properties of the masonry material, whereas structural steel was considered elastic. Exponential and parabolic curves described the tensile and compressive behaviors, whereas an elasto-plastic approach characterized the shear-sliding mechanism. The values of tensile and compressive strengths were taken from the mechanical characterization. In this sense, the tensile and compressive strengths presented values of 0.31 MPa and 9.10 MPa. The compressive fracture energy presented a value of 14.56 N/mm, and the tensile fracture energy was assumed to be equal to 0.012 N/mm. For the shear-sliding behavior, the friction coefficient was also taken from the mechanical characterization; however, cohesion was initially assumed equal to the tensile strength. A summary of the nonlinear mechanical properties of the masonry material is reported in Table 6.

Compressive strength $f_c$ [MPa]	Compressive fracture energy $G_c$ [N/mm]	Tensile strength $f_t$ [MPa]	Tensile fracture energy $G_c$ [N/mm]	Cohesion $c$ [MPa]	Friction coefficient $\mu$ [-]
9.10	14.56	0.31	0.012	0.31	0.785

Table 6. Nonlinear mechanical properties for fired brick masonry material

For the dynamic nonlinear analyses, the numerical solution used in this investigation corresponded to the Newmark Method with average acceleration, whereas damping was defined following a Rayleigh approach. Two vibration modes with cumulative effective masses of approximately 71% and 84% and a damping ratio of 2.5% were considered for the definition of the Rayleigh damping matrix. Following the experimental campaign, three subsequent nonlinear dynamic analyses were conducted to account for damage accumulation. Such numerical simulations were associated with the shaking table tests with 25%, 50%, and 75% of seismic input. The signals used for the numerical simulations corresponded to the acceleration registered by the transducers placed at the shaking table, which were subjected to a baseline correction and a filtering process. The results presented correspond to the numerical simulations of the 75% of seismic input whose history of acceleration is depicted in Figure 11.

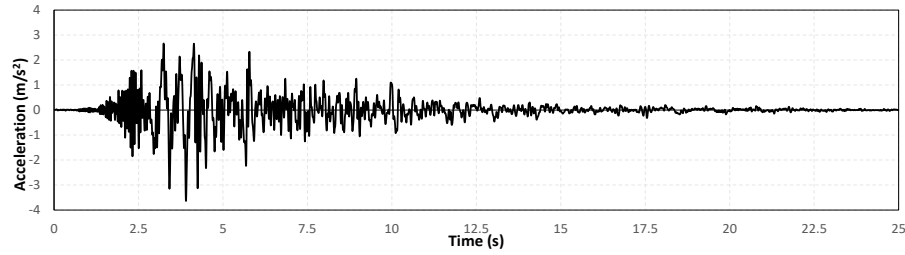


Figure 11. History of acceleration registered at the base of the shaking table for the 75% seismic input.

The comparison between experimental and numerical results was focused on the displacement histories at the points with optical cameras. Considering the initial value of tensile strength reported in the mechanical characterization, it was possible to observe a significant variation in maximum relative displacements. In the experimental campaign, the maximum relative displacement at the top of the moveable edge (measured point OC3) presented an approximate value of approximately 18 mm, whereas the numerical one was around 8.7 mm (see Figure 12a). The tensile strength and cohesion values were reduced to improve the correspondence between experimental and numerical displacements. For this purpose, tensile strength and cohesion with values equal to 0.248 MPa, 0.186 MPa, 0.124 MPa, 0.062 MPa, and 0.031 MPa were considered for further analyses. The numerical results experienced a significant improvement when considering low values of such mechanical properties. As illustrated in Figure 12b, there is a reasonable agreement regarding maximum displacement, assuming a value equal to 0.031 MPa. Nonetheless, the residual displacements observed in the experimental shaking table tests were not reproduced with the numerical model, which may be a limitation of the proposed modeling approach.

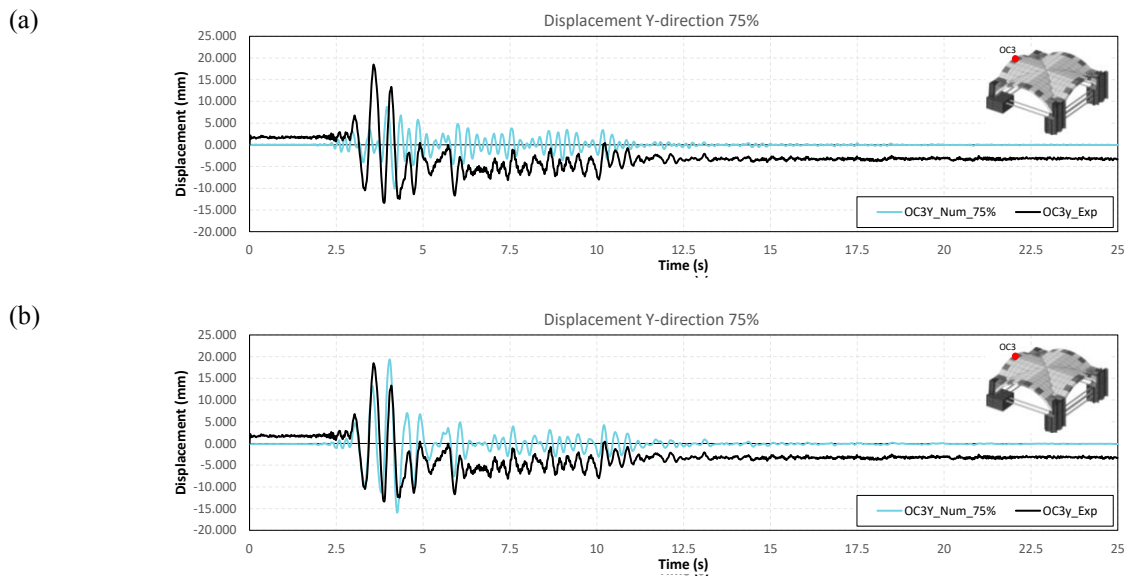


Figure 12. Comparison between experimental and numerical results for the 75% of seismic input considering tensile and cohesion equal to: (a) 0.310 MPa and (b) 0.031 MPa.

As illustrated in Figure 13, the collapse mechanism of the DME model of the unstrengthened cross vault consisted of plastic hinges along the masonry web and diagonal cracking. Additional damage was identified in the connection between the masonry cross vault and the infill. It was observed that the experimental and numerical collapse mechanisms presented an acceptable agreement. The crack pattern obtained after the shaking table tests coincides with

the appearance of plastic strains in the numerical model. These results demonstrate the capability of the DMEM to simulate the complex response of masonry cross vaults with an acceptable degree of accuracy.

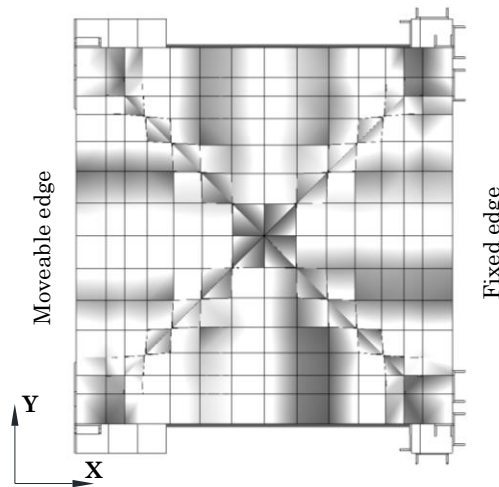


Figure 13. Numerical collapse mechanism of the numerical model of the unstrengthened masonry cross vault.

## 6 CONCLUSIONS

- This paper presents the simulation of the seismic response of an unstrengthened masonry cross vault using an innovative numerical tool known as the Discrete Macro-Element Method (DMEM).
- The proposed method is described by a simplified mechanical scheme that allows the simulation of the collapse mechanism of masonry structures with a reduced number of DOFs and computational demand.
- The numerical model could simulate the dynamic properties of the unstrengthened masonry cross vault with an acceptable degree of accuracy. This process involved applying a sensitivity analysis which evidenced that the vault's mechanical response does not accurately agree with the material's mechanical properties.
- The unstrengthened masonry cross vault's seismic response was investigated through nonlinear dynamic analysis using the DMEM. A reasonable agreement between the experimental and numerical models was obtained in terms of history of displacement and collapse mechanism. It is worth noting that the proposed model still presents some limitations, especially in simulating the residual displacement.

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