Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2023) 4484-4491

ECCOMAS Proceedia

COMPDYN 2023
9th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Athens, Greece, 12-14 June 2023

EXPLORATION OF DYNAMIC LOADING STRUCTURAL TOPOLOGY OPTIMIZATION AND APPLICATION TO BUILDING LATERAL LOAD RESISTING FRAME

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Abstract. *The dynamic response topology optimization problems are usually computationally expensive, especially for time domain solution scheme involving additional iterations of Newmark beta time integration. In our work, frequency domain solution scheme as well model order reduction techniques for time domain strategy have been explored. Structural optimization for minimizing dynamic compliance has been studied for several standard examples. Application of this strategy to lateral load resisting system of a building subjected to earthquake ground excitation has been studied. The effect of geometric nonlinearity has also been studied for better accuracy of the solution.*

Keywords: Dynamic Structural Optimization, Bracing layout, Geometric non linearity

Peer-review under responsibility of the organizing committee of COMPDYN 2023.
doi: 10.7712/120123.10735.21236

1 INTRODUCTION

Dynamic response topology optimization has gained significant attention in recent years due to its potential to design lightweight and durable structures that can withstand dynamic loads and vibrations. The objective of dynamic response topology optimization is to optimize the distribution of material in a structure to minimize its weight while maintaining its perfor-mance under dynamic loads. This is achieved by incorporating dynamic loading and vibration considerations into the optimization process. The optimization can be performed using various techniques such as mathematical programming, level set, and evolutionary algorithms. Several research studies have investigated the optimization of different types of structures, including trusses, plates, and shells. Constraints such as natural frequency, stress, and displacement can be considered in the optimization process to ensure the optimal design satisfies performance requirements. The resulting optimized structures can offer several benefits, including reduced weight, improved performance, and reduced manufacturing costs. However, dynamic re-sponse topology optimization remains a complex and challenging problem due to the high computational cost and the difficulties in dealing with geometric nonlinearities and material uncertainties. Therefore, further research is required to develop efficient and effective optimi-zation techniques that can handle these challenges.

Geometric nonlinear dynamic loading topology optimization is a technique used to optimize the shape of a structure subjected to dynamic loads, considering geometric nonlinearities. Geometric nonlinearity refers to the changes in shape of a structure that can occur under load, which can significantly affect its behaviour. Dynamic loading refers to loads that change over time, such as the impact of a moving object or the effect of wind on a structure. These types of loads can cause a structure to vibrate, leading to potential failure if the design is not optimal.

In [1] a method for dynamic response topology optimization of structures with contact constraints using the level set method has been developed. The proposed method is applied to optimize the design of a brake disc under dynamic loading, and the results demonstrate that the method is effective in finding an optimized design that can withstand dynamic loads and vibrations. In [2] approximate analysis techniques for topology optimization under multifrequency harmonic excitation has been studied. Use of model order reduction techniques for computational efficiency has been shown in [3]. Structural design of building components using spectral approach has been proposed in [4] for earthquake excitation and optimitum topology has been obtained.. Ran-dom vibration and uncertainties have been considered in [5, 6] for dynamic response topology optimization. Geometric nonlinear analysis for topology optimization has been presented in [7] for structures subjected to dynamic loading using equivalent static loads. Solutions for structures involving nonlinearity has been performed in [8] using moving morphable components. Several researches have studied geometric nonlinear topology optimization including SIMP, BESO, energy interpolation and other techniques [9, 10, 11, 12]. From literature, it has been observed that dynamic loading topology optimization studies have been explored only recently and optimizations considering finite deformation are rare.

The goal of this paper is to explore the dynamic loading topology optimization of structures subjected to dynamic excitation including sinusoidal wave and earthquake ground motion and use it for optimizing lateral bracing. Also, the effect of geometric nonlinearity is briefly summarized.

2 TOPOLOGY OPTIMIZATION

The well-known density-based method [13] is used in this study for the dynamic response topology optimization problem. Using a modified solid isotropic material with penalization (SIMP) interpolation scheme, the density-based topology optimization approach interpolates each element's Young's modulus E_e and structural density (or mass density) ρ with element e. This polynomial interpolation model [14], which helps avoiding localized modes in dynamic analysis is as follows:

$$E_e = E_0\left(\alpha\eta_e^p + (1-\alpha)\eta_e\right) \tag{1}$$
$$\rho_e = \rho_0\eta_e$$

where p is a penalization number to prevent intermediate densities and E_0 and ρ_0 are the Young's modulus and structural density of the solid material, α is a positive real number smaller than 1.0. For dynamic loading, the objective function considered here (as in [3]) is the mean dynamic compliance given by:

$$c(u,\eta) = \frac{1}{t_f}\int_0^{t_f} f^T(t)u(t)dt \tag{2}$$

where $f(t)$ is the applied force vector and $u(t)$ is the displacement response vector. Finally the optimization problem [3] is configured as:

$$\begin{array}{ll} \min_\eta & c(u,\eta) \\ \text{s.t.} & \boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{f}(t), t \in [0, t_f] \\ & g(\eta) = V(\eta) - V_{\max} = \sum_{\mathrm{e}=1}^{N} \eta_e v_e - V_{\max} \leq 0 \\ & \boldsymbol{0} < \eta_{\min} \leq \eta \leq \boldsymbol{1} \end{array} \tag{3}$$

where M, C, and K are matrices representing the global mass, damping, and stiffness, respectively. The number of elements in the structure is denoted by N. The volume of each individual element is represented by v_e, while V_{max} is the total prescribed volume of material. To prevent singularity of the stiffness matrix during the topology optimization process, a positive lower bound vector η_{min} is assigned to η. The displacement vector $u(t)$, velocity vector $\dot{u}(t)$, and acceleration vector $\ddot{u}(t)$ can be determined by solving the dynamic equilibrium equation, given the value of the design variable vector η and the initial conditions $u(0) = u_0$ and $\dot{u}(0) = \dot{u}_0$.

2.1 Sensitivity Analysis

Considering J_d as the objective function and adding dynamic equilibrium equation multiplied by lagrange multiplier, we can compute adjoint sensitivity [3] which brings huge computational efficiency compared to direct sensitivity. And this is very much essential in case of dynamic topology optimization problems as the finite element solution of the equilibrium equation itself involves iterative Newmark Beta algorithm.

$$\begin{aligned} \frac{\partial J_d}{\partial \eta_e} &= \frac{\partial J_d}{\partial \eta_e} + \int_0^{t_f} \lambda^T \frac{\partial}{\partial \eta_e}(\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} - \boldsymbol{f})dt \\ &= \int_0^{t_f}\left(\frac{\partial c}{\partial \eta_e} + \frac{\partial c}{\partial \boldsymbol{u}}\frac{\partial \boldsymbol{u}}{\partial \eta_e}\right)dt \\ &+ \int_0^{t_f} \lambda^T \frac{\partial}{\partial \eta_e}(\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u})dt \end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\partial J_d}{\partial \eta_e} = &\int_0^{t_f} \left(\boldsymbol{\lambda}^T \frac{\partial \boldsymbol{M}}{\partial \eta_e} \ddot{\boldsymbol{u}} + \lambda^T \frac{\partial \boldsymbol{C}}{\partial \eta_e} \dot{\boldsymbol{u}} + \lambda^T \frac{\partial \boldsymbol{K}}{\partial \eta_e} \boldsymbol{u} + \frac{\partial c}{\partial \eta_e} \right) dt \\ &+ \int_0^{t_f} \left(\frac{\partial \boldsymbol{u}}{\partial \eta_e} \right)^T \left(\boldsymbol{M}\ddot{\boldsymbol{\lambda}} - \boldsymbol{C}\dot{\boldsymbol{\lambda}} + \boldsymbol{K}\boldsymbol{\lambda} + \frac{\partial c}{\partial \boldsymbol{u}} \right) dt \\ &+ \left[\left(\frac{\partial \boldsymbol{u}}{\partial \eta_e} \right)^T (-\boldsymbol{M}\dot{\boldsymbol{\lambda}} + \boldsymbol{C}\boldsymbol{\lambda}) + \left(\frac{\partial \dot{\boldsymbol{u}}}{\partial \eta_e} \right)^T \boldsymbol{M}\boldsymbol{\lambda} \right] \Bigg|_{t=t_f}\end{aligned} \tag{5}$$

Choosing λ in a way to eliminate derivatives of state variable u with respect ro design variable η_e leads to computation of λ and finally the adjoint sensitivity analysis.

2.2 Frequency Domain Approach

Here, a frequency domain method has been used for computational efficiency and is adopted from [15]. Assuming a response excitation under a single harmonic excitation, $\mathbf{f}(t) = \mathbf{F}e^{i\omega t}$, where $\mathbf{F}$ is the maximum amplitude and ω is the forcing frequency of the harmonic response. The maximum displacement is the solution of the state equation $\mathbf{F} = \mathbf{SU}$ where $\mathbf{U}$ is the peak amplitude of the displacement response and the dynamic stiffness is $\mathbf{S} = (\mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M})$.

The formulation of topology optimization for dynamic compliance minimization under a forcing frequency ω using the dynamic state equation is as follows:

$$\begin{array}{lll} \text{minimize} & \Phi_D(\rho, \mathbf{U}) = \left|\mathbf{F}^T\mathbf{U}(\rho)\right| & \\ \text{subject to:} & \int_\Omega \rho dV - \bar{V}0 & \\ & 0 \leq \rho_e \leq 1, & \forall e \in \Omega \\ \text{with:} & \mathbf{F} = \mathbf{SU} & \end{array}$$

where Φ_D is the objective function and $|.|$ stands for the complex modulus. The aforementioned topology optimization formulation is essentially equivalent to the standard static compliance topology optimization. The important feature is the introduction of the complex state equation and displacement vector $\mathbf{U}$. in the dynamic compliance. When the dynamic stiffness becomes closer to the structural stiffness matrix $\mathbf{S}(\omega = 0) = \mathbf{K}$, the challenge is reduced to topology optimization for minimising of static compliance. As the input driving frequency approaches one of the inherent frequencies of the structure, i.e. resonance, the dynamic compliance dramatically increases. Furthermore take note that several definitions of the dynamic compliance have been put forward, such as average power of the excitation $\Phi_D = \left|\mathrm{F}^T\mathrm{U}_\mathbf{I}\right|$ and total response $\Phi_D = \mathbf{U}^T\overline{\mathrm{U}}$, where $\mathbf{U}$ is the conjugate of the displacement vector, among others.

Extension to multi-frequency input excitations, such as earth-quake ground motions, by representing the excitation as a linear superposition of harmonics can be performed as follows:

$$\Phi_{TD}(\rho, \mathbf{U}) = \int_{\omega_a}^{\omega_b} \left|\mathbf{F}^T\mathbf{U}\right| d\omega \tag{6}$$

2.3 Nonlinear Dynamic Topology Optimization

In case of geometric non linear structures the stiffness matrix K will also depend upon the current sate of displacement in addition to the density design variable. So the equation now becomes:

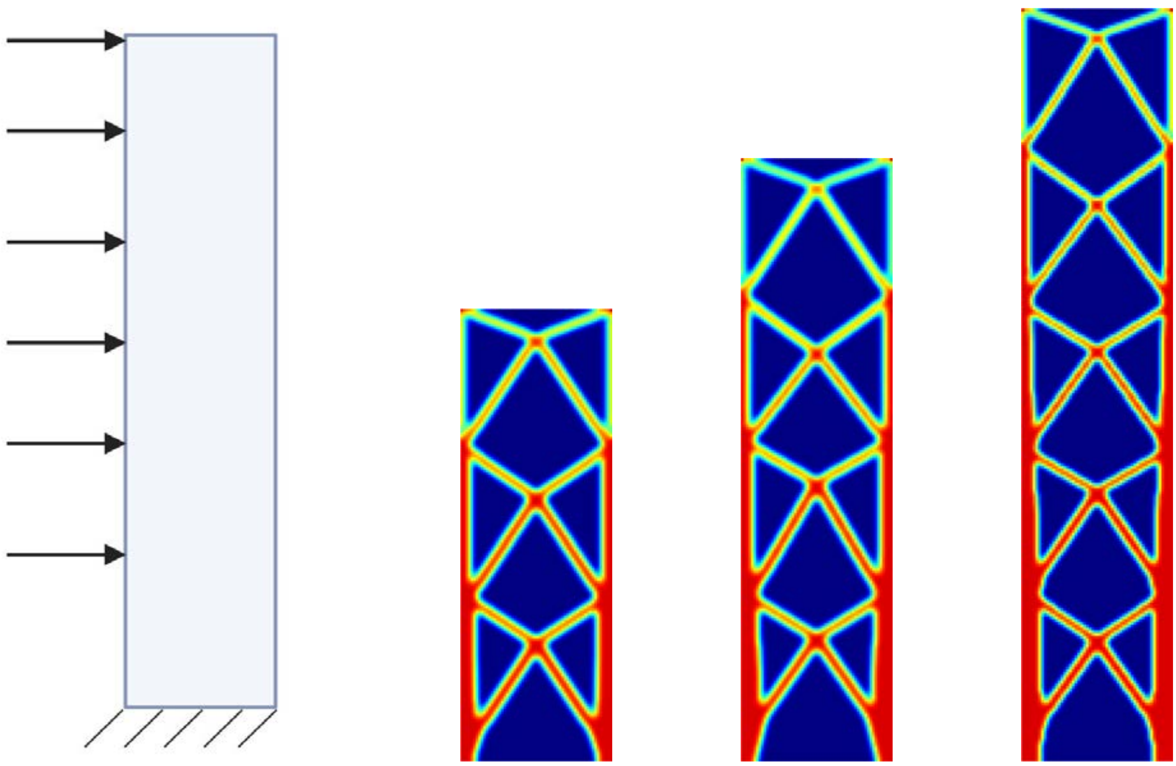
$$\begin{array}{ll} \min_\eta & J_d \\ \text{s.t.} & \boldsymbol{M}(\boldsymbol{\rho})\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}(\boldsymbol{\rho})\dot{\boldsymbol{u}}(t) + \boldsymbol{K}(\rho, u)\boldsymbol{u}(t) = \boldsymbol{f}(t), t \in [0, t_f] \\ & g(\eta) = V(\eta) - V_{\max} = \sum_{e=1}^{N} \eta_e v_e - V_{\max} \leq 0 \\ & \boldsymbol{0} < \eta_{\min} \leq \eta \leq \boldsymbol{1} \end{array} \tag{7}$$

The equilibrium equation is expressed in terms of residual as:

$$R = F_{ext} - F_{int} = 0 \tag{8}$$

3 RESULTS

Here a building shear wall subjected to lateral loading is considered for structural optimization ([15]) as shown in Fig. 1(a). Modal compliance in frequency domain is optimized to obtain the efficient layout subjected to volume fraction of 0.2. The building is fixed in the bottom. Three base to height ratio (1:3, 1:4, 1:5) have been studied to see the output shapes. The shapes are shown in Fig. 1(b), 1(c) and 1(d). The bracing system comes similar to eccentric bracing. Next a time domain analysis (adopted from [16]) has been conducted and output shape is shown in Fig. 2 for a three storey building, with masses concentrated in three storeys. It is to be noted that the blue colour space is actually void i.e. free of any material. Figure 3 shows a small comparison of linear analysis and geometric non linear optimization results. The difference is minimal for considered earthquake loading although it is being investigated further.



(a) (b) (c) (d)

Figure 1: Configuration of building lateral load resisting frame subjected to lateral loading

Figure 2: Configuration of building lateral load resisting frame subjected to lateral loading

4 CONCLUSIONS

In this paper dynamic response topology optimization using SIMP approach has been studied for dynamic compliance minimization subjected to volume fraction in time and frequency domain. Time domain analysis using Newmark-Beta algorithm takes significantly large time compared to frequency domain analysis. The output shapes are more or less same. The case study (taken from literature) of a building shear wall frame subjected to earthquake ground motion has been studied. Incorporation of geometric non linearity. The incorporation of geometric non linearity has shown minimal changes in the output layout for the considered earthquake loading although change of loading might have majot impact and needs further study.

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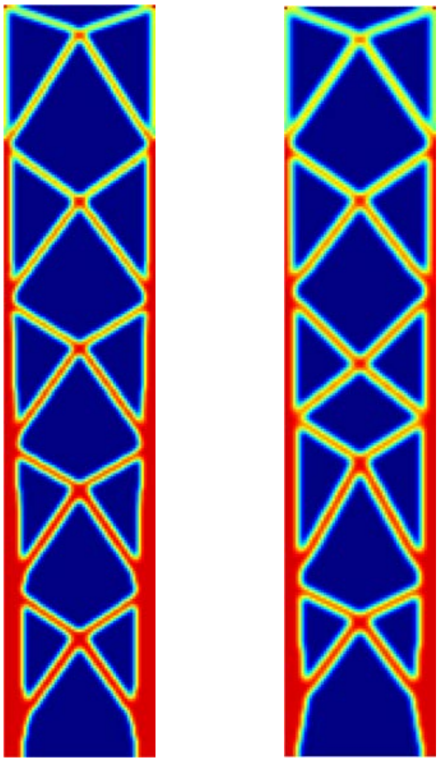

(a) (b)

Figure 3: (a) Linear (b) Non-linear analysis

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