Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2023) 2452-2464

ECCOMAS
Proceedia

COMPDYN 2023
9th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Athens, Greece, 12-14 June 2023

A NOVEL FE MODEL FOR THE TOWER OF PISA

Dionis Butuc[1], **Anna De Falco**[1], **Carlo Resta**[2], **Nunziante Squeglia**[1] **and Laura Vignali**[1]

[1]Dept. of Civil and Industrial Eng., University of Pisa
Largo Lucio Lazzarino 1, Pisa – Italy
e-mail: dionisbutuc@gmail.com, anna.de.falco@unipi.it, nunziante.squeglia@unipi.it,
laura.vignali@phd.unipi.it

[2] Dept. of Energy, Systems, Territory and Construction Eng., University of Pisa
Largo Lucio Lazzarino 1, Pisa – Italy
e-mail: carlo.resta@ing.unipi.it

Abstract. *The Leaning Tower of Pisa, in Italy, is a structure unlike any other; for its peculiar shape and position, but also on account of its historical and cultural relevance. A large number of studies focused on different aspects of its safeguard, and during the 20th century interest by the scientific community surged, in response to a worrying increase in the tilt of the structure. Investigations culminated in a geotechnical stabilisation intervention, carried out around the turn of the century by an international Committee for the Safeguard of the Leaning Tower. Numerous numerical and experimental studies were carried out, simulating the behavior of the ground and that of the structure; consolidation interventions were implemented to prevent structural failures; provisional structures were built and a monitoring system was installed to measure relative displacements and crack opening. Despite the clear success of the intervention, there are still some open questions concerning the structural safety of the Tower. This work presents a new model of the monument, designed to be the most accurate yet, which will be used to help scholars gain insight into the structural behavior of the Tower. As a test and a showcase of the model's use, the results of two analyses regarding the effects of tilt on damage and bending are also reported.*

Keywords: Finite Element Model, Cultural Heritage, Leaning Tower of Pisa, masonry, monitoring

ISSN:2623-3347 © 2023 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of COMPDYN 2023.
doi: 10.7712/120123.10574.21247

1 Introduction

1.1 A history of the Tower and its tilt

The Leaning Tower of Pisa (Figure 1a) is part of the medieval monumental complex of Piazza del Duomo, comprising the Cathedral of Santa Maria Assunta, the San Giovanni Baptistery and the Camposanto Monumentale. Its unique inclination, which during the monument's history reached a maximum value of 5.5 degrees southward, is the basis of both its fortune and its vulnerability.

The Tower is an eight-storey hollow cylinder of marble (Figure 1b), surrounded by colonnades. Its height is about 54 m above the ground level, the external diameter is 16 m and the wall is about 4 m thick. The masonry foundations of 19.6 m in diameter rest on weak, highly compressible soils, which can be grouped in three complexes having different mechanical characteristics. Complex A, ranging to a depth of 10 m, is the most compressible layer, made of silts, clays and sands, with a water table between 1 m and 2 m deep; Complex B consists of very soft sensitive marine clays up to a depth of 40 m; Complex C is a dense sand layer extending to considerable depth [1].

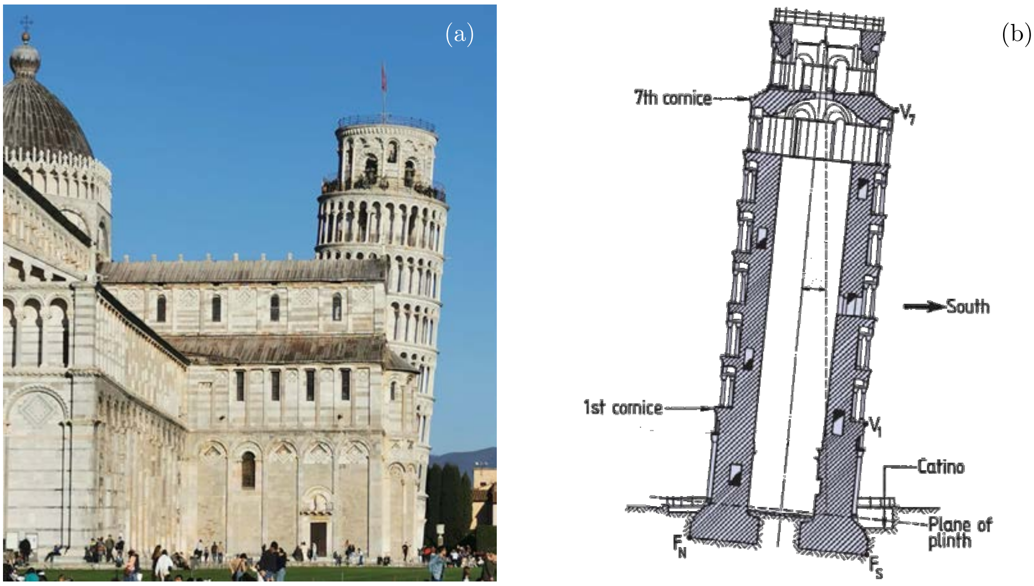

Figure 1: The Leaning Tower of Pisa, seen here (a) playing hide-and-seek (badly) behind the south transept of the Cathedral; (b) vertical section on the plane of maximum tilt (adapted from [2]).

The surface of separation between the first two layers right underneath the foundation shows a 2 m depression induced by the weight of the Tower, which amounts to about 14000 tons.

The construction of the monument was probably carried out based on a project by Bonanno Pisano and lasted two hundred years, starting from 1173. A first interruption in the construction occurred five years later, when the fourth floor was reached. The reason for this is unknown, but it is certain that this 100-year standstill avoided a probable collapse of the Tower, due to the soil in complex B, which would not yet have been strong enough to support the Tower's load. When work recommenced, the strength of the clay had increased due to consolidation under the weight of the Tower, but another stop occurred in 1278, when construction had reached the seventh stage. The bell chamber was completed almost two hundred years after the work had begun, and, in the meantime, a further consolidation of the underlying clay had probably taken place. Attempts by the builders to recenter the centre of gravity over the base at each floor resulted in the characteristic curved shape of the Tower.

Based on the modifications made to the Tower by the ancient builders, it is possible to trace the evolution of the inclination, which seems to have already been 0.6°, in the south direction, in 1278. Since then, the inclination has continued to grow, and, in this regard, three events occurred in the history of the Tower that should be cited as they each caused a sudden increase in tilt:

1. in 1838, architect Alessandro Gherardesca excavated the soil around the base of the Tower to build the Catino, which is a masonry basin created to bring to light the long-buried entrance following a subsidence of about three meters. On that occasion, about a 0.4 degree increment of inclination occurred.

2. In 1935, the Rodio company waterproofed the Catino, due to the presence of water springing from the foundation. During the works, the inclination had a sudden increase of about 31 arc seconds.

3. Around 1970, groundwater pumping operations significantly increased the rate at which tilt was increasing. For this reason, they were immediately prohibited.

From 1930 to the 1990s the rate of tilt increased from 3 to 6 arc seconds per year, reaching a total of 5.5°. In 1990 the Leaning Tower of Pisa was closed to the public, fearing that its foundations were about to become unstable.

Preserving this valuable historic monument required noninvasive and reversible stabilisation measures. For this reason, researchers and scholars worked hard to find a suitable solution for the purpose. In 1993, as a temporary measure, 600 tonnes of lead weights were applied to the northern side of the foundation on a post-tensioned removable concrete ring. This prompted a reduction in tilt of about 60 arc seconds, also reducing the overturning moment by about 10%. At that time, concerns about the safety of the masonry structure began to arise, and, in 1992, lightly post-tensioned steel tendons were placed around the Tower at the first cornice and at intervals up the second floor. In September 1995, the unsuccessful attempt to install temporary ground anchors caused an unstable response of the Tower, and lead weights were increased to 900 tonnes to control the movements.

Concern for stress in the masonry and instability of the foundation led to the search for a permanent solution to reduce the inclination, and, in view of this, many possible ways were studied to induce a controlled subsidence of the northern side. After installing safeguard cables attached to the third floor of the Tower and anchored, at a distance of 100 meters, to two massive steel A-frames, a preliminary soil extraction was carried out in 1999, until the northward rotation reached about 130 arc seconds. Building on the success of this operation, full underexcavation was undertaken in 2000. The intervention led to a total decrease in inclination of about 0.5°. The current tilt of the Tower is therefore of about 5 degrees.

1.2 Monitoring efforts

Since the end of the 13th century, 17 committees succeeded one another in the study of the Leaning Tower, trying to protect it against the perils brought on by its tilt. However, only in the 20th century (probably as a consequence of the St. Mark's Bell Tower having collapsed, in Venice, in 1902) a periodic control of the movements of the Tower started; before then, the only two measurements of tilt were the one carried out with the plumb line by Cresy and Taylor in

1817 (registering approximately 4.9°) and that by De Fleury in 1859 (registering about 5.3°). By comparing these two values, the increase caused by the excavation of Catino is evident [3].

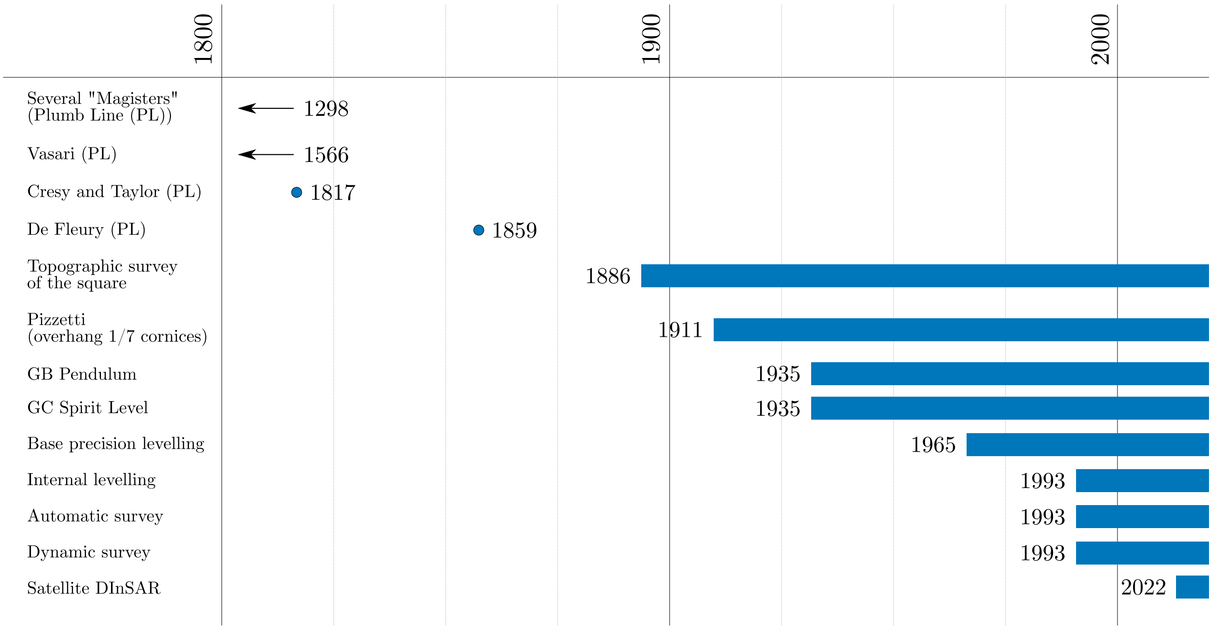

Figure 2: Timeline of monitoring systems installed on the Tower over the years.

The investigation into the inclination of the Tower is part of a broader context which consists in the study of the general subsidence of the whole Piazza del Duomo, obtained through precision levelling, which has been going on since 1908 (Bernieri Committee). Over the centuries, the number and position of levelling benchmarks have obviously changed, and since 1992 the reference point, initially located at the entrance of the Baptistery (IGM-CV 1886), has become more reliable. The current one is materialised by a 60 metre-long invar rod, anchored to the deep sand layers of Complex C [4].

The benchmarks of the Tower were installed on the foot of four of the ground floor columns by the Technical Board of 1927; in 1965 the Istituto Geografico Militare (IGM, *Military Geographical Institute*), entrusted by the Polvani Committee, redesigned the levelling network and added 15 bronze bolts (one for each column), so that the base of the Tower carried 19 benchmarks. During the stabilisation works of the 1990s, some points of measurement in the Catino and 9 internal staffs (at an height of 1.5 metres from the base) were also included.

Next to the levelling measurements, the Bernieri Committee started a long tradition of evaluating of the out of plumb between the seventh and first order of the Tower by observation with a theodolite, a technique also referred to as Pizzetti's method. Since 1934, these values can be compared with those obtained by the Girometti-Bonechi pendulum (GB) and the spirit level of Genio Civile (GC), which were installed to increase the frequency of measurements and to evaluate the difference between the rotation of the foundation (captured by GC) and the deformation of the Tower body (known thanks to GB). The Polvani Committee added other instruments in 1965: four levels at first floor (Salvadori levels), an anemometer and a thermometer.

In 1988, the collapse of the Civic Tower of Pavia fed fears regarding the fate of the Tower, so an International Committee for the safeguard of the Pisa Tower was created. Its members decided to develop a continuous monitoring system, which remained active during all stabilisation works, providing hourly measurements. The network was made of 25 deformometers to detect any change in existing cracks, two levelling circuits at two different floors, 11 inclinometers, two pendulums (on the north and south sides) with telecoordinometers, 22 wire extensometers to analyse the changes in the shape of the Tower's body, 12 thermometers and a weather station on the top floor to correlate the tilt with environmental variables. Five seismometers were also installed for dynamic assessments.

Once stabilisation was completed, the monitoring system was appropriately modified and simplified to adapt it to the different needs of controlling the behavior of the Tower over time. For this reason, the instruments measuring inclination are now the GB pendulum, the southern pendulum (northern one was dismissed) and an inverted pendulum added in 2001 on the east side (suitable for measuring rotations of the foundation).

As of today, the whole Piazza del Duomo is the object of a ministerial project for the monitoring and preservation of cultural heritage, so the University of Pisa, in collaboration with Opera della Primaziale Pisana, will implement the monitoring system of the Tower by adding new accelerometers, electro-levels and thermometers. Movements of the square will be also observed by satellite radars, and the results will be compared with a terrestrial measurement campaign [5]. A summary of all monitoring techniques employed for the study and safeguard of the Tower can be found in Figure 2.

1.3 Open questions

Despite the clear success of the intervention, several open questions remain. Two of them, concerning the structural safety of the Tower before and after the stabilisation, can be summarised as follows:

1. had the intervention never taken place, and therefore had the inclination progressed, what would have been the consequences for the monument? In other words, which is the influence of the Tower's tilt on damage to the masonry?

2. What influence does the tilt have on the Tower's bending?

To gain insight into these questions and more generally to foster research regarding the Tower's structural health, a new Finite Element model of the Leaning Tower has been created and its first applications are presented in this study. The model's 3D geometry is developed on the basis of a very refined point cloud obtained via a laser scanner survey capturing an unprecedented level of detail, while at the same time keeping the mesh as simple as possible. It is able to reproduce the inclination of the structure and non-straightness of the shape of its stem, the interior and exterior shape of the Tower, as well as layered structure of the walls, and even the stairway built inside the wall. The 'Catino' is also included in order to evaluate its contribution to the stability of the Tower. The ground is modeled on the basis of the knowledge acquired during the geotechnical studies and works. The calibration of the model parameters, which now take values from previous studies, will be carried out as soon as data from the new monitoring system that will be installed on the Tower will be available. This model will be improved by adding non-linear constitutive laws of materials to provide answers regarding future scenarios, including seismic ones.

In this brief work, we present the model's characteristics, as well as preliminary results regarding the two open questions outlined earlier.

2 Materials and Methods

2.1 FE model of the Tower

A Finite Element (FE) model of the Leaning Tower was produced to improve on existing simulation tools. Its creation was possible thanks to a vast wealth of experimental data inherited from the numerous investigators that dedicated their efforts to study the monument.

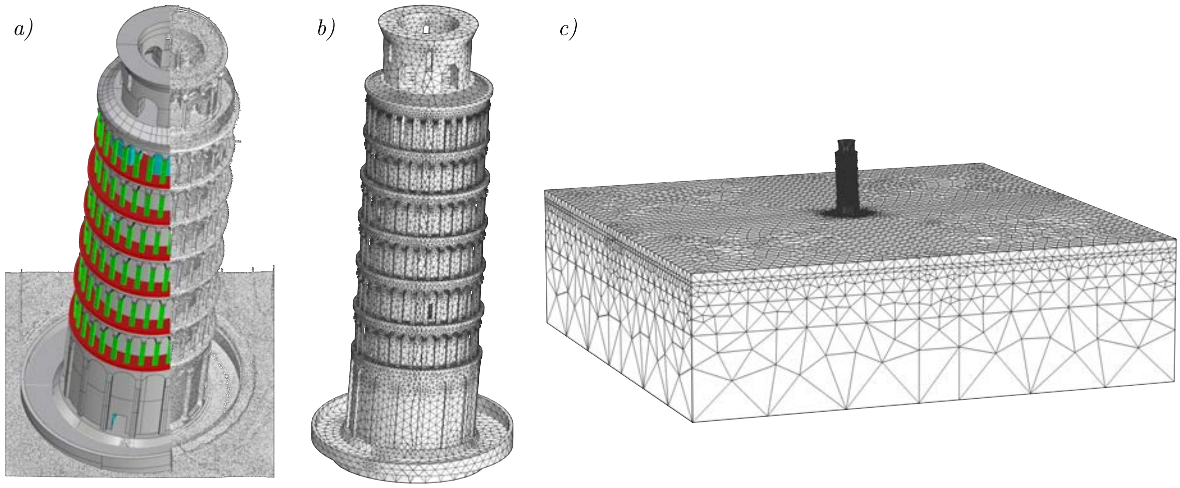

Figure 3: Generation of the FE model of the Tower: (a) reference point cloud data from laser scanner survey and 3D CAD model generated from it; (b) FE mesh; (c) overall view of the FE model.

The FE model was designed in order to satisfy two different — and apparently conflicting — necessities: to represent the actual geometry of the Tower as accurately as possible, and to require a reasonable amount of computational resources when used for nonlinear analyses. The first objective was pursued by designing the 3D CAD of the structure based on a detailed point cloud (see Figure 3a), obtained from and external and internal laser scanner survey of the Tower. At the same time, specific architectural elements were simplified in order to reduce the complexity of the analysis mesh. Notably, columns were replaced with ones of same-inertia square cross-section, and details such as steps were smoothed out in the top portion of the Tower.

The *Catino* was also modelled, in order to allow studying the effects of its interaction with the Tower.

To avoid an excessive simplification of the Tower's base restraint, a portion of soil under the structure has also been modelled (Figure 3c). In the present state of things, it is made up of four layers, corresponding to the three Complexes, the most superficial of which (Complex A) is again split according to the experimentally determined properties [1].

The complete model of the Tower consists in 346903 serendipity tetrahedral finite elements (Figures 3b and 3c). Of these, 88053 (about 25%) are used to represent the soil layers and 2635 for the Catino.

Figure 4 shows the different materials employed during the first analyses detailed in the present work. Material properties are summarised in the following Table 1. A nonlinear, damaging constitutive law, Mazars' scalar damage model, was employed for the Tower's masonry and columns. More details regarding on the matter can be found in the following Section 2.2. A linear elastic behaviour was instead assigned to the remaining portions of the model. The properties of the Tower's materials were chosen based on the results of experimental campaigns carried out by the Commissione Polvani [6]. In a similar fashion, property values for the subsoil model (which is considered linear elastic for these first investigations) are adapted from the site's stratigraphy, reported by Fiorentino *et al.* [1].

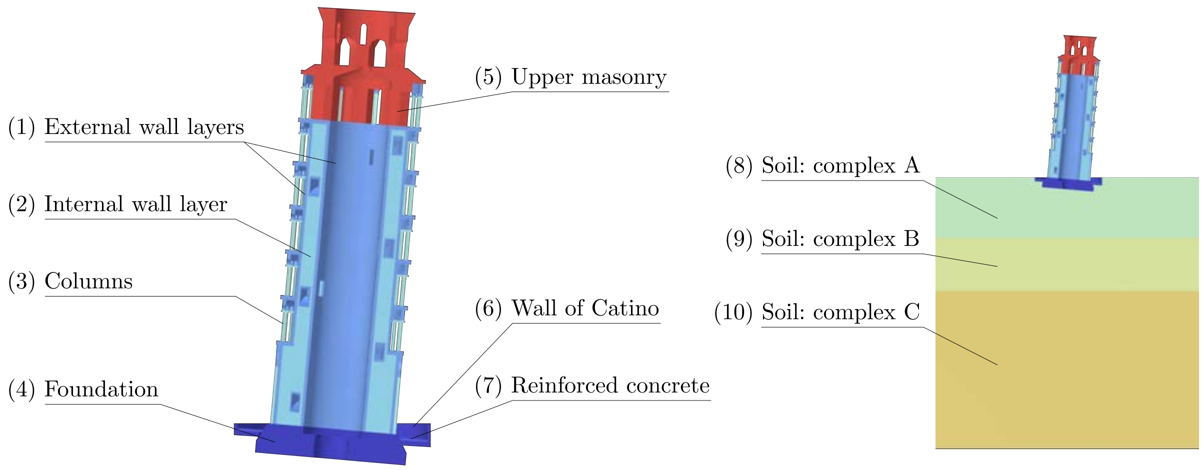

Figure 4: Materials used in the FE model.

Table 1: Mechanical properties of the materials used in the FE model.

	E [MPa]	ν [-]	f_c [MPa]	f_t [MPa]	γ [kN/m^3]
(1) External wall layers	$5.0 \cdot 10^4$	0.2	6.0	0.5	2400
(2) Internal wall layer	$7.0 \cdot 10^3$	0.15	3.0	0.2	2400
(3) Columns	$8.0 \cdot 10^4$	0.36	10.0	1.0	2400
(4) Foundation	$1.0 \cdot 10^4$	0.15	-	-	2420
(5) Upper masonry	$1.2 \cdot 10^4$	0.25	3.0	0.2	2400
(6) Wall of Catino	$5 \cdot 10^4$	0.2	-	-	2400
(7) Reinforced concrete	$2.5 \cdot 10^4$	0.2	-	-	2500
	V_S [m/s]	V_P [m/s]	γ [kN/m^3]		
(8) Soil: complex A	180	1650	18.50		
(9) Soil: complex B	230	1730	18.50		
(10) Soil: complex C	340	1730	20.52		

2.2 Mazars' scalar damage model

Mazars' constitutive law [7] was employed in this study. Although originally developed for concrete, as of late Mazars' material has also been successfully applied to simulate damage in masonry structures [8]. The material is isotropic. It tracks the evolution of damage thanks to a state variable d, which updates the original stiffness tensor Λ_0 to a damaged one $\Lambda(d)$:

$$\Lambda(d) = (1 - d)\Lambda_0. \tag{1}$$

Damage itself ranges from a starting value of 0 to a value of 1, when the material is no longer capable of sustaining any stress. Its value descends from the assumption that extensions cause cracking, so it depends on the maximum positive strains reached in any given point, and it is irreversible (i.e., there is no stiffness recovery). It can be expressed as a combination of damage due to tension and (indirectly, through Poisson deformations) to compression:

$$d = \alpha_t d_t + \alpha_c d_c. \tag{2}$$

Combination coefficients $\alpha_t, \alpha_c \in [0, 1]$ and:

$$\alpha_t = \sum_{i=1}^{3} \frac{\langle \epsilon_i^{(t)} \rangle \langle \epsilon_i \rangle}{\tilde{\epsilon}^2}, \quad \alpha_c = \sum_{i=1}^{3} \frac{\langle \epsilon_i^{(c)} \rangle \langle \epsilon_i \rangle}{\tilde{\epsilon}^2}. \tag{3}$$

Here, ϵ_i are the principal strains in a given point, while $\epsilon_i^{(t)}$ and $\epsilon_i^{(c)}$ are the eigenvalues of special strain tensors obtained considering only the tensile and compressive components of the stress tensor, respectively. Macaulay brackets, as usual, correspond to the ramp function.

$\tilde{\epsilon}$ is the *equivalent strain*, defined as:

$$\tilde{\epsilon} = \sqrt{\sum_{i=1}^{3} \langle \epsilon_i \rangle^2}. \tag{4}$$

Damage components d_t and d_c can be written as follows:

$$d_t(\tilde{\epsilon}) = 1 - \frac{k_0 (1 - A_t)}{\tilde{\epsilon}} - A_t e^{B_t(\tilde{\epsilon} - k_0)}, \tag{5}$$

$$d_c(\tilde{\epsilon}) = 1 - \frac{k_0 (1 - A_c)}{\tilde{\epsilon}} - A_c e^{B_c(\tilde{\epsilon} - k_0)}, \tag{6}$$

and scalar parameters A_t, B_t and A_c, B_c can be derived from experimental uniaxial tests on the material (or on equivalent simulations). To better illustrate the behaviour of Mazars' material, Figure 5 shows the results of a simulated uniaxial test on the same material used to model the external wall of the Tower, in tension and compression.

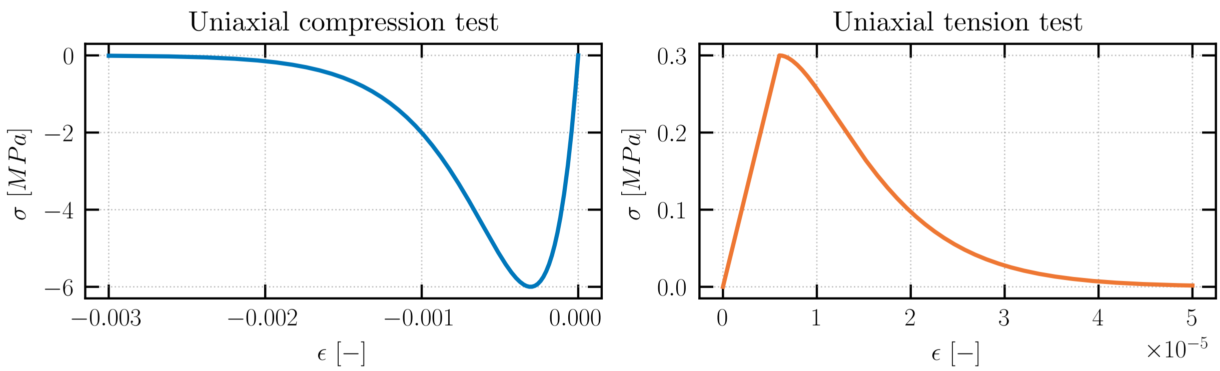

Figure 5: Model behaviour of uniaxial compression and tension test on sample with Mazars' material used to simulate the external wall of the Tower.

3 Results and discussion

3.1 Effect of tilt on damage

At the time of stabilisation, it had been clear for a while that the Tower's tilt was causing issues due to excess of compression on the southwestern side [6]. The most critical section was identified at the first floor, in the southwestern side, where the sudden shrinking of the wall's cross section and the presence of the stairwell also contributed to high levels of compressive stresses.

It was in fact subject to a first provisional intervention during the early nineties and was conclusively secured through targeted injections and the installation of hoops.

The area also proves the most suitable to evaluate the model's damage and stress distribution under self-weight and similarly to compare the effect of different tilt levels.

Figure 6 shows the model's results in terms of damage per Mazar's consitutive law (Figure 6a) and in terms of vertical stresses (Figure 6b). As validation, the latter can be compared with the same variable evaluated with a previous two-dimensional FE model of the critical section, designed by Prof. Giorgio Macchi in 1994 [9] and shown in Figure 6c. The comparison indicates a similar — and expected — concentration of stress on the two external wall layers, as well as a critical area across the external layer and the stairwell's contours. This is more clearly visible in the resulting damage in Figure 6a, where cracking appears in the corresponding zone. This area is in fact characterized by visible cracks.

The Tower's tilt history, as seen in Figure 7, encompassed all angles between zero and 5.5 degrees. Insight into the effect of tilt and self-weight on the masonry can be gained by comparing damage values resulting from the same FE model with different inclinations. Apart from the current tilt, a value of seven degrees was chosen for the comparison, as an extreme representation of a hypothetical future situation in which stabilisation never took place.

Figure 8 shows the results of the comparison, concentrating on the first three floors of the Tower since damage is mostly concentrated in that area. Compared to the current situation, shown in Figure 8a, an increase in tilt up to 7 degrees leads to the appearance of troublesome cracks on the northeastern side, extending from the outer layer of the wall to the internal filling (Figure 8b). Apart from that, the damaged area in and around the critical section also shows an expansion, and the infill wall closest to the foundation also show suffering from the increased load.

3.2 Effect of tilt on the Tower's bending

As briefly described in Section 1.2, several different instruments and techniques have been employed to monitor the Tower's tilt. Figure 9 shows in greater detail the measurements obtained along the years.

It can be easily seen that, after 1935, two separate series of measurements emerge. The first one, comprising the GC bubble level and precision levelling, can be thought of as a measure of tilt in the Tower's base. The second series, comprising all remaining data, corresponds instead to the Tower's overall tilt, measured between the first and seventh cornices. In the plot, the two data series have been set up so that, in 1935, they have corresponding values. That is to say, that any further discrepancies between the series have to be interpreted as an increase in the difference between overall tilt and tilt at the base.

An analysis, both experimental and numerical, of this discrepancy, may help to gain insight into the relationship between the Tower's tilt and bending.

A first deduction, of an empirical nature, can be done by observing that the discrepancy

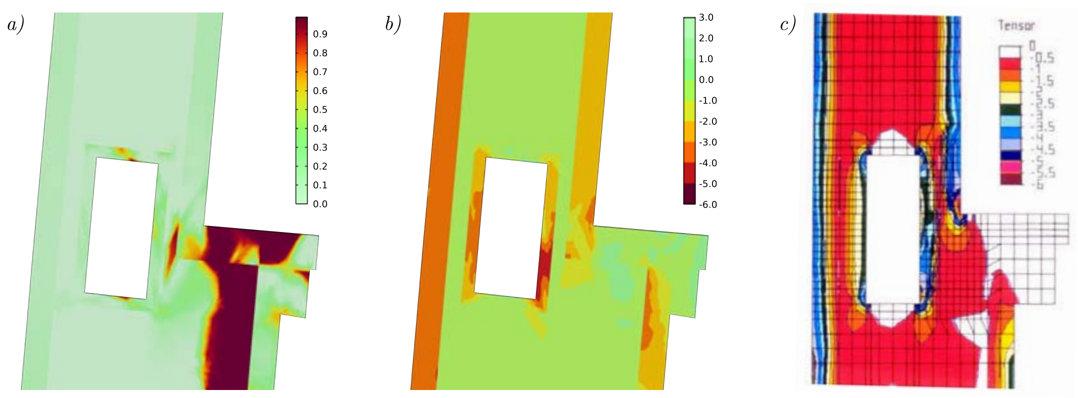

Figure 6: Effects of self weight: (a) damage per Mazars' constitutive law; (b) vertical stresses [MPa]; (c) vertical stresses from [9] [MPa].

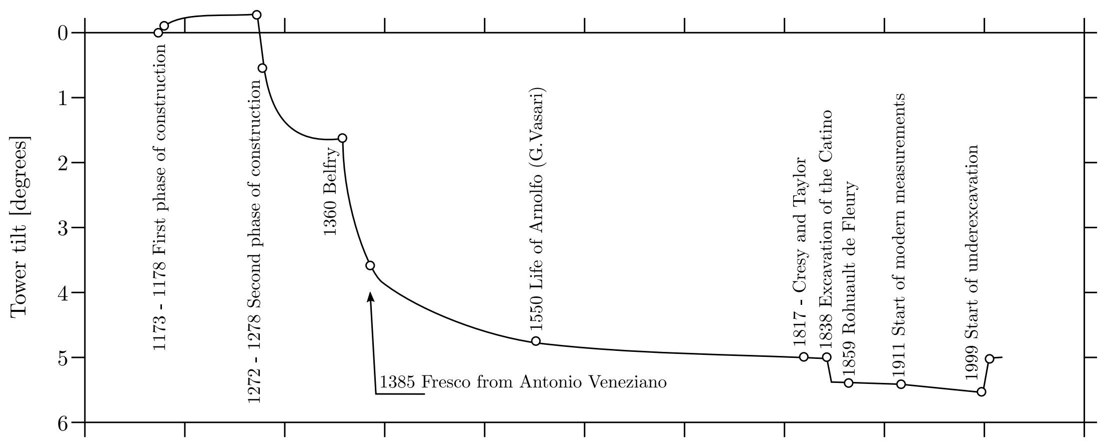

Figure 7: Reconstruction of the Tower's tilt history, from [10].

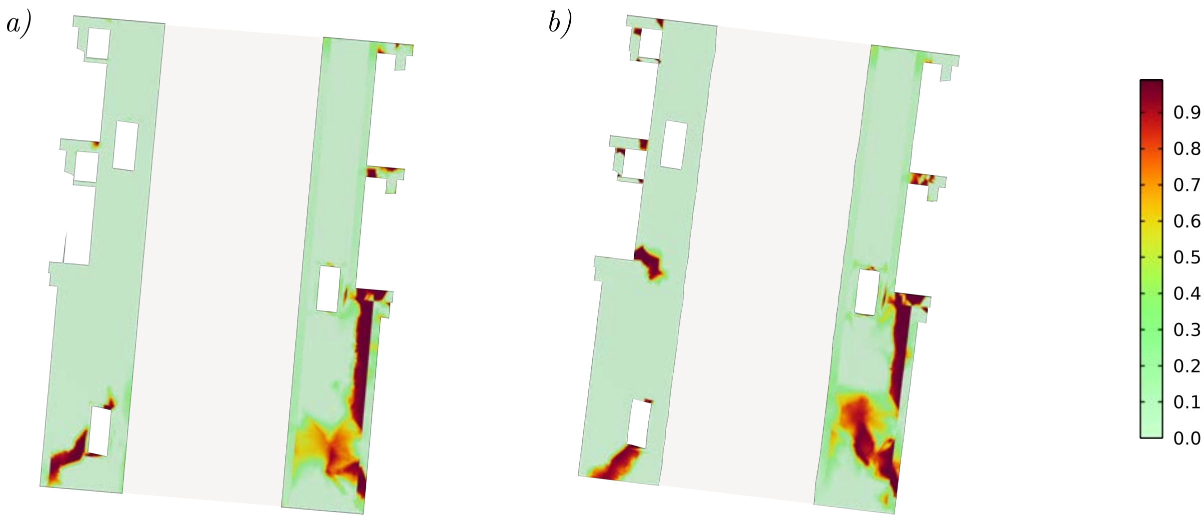

Figure 8: Effects of increasing tilt: damage on the first three floors of the Tower, per Mazars' constitutive law, due to self weight on models with 5° and 7° tilt.

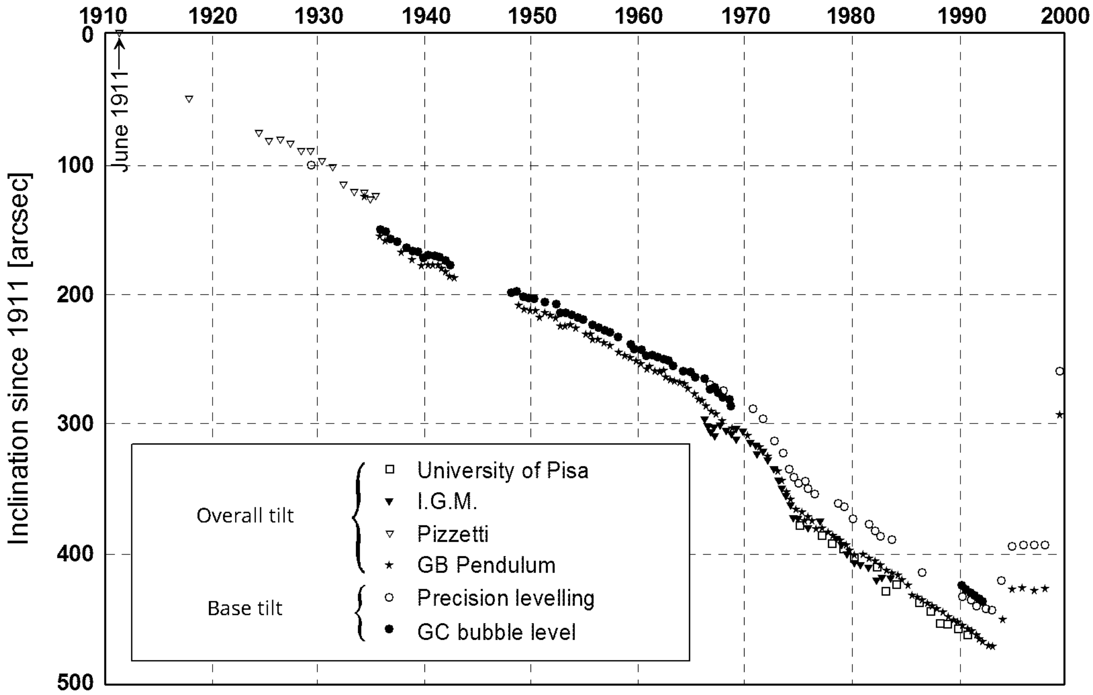

Figure 9: Measurements of the Tower's tilt since 1911, from [10].

between the two series of measurements remained more or less unaltered after the preliminary stabilisation intervention had taken place (1999). The backward rotation gained during the intervention (130 arcsecs circa) should have led to a reduction in bending and thus to a reduction of the aforementioned tilt discrepancy. The latter did not happen, which could suggest that slow, anelastic deformations were involved, instead of elastic ones.

To confirm this, the overall tilt of the Tower was evaluated, in the FE model, reproducing Pizzetti's technique, namely, by considering the relative positions of the first and seventh cornices. Once again, the model with a tilt of seven degrees was employed as a reference, extreme case. Comparing the seven- and the five-degree models, we found that a two degree rotation only led to a recovery of 39 arcseconds in overall tilt. This suggests that the effect of a mere one twelfth of a degree rotation of the Tower (as experienced between 1935 and 1990) would not be able to cause elastic deformations leading to an increase in bending of about 50 arcseconds, as seen in Figure 9. The FE model thus substantiates what had been inferred by measurement data alone, i.e., that elastic deformations are not the cause of increasing discrepancies in the Tower's overall and base tilt.

4 Conclusions

The Leaning Tower of Pisa, despite a long history of studies concerning its structural health, continues to offer numerous research questions, some of which remain open to this day.

We presented here a new FE model of the monument, built on the large trove of data from past studies to support future research efforts on the topic. The model, which also takes advantage of newly available data and computational resources, pursues a twofold objective: a very accurate representation of the Tower's geometry, as well as a relative computational simplicity. As a brief showcase of the opportunities offered by this model, we presented preliminary results regarding two open research questions. Firstly, the effects of different tilt values on the well-being of the Tower's masonry were evaluated, employing a nonlinear, damaging constitutive law for the analyses. Results showed that a further increase in the monument's tilt could have caused damage to appear in areas previously considered "safe", and that conversely an hypothetical absence of tilt would still cause cracking in the masonry due to excessive loads. The source of discrepancies between the Tower's overall tilt and the inclination of its base was then investigated, with numerical results pointing in the direction of slow onset deformations in the Tower's shaft.

The installation of a new monitoring system on the Tower is scheduled for the near future, so future research may start by taking advantage of this new source of data, to reduce uncertainty regarding material parameters of the model, and to investigate the structural role of the Catino and its relationship with the Tower.

REFERENCES

[1] Gabriele Fiorentino, Giuseppe Quaranta, George Mylonakis, Davide Lavorato, Alessandro Pagliaroli, Giorgia Carlucci, Fabio Sabetta, Giuseppe Della Monica, Giuseppe Lanzo, Victoria Aprile, Giuseppe Carlo Marano, Bruno Briseghella, Giorgio Monti, Nunziante Squeglia, Raffaello Bartelletti, and Camillo Nuti. Seismic Reassessment of the Leaning Tower of Pisa: Dynamic Monitoring, Site Response, and SSI. *Earthquake Spectra*, 35(2):703–736, May 2019.

[2] John B. Burland. The Stabilization of the Leaning Tower of Pisa. *Journal of Architectural Conservation*, 8(3):7–23, January 2002.

[3] Nunziante Squeglia and Giuseppe Bentivoglio. Role of Monitoring in Historical Building Restoration: The Case of Leaning Tower of Pisa. *International Journal of Architectural Heritage*, 9(1):38–47, January 2015.

[4] Gabriella Caroti, Andrea Piemonte, and Nunziante Squeglia. 100 Years of Geodetic Measurements in the Piazza del Duomo (Pisa, Italy): Reference Systems, Data Comparability and Geotechnical Monitoring. 2019.

[5] A. De Falco, C. Resta, and N. Squeglia. Satellite and on-site monitoring of subsidence for heritage preservation: A critical comparison from Piazza del Duomo in Pisa, Italy. In *Geotechnical Engineering for the Preservation of Monuments and Historic Sites III*, pages 548–559. CRC Press, London, first edition, May 2022.

[6] Commissione per il consolidamento della Torre pendente di Pisa. *Ricerche e Studi Su La Torre Pendente Di Pisa e i Fenomeni Connessi Alle Condizioni d'ambiente*. Ministero dei lavori pubblici, Firenze, Italy, 1971.

[7] Jacky Mazars. A description of micro-and macroscale damage of concrete structures. *Engineering Fracture Mechanics*, 25(5-6):729–737, 1986.

[8] Anna De Falco, Matteo Mori, and Giacomo Sevieri. Mazars' damage model for masonry structures: A case study of a church in Italy. In *Proceedings of the 2018 COMSOL Conference in Lausanne*, page 7, Lausanne, October 2018. COMSOL Multiphysics.

[9] Giorgio Macchi and Simone Ghelfi. Studi ed interventi del Comitato Internazionale: aspetti strutturali. In *La Torre restituita: gli studi e gli interventi che hanno consentito la*

stabilizzazione della Torre di Pisa, volume 1 of *Bollettino d'arte*, pages 183–213. Ministero per i beni e le attività culturali, Roma, Italy, libreria dello stato edition, 2005.

[10] N. Squeglia and C. Viggiani. *An Update on the Tower of Pisa*, pages 667–676. CRC Press, London, first edition, June 2022.