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A CAT BOND-BASED SOLUTION FOR SEISMIC RISK TRANSFER IN ITALY

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Abstract

Natural and man-made disasters are source of significant concern for privates and public authorities worldwide since they commonly imply relevant costs for repairing damaged structures and infrastructure and for a rapid recovery of the involved region's economy. In this context, the Catastrophe Bonds (CAT bonds) are risk-linked securities adopted by insurers to transfer potential high losses to the capital markets. Despite their growing importance, CAT bond pricing formulations and risk-managing solutions based on this financial tool are still limited. For these reasons, this paper wants to propose a general methodology for designing a CAT bond-based loss-coverage scheme for a distributed portfolio, with a pricing formulation able to consider uncertainties deriving from model parameters. The framework is applied to the residential building stock of Italy, proposing an ad-hoc CAT bond-based coverage scheme that consider three different levels of default risk.

Keywords: seismic risk, CAT-bond, risk transfer, structural engineering, earthquake engineering

1 INTRODUCTION

Natural and man-made are source of significant concern for privates and public authorities worldwide since they commonly imply relevant costs for repairing damaged structures and infrastructure and for a rapid recovery of the involved region's economy [1]. Every year, floods, earthquakes, tornadoes, and windstorms [2] cause billion dollars losses [3]. Furthermore, losses are expected increasing over time, given the growing urbanization and concentration of population ([4] and [5]). When dealing with natural disasters, losses are strictly spatially and temporally correlated, leading to potential huge losses that are difficult to be covered by insurances or public authorities. For this reason, reinsurance companies, i.e. companies that take on all or part of the risk covered under a policy issued by a first level of insurance companies, that usually have large portfolios, or public governments need to provide coverage of these potential high losses by adopting sophisticated so-called Alternative Risk Transfer products (ART) [6]. One ART solution is represented by the insurance-linked securitization, an alternative way for transforming catastrophe risk into securities (i.e., catastrophe bonds) and selling them to financial entities able to absorb such high levels of losses (i.e., the financial market). CAT bonds offer a significant supply for reinsurance surpassing the capacity of traditional providers and are therefore well suited to provide coverage for substantial losses [7]. So, in case of default, the principal is used to pay the losses of the issuing company, otherwise the capital is returned to the investor at maturity and coupons are also paid as counterweight to the assumed risk. An important aspect in designing an earthquake CAT bond, is the definition of the trigger event. Usually, a common trigger is the exceedance of a loss threshold, that is the one adopted in this study. Recently, [8] proposed a risk-based CAT bond pricing procedure able to consider the propagation of parameter uncertainties on the default probability of a CAT bond and on the pricing, while [9] showed a general methodology for addressing the design of a CAT bond-based coverage for a spatially distributed portfolio. This paper presents the outcomes of [9] in which a CAT bond-based coverage scheme against losses induced by seismic events all over the entire national boarders was priced for the residential building stock in Italy. Further details can be found in [9].

2 PROPOSED FRAMEWORK

The design of a possible CAT bond-based coverage scheme, can be subdivided in four main steps, illustrated in Figure 1. The procedure is general and can be easily adapted for designing a CAT bond-based coverage against different hazard by different issuing companies.
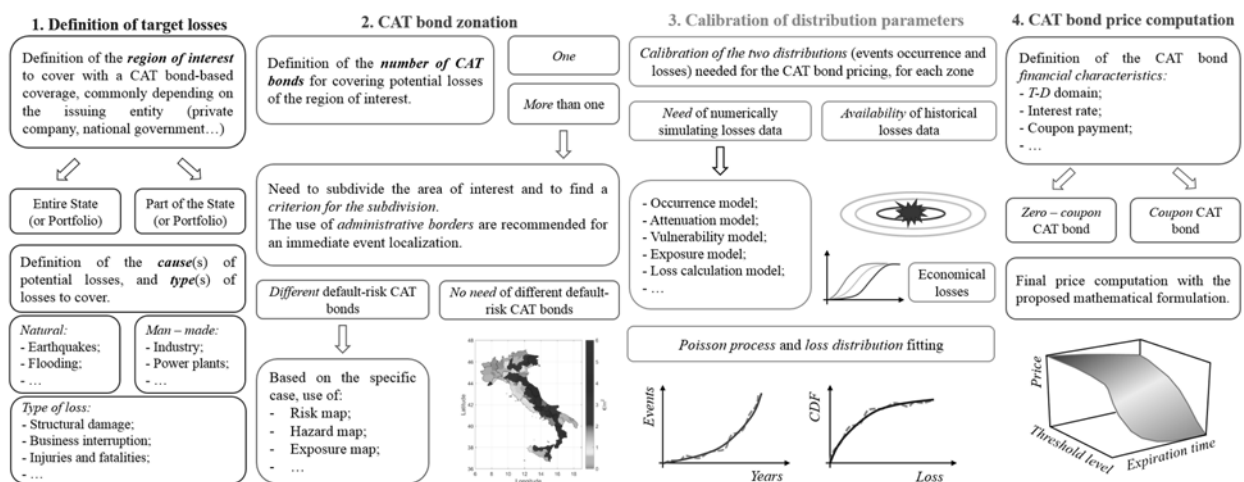

Figure 1: General framework for the designing of a CAT-bond based coverage scheme.

2.1   Definition of the target losses

In the first step of the proposed procedure, the issuing company has to identify the spatially distributed portfolio that has to be covered with the CAT bonds. In most of cases, public governments may want to cover the entire national territory, while private company may want to cover the insured portfolio. In many cases, the issuing company can decide to cover only a part of the portfolio, commonly the most exposed at risk. Secondly, the kind of losses to be covered with the CAT bond has to be identified. This decision is very case specific and particularly important, since the trigger event is based on the exceedance of this losses. Commonly, public authorities may consider to cover losses associate to direct structural damage on buildings, while a private issuing company, which offers multi-loss and multi-hazard coverage, has to carefully evaluate the losses to be insured.

2.2   Definition of the covered area

When the portfolio is distributed over a wide territory, different risk levels can be identified within the same region. In this context, a common practice is to practice is to tailor CAT bonds associated to different risk levels, in order to meet the needs of different types of investors. An area with many significant events leads to calibrating high-risk CAT bonds with related high gains for risk-seeking investors, while with rare and lowly impacting losses leads to low-risk CAT bonds. If a portfolio is uniformly distributed on a wide area with a quite homogeneous vulnerability and exposure, the subdivision can be guided by the hazard of interest. In this latter case, CAT bond default risk will be strictly connected to the hazard.

2.3   Calibration of the distributions' parameters

The third phase consist in the calibration of the distribution's parameters. In particular, the two distributions involved in the price computations are distribution of the expected losses and the Poisson process, representing the occurrence of the events. Regarding the loss distribution, rarely enough historical data of extreme events are available, and thus computer simulations are needed to predict potential losses that can arise for the portfolio of interest. Furthermore, when historical data are available, often they refer to old events for which structural vulnerability and exposure were different from the current ones, highlighting the need of simulations. Based on the specific considered loss, suitable loss models must be adopted.

2.4   CAT bond computation

This work adopts stochastic processes for CAT bond pricing. in this case, one common method is to model the credit default probability which follows the way of pricing credit derivatives in finance, and to assume the time to be continuous. The catastrophe process is thus modelled as a compound doubly stochastic Poisson process $M(s)$, where the potentially catastrophic events follow a doubly stochastic Poisson process, and the associated losses X_i are assumed independent and generated from a common cumulative distribution function (CDF). Clearly, this distribution function has to correctly fit the observed claims. The CAT bond's default occurs when the accumulated losses $L(t)$ exceed the money threshold level D before the expiration time T. Under these assumptions, the price for *zero-coupon* V_t^{zc} (i.e. debt security that does not pay interest but renders profit only at maturity) and coupon V_t^c CAT bond (i.e. debt security that includes attached coupons and pays periodic interest payments during its lifetime and its nominal value at maturity), can be computed as discounted expected value of the future payoff. More formally, the credit default probability can be computed as

$P_f(T,D;\boldsymbol{\Theta}) = P\big[L(T;\boldsymbol{\Theta}) \geq D\big]$, where $\boldsymbol{\Theta}=[\boldsymbol{\Theta}_P, \boldsymbol{\Theta}_L]$ represents the parameters characterizing the Poisson process $\boldsymbol{\Theta}_P$ and the loss distribution $\boldsymbol{\Theta}_L$. The inclusion of $\boldsymbol{\Theta}$ allows the analyst to take into account in the formulation the uncertainty of the model parameters and thus computing also the P_f and price bounds. Thus, conditioning on the number of events, and considering the independence between the Poisson point process and the incurred losses previous equation becomes $P_f(T,D;\boldsymbol{\Theta}) = \sum_{n=1}^{\infty} \big[1 - F_X^n(D;\boldsymbol{\Theta}_L)\big] \cdot P\big[M(T;\boldsymbol{\Theta}_P) = n\big]$, where $F_X^n(D;\boldsymbol{\Theta}_L)$ is the n-fold convolution of the loss distribution evaluated in D and represents the CDF of $X_1 + X_2 + \ldots + X_n$ ([10], [11]). This formulation is general and can be applied to every loss distribution type. Fig. 2 shows the procedure for CAT bond pricing based on a fixed accepted level of risk. First, the issuer defines a quantile q on the P_f distribution and finds the related CAT bond pricing surface, characterized by a constant risk value for each T-D combination. This procedure allows computing the entire P_f and V_t^{zc-c} distribution, or the value corresponding to a specific quantile q, for each T-D combination.
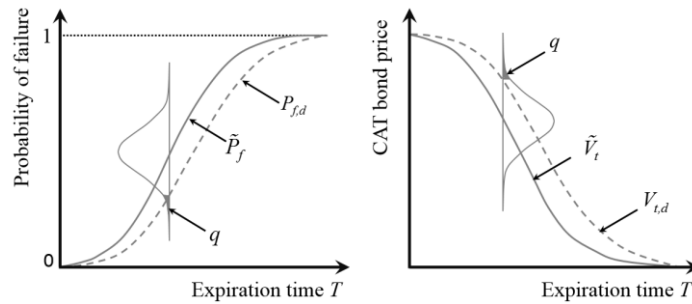

Figure 2: Relationship between P_f, $P_{f,d}$, $\tilde{P}_f$ and V_t, $V_{t,d}$ given a quantile q.

Following [12], the solid line in Fig. 2 represents a predictive $P_f(T,D)$ or point $\tilde{P}_f(T,D)$ estimate of $P_f(T,D;\boldsymbol{\Theta})$: $P_f(T,D)$ is computed as expected value of $P_f(T,D;\boldsymbol{\Theta})$ over $\boldsymbol{\Theta}$, while $\tilde{P}_f(T,D)$ is obtained by using a point estimate of $\boldsymbol{\Theta}$ (i.e. $\boldsymbol{\Theta}=\tilde{\boldsymbol{\Theta}}$, where $\tilde{\boldsymbol{\Theta}}$ could be the mean or median). Similarly to P_f, $V_t(T,D)$ (or $\tilde{V}_t(T,D)$) is a predictive (or point) estimate of the CAT bond price obtained from $P_f(T,D)$ (or $\tilde{P}_f(T,D)$). For each T-D combination, q is the probability that the default probability P_f is smaller than the probability $P_{f,d}$ assumed for the pricing design as the fixed risk, where d in the subscript stands for design value, and represented in Fig. 2 by a dotted line. $P_{f,d}$ is then needed for the calculation of the related CAT bond design price $V_{t,d}$ on the price distribution V_t. Assuming a quantile of the P_f distribution implies considering the same probability for the bond to be under-priced. Formally, this condition is given by

$$P\big[P_f < P_{f,d}\big] = P\big[V_t > V_{t,d}\big] = q \tag{1}$$

For computing $P_{f,d}$ for a given quantile q, the P_f distribution is thus needed. Since nested reliability calculations are required for the computation of the $P_f(T,D;\boldsymbol{\Theta})$ distribution due to

uncertainties in the model parameter, approximated quantiles obtained by first-order analysis can be used [12]. The design default probability $P_{f,d}$ can thus be calculated as

$$P_{f,d}(T,D) = \Phi\left[-\beta(T,d) - k \cdot \sigma_\beta(T,D)\right] \tag{2}$$

where $\Phi(\cdot)$ is the standard normal cumulative density function, $\beta(T,d)$ is the reliability index calculated as $\beta(T,d) = \Phi^{-1}\left\lfloor 1 - P_f(T,D)\right\rfloor$ (or similarly $\beta(T,d) = \Phi^{-1}\left\lfloor 1 - P_f(T,D)\right\rfloor$) and $k \cdot \sigma_\beta$ represents the quantile of the β distribution reflecting the acceptable level of risk. From the assumed quantile q, the constant term k can be computed as $k = \Phi^{-1}(1-q)$. Following [12] the variance $\sigma_\beta(T,D)$ of the reliability index $\beta(T,D;\Theta)$ can then be approximated by using a first-order Taylor series expansion around $\mathbf{M}_\Theta$, where $\mathbf{M}_\Theta$ is the mean vector Θ

$$\sigma_\beta^2(T,D) \approx \nabla_\Theta\beta(T,D)^T \Sigma_{\Theta\Theta}\nabla_\Theta\beta(T,D) \tag{3}$$

where $\Sigma_{\Theta\Theta}$ is the covariance matrix of the model parameters and $\nabla_\Theta\beta(T,D)$ is the gradient column vector of $\beta(T,D;\Theta)$ at $\mathbf{M}_\Theta$. The vector $\mathbf{M}_\Theta$ can be estimated either with the maximum likelihood estimation method or, more precisely, with the Bayesian updating technique, as the posterior mean vector. As for $\mathbf{M}_\Theta$, the covariance matrix can be computed in a simplified way as the negative of the inverse of the Hessian of the log-likelihood function [13] or, again, more precisely with the Bayesian updating technique. The gradient of β in Equation (3) is computed applying the chain rule to the definition of reliability index, while the gradient of P_f can be computed numerically using the definition of derivative. Once $P_{f,d}$ is calculated, the corresponding CAT bond price can be computed according to [8] as discounted expected value of the future payoff under the risk-neutral measure (or equivalent martingale measure), considering an arbitrage-free opportunities financial market. For both *zero-coupon* and *coupon* CAT bond, the bond principal is assumed to be completely lost, in case the bond is triggered. Given the threshold D, the price of the zero-coupon CAT bond $V_{t,d}^{zc}$ paying the principal Z at maturity time T and correspondent to the assumed quantile q is

$$V_{t,d}^{zc}(T,D) = E\left[e^{-\int_t^T r(\xi)d\xi|} F_t\right] \cdot Z\left[1 - P_{f,d}(T,D)\right] \tag{4}$$

where $r(\xi)$ represents the stochastic discount factor. Finally, the price of the *coupon* CAT bond $V_{t,d}^c(T,D)$ paying the principal value PV at maturity, and *coupon* payments $C(s)$, which cease if the bond is triggered, can be obtained as

$$V_{t,d}^c(T,D) = E\left[e^{-\int_t^T r(\xi)d\xi} \mid F_t\right] \cdot PV\left[1 - P_{f,d}(T,D)\right] +$$
$$+\int_t^T E\left[e^{-\int_t^T r(\xi)d\xi_t} \mid F_t\right] C(s)\left[1 - P_{f,d}(s,D)\right]ds \tag{5}$$

Note that when k is assumed equal to +/-1, the approximate 15% and 85% percentile bounds of P_j and consequently of V_t^{zc} (or V_t^c) containing 70% of the probability, are computed. The complete mathematical derivation of the pricing technique here summarized can be found in [8] and [9].

3 CASE STUDY

The exposed framework is applied to design a coverage scheme for the entire residential building asset of Italy considering seismic events as relevant natural hazard. In this application, the Italian Government is taken as the issuing entity, which adopts CAT bonds for a full risk-transfer, considering as lower bound seismic events with magnitude $M_W \geqslant 4.5$. The region of interest is represented by the Italian peninsula, and the target losses are represented by the potential direct costs to be sustained for repairing seismic damage to the Italian residential building stock. First, Italy is divided in three zones based on the seismic risk maps developed by [14]. This zonation (Fig. 3a), based on the seismic risk map and adopting administrative borders, assures an almost constant combination of events frequencies and amount of losses within each zone, and the exact attribution of each event to the corresponding zone. The calibration of the Poisson process and loss distribution parameters is based on the numerical simulation of 100'000 years of seismicity within the national territory, because of the limited number of real losses and claim data. For the generation of 100'000 years of seismic events, the seismogenic source zone model ZS9 of [15] is adopted, together with the seismogenic zone parameters of [16]. The shaking scenario associated to each generated event, is computed in terms of peak ground acceleration with the ground motion prediction equation proposed by [17]. According to [14], the seismic vulnerability of the Italian residential building stock is characterized by setting a building taxonomy consisting in 8 taxonomy classes (TCs): (i) masonry structures built before 1919, (ii) masonry structures built post 1919, (iii) gravity load designed reinforced concrete (RC) structures with 1-2 storeis, (iv) gravity load designed RC structures with 3+ storeis, (v) seismic load designed RC structures with 1-2 storeis, (vi) seismic load designed RC structures with 3+ storeis, (vii) gravity load designed masonry-RC structures, (viii) seismic load designed masonry-RC structures. References and parameters of each class fragility curve can be retrieved in [14]. The exposure model of the national residential building stock is defined at municipality-level granularity and data are retrieved from the 15th census database of the National Institute of Statistics [18]. Fig. 3b and 3c illustrates 100'000 years of simulated seismicity for the seismogenic zone 905. For the calibration of the three sets of distributions parameters, earthquakes occurred inside of each CAT bond zone border were then selected. Fig. 3d shows the selected events for each zone, resulting in 126'414 in Zone 1, 151'245 in Zone 2 and 38'380 in Zone 3. Among the three, Zone 2 has the highest intensity since more events occur in it, in the same time window. Lognormal CDFs were fitted on the cumulative losses to obtain the loss distribution parameters for each zone (Fig. 3e). In the present work, CAT bond price is evaluated at time $t = 0$, assuming a principal equal to 1 €. Two different products were considered for the pricing, a *zero-coupon* and a *coupon* CAT bond, both with a full loss of the principal in case of bond triggering. In the first case, the *zero-coupon* CAT bond is assumed to be priced at 3.5% over LIBOR so that if no trigger event occurs, the total yield is 6%, and consequently $Z = 1.06$ €. For the coupon CAT bond, the yearly coupon payments $C(s) = 0.06$ € and $PV = 1.00$ € are considered. A continuous discount rate r equivalent to LIBOR = 2.5% is assumed constant and equal to $ln(1.025)$ [19]. Expiration time and threshold level are considered respectively ranging between [0.25, 5] years and [0.1, 10] *bn* €, guaranteeing in this way a sufficiently broad *T-D* domain for show-

ing the variation of CAT bond price for a wide range of possible combinations. The bond for a zone is triggered when the accumulated losses caused by earthquakes occurred within the zone are greater than the set threshold before the set expiration time.
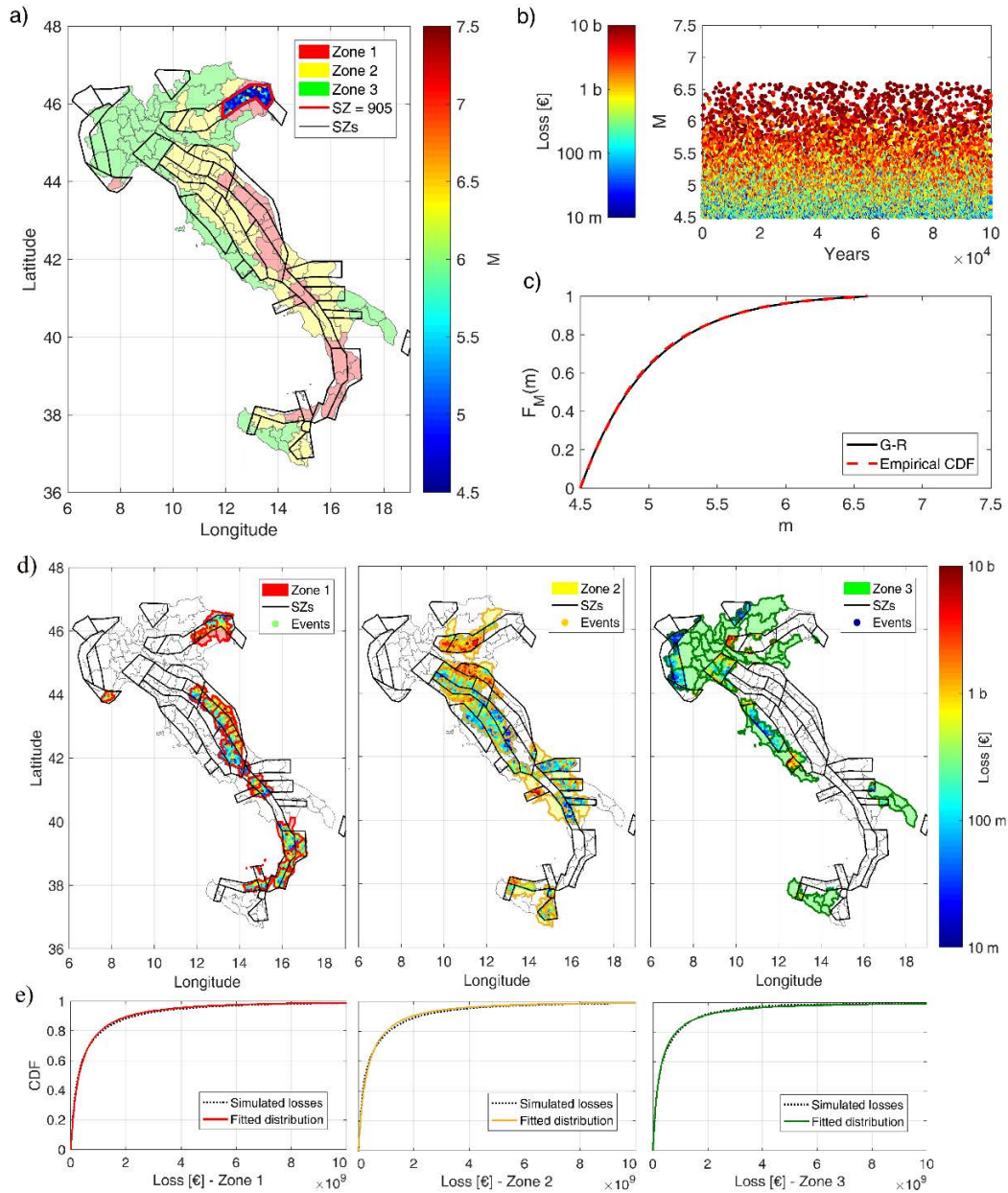

Figure 3: Proposed CAT bond zonation (a), 100'000-years simulated seismicity for SZ #905 (b-c), selected events for each Zone (d) and loss data fitting with lognormal distribution (e) (adapted from [9]).

Fig. 4 shows the probability of failure P_f surfaces for Zones 1, 2 and 3, together with the bounds deriving from considering the parameters uncertainties and containing the 80% of the probability. Two cross sections of the surface are also shown, corresponding to planes with T = 2 years, and D = 3 bn €. As a general behaviour common for all the three zones, for a given expiration time T, P_f decreases as the threshold level D increases, whereas for a given threshold level D, P_f increases from 0 to 1 over time. P_f of Zone 1 and Zone 2 are comparable

since despite a slightly lower expected loss, Zone 2 has a higher Poisson intensity due to a wider zone area and consequently more events inside. Zone 3 has the lowest P_f due to a combination of lower expected losses and less expected events.
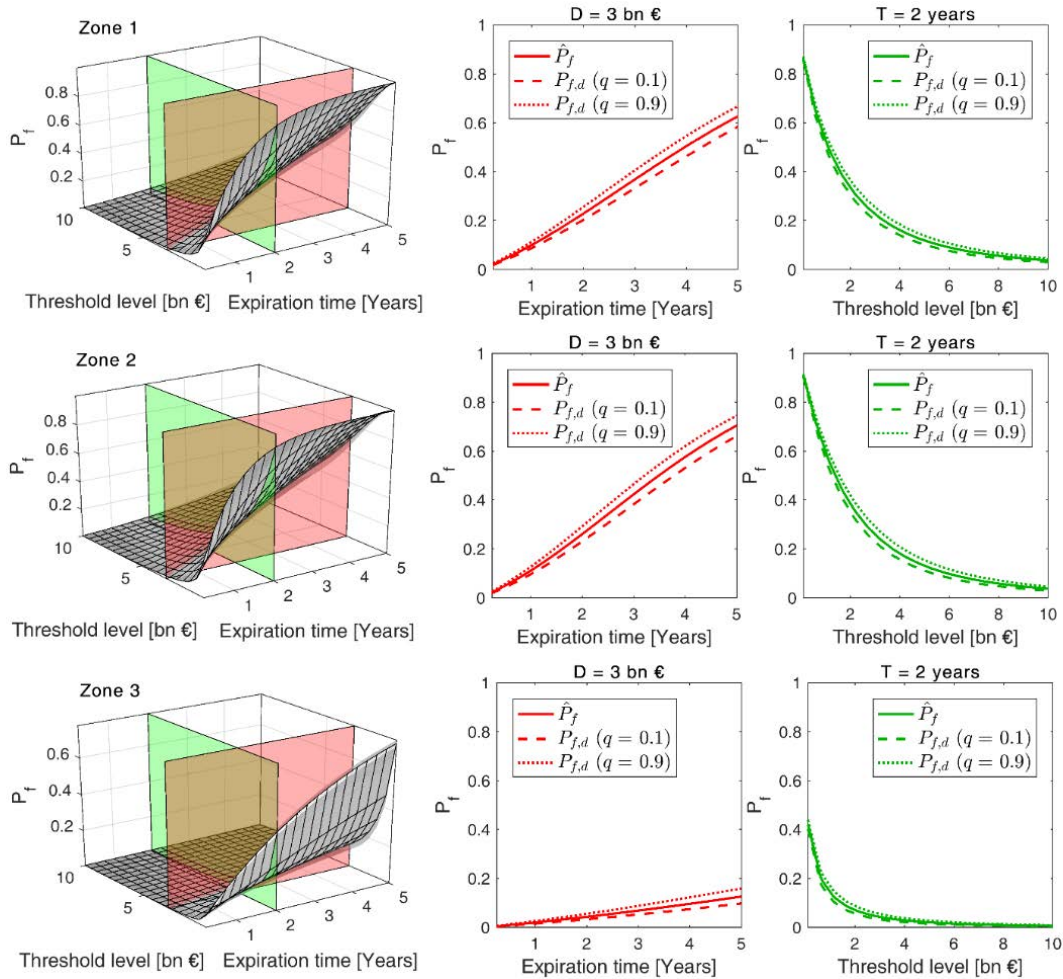

Figure 4. Failure probability surface for the three Zones (adapted from [9]).

Fig. 5a shows the zero-coupon CAT bond pricing surfaces V^{zc} paying $Z = 1.06$ € at maturity, for each Zone. In this case, for a given threshold level D, the CAT bond value decreases over time, whereas for a set expiration time T, the CAT bond value increases as the threshold level D increases. The prices reflect the related failure probabilities: price of Zone 3 is the highest since it is associated with the lowest probability of exceed a given money. Higher gains provided by the bonds are associated to higher failure probabilities. Fig. 5b illustrates the case of the coupon CAT bond, evidencing how the overall trend is similar to the zero-coupon one due to the high ratio intercurrent between the principal and the entity of coupons. Numerical results are the combination of two contributions: as time passes, the chance of receiving more coupon payments is bigger, but at the same time, the possibility of losing the principal increases. Both the zero-coupon CAT bond and the coupon CAT bond price reflect the different seismic risk-levels of the three zones. For a given T-D combination, the price for a bond in Zone 1 and Zone 2 is the lowest while the price in Zone 3 is the highest.
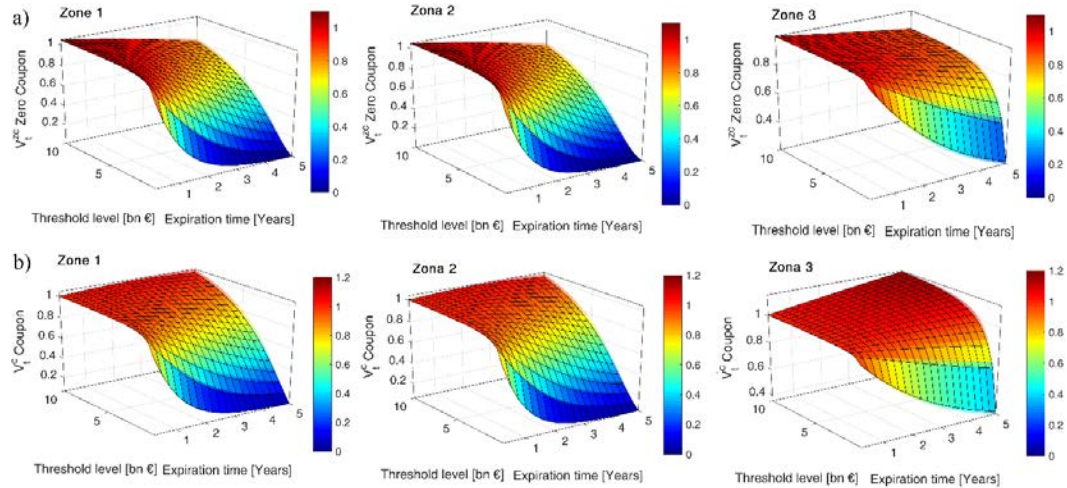
Figure 5. Zero-coupon (a) and coupon (b) CAT bond price for the three Zones (adapted from [9]).

Finally, a sensitivity analysis on the role of the parameters uncertainties on the final variability is performed. In particular, the aim of this analysis is to evaluate the uncertainty of which parameter mostly influences the overall dispersion. The analysis is performed for the *Zero-coupon* CAT bond price of Zone 1, in correspondence of four points of the *T-D* domain:

a) $T = 2$ years – $D = 2$ bn €

b) $T = 4$ years – $D = 2$ bn €

c) $T = 4$ years – $D = 6$ bn €

d) $T = 2$ years – $D = 6$ bn €.

In particular, for each *T-D* combination, six covariance matrices have been assumed for testing all the possible combinations of setting equal to zero the variances. AA shows that most of the total variability comes from the uncertainty of the parameter of the lognormal distribution representing losses due to earthquake, and this happens in all the four investigated points of the pricing surface.
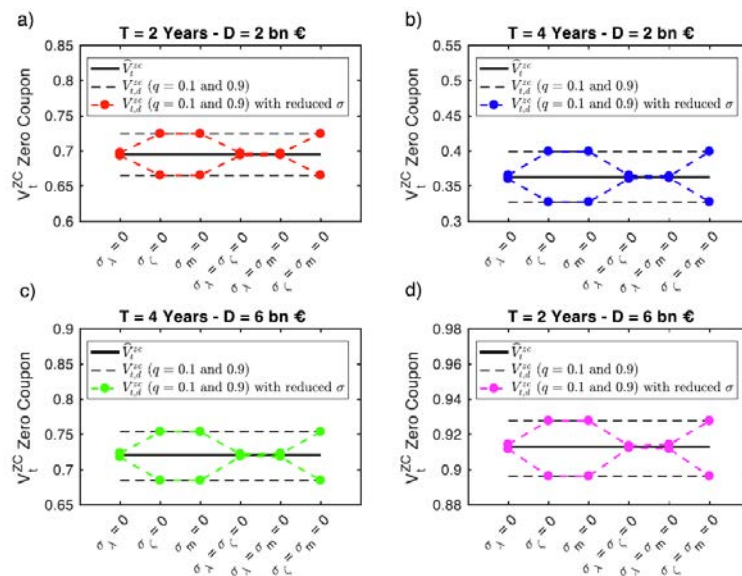

Figure 6. Sensitivity analysis for four points of the T-D domain (adapted from [9]).

4   CONCLUSION

This paper presented a general framework for designing a CAT bond coverage system for a distributed portfolio subject to significant losses arising from different possible sources, commonly natural hazards. The flexibility of the proposed methodology allows its adoption by different issuing entities, against various types of losses induced by natural or man-made hazards. For the CAT bond price computation, this paper adopts the mathematical formulation for CAT bond pricing based on a reliability assessment of the P_f underlying the pricing process. In this way, it is possible to obtain a complete knowledge of the default probability and CAT bond price distribution, for a given combination of loss threshold and expiration time. The related CAT bond pricing surface is characterized by a constant reliability for each expiration time T - threshold level D combination. The general framework is applied to a case-study in which a possible CAT bond-based coverage configuration is designed for the residential building portfolio of Italy against earthquake-induced structural losses. In the application, the Italian territory was subdivided in three zones, based on the Italian seismic risk map, and three different CAT bonds, characterized by different levels of default risk, were priced. The outcomes showed the effect of the CAT bond zonation on the final price computation, and the importance of considering uncertainty in the model parameters in defining a CAT bond pricing. This work can be considered the first original attempt currently retrievable in scientific literature aimed at a rational management of significant losses induced to the Italian residential building stock by seismic events.

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