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SEMI-ANALYTICAL SOLUTIONS FOR EVALUATING THE FIM FOR PILE FOUNDATIONS

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Abstract

In this contribution, the kinematic filtering effect of free-head and fixed-head single piles in layered soils is investigated to define new simplified formulas for evaluating the foundation input motion (FIM). These solutions have been derived numerically by employing a boundary element method code (KIN SP, Stacul and Squeglia [11]) and a finite element method code (VERSAT-P3D, Wu [12]). The proposed solutions allow to compute the transfer functions in translation (I_u) and rotation (I_θ), representing the ratio between pile-head and free-field motion as a function of the excitation frequency, for the following soil profiles: homogeneous, two-layered, parabolic and Gibson soil profiles. These solutions have been developed assuming a linear elastic behavior for the pile material and a linear viscoelastic soil model. Nevertheless, these solutions are expected to be valid also in the case of non-linear soil response and large earthquake-induced shear strains. In fact, as shown in a recent work (Stacul et al. [10]), kinematic soil-pile interaction is a stiffness-controlled mechanism while dynamic and non-linear effects simply modify soil response at free-field conditions.

Keywords: pile-soil kinematic interaction, soil-structure interaction, free-head single pile, fixed-head single pile, Foundation Input Motion.

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1 INTRODUCTION

Pile-soil kinematic interaction originates while soil deformations, due to the passage of seismic waves, try to impose a displacement field to the embedded pile which in contrast is not completely able to follow soil displacements due to its own flexural stiffness. As a result, two important effects arise: a) the development of kinematically induced internal forces (shear and bending) along the pile shaft, b) the modification of seismic motion at the foundation level compared to that at free-field conditions. Both phenomena have been studied by several authors [1-10] and this contribution is mainly focused on the issue at point b).

The latter is an important aspect when studying the seismic response of a soil-pile foundation-structure system, in fact, the so-called FIM (Foundation Input Motion) is the result of pile-soil kinematic interaction mechanism which modifies the free-field motion (Fig. 1).

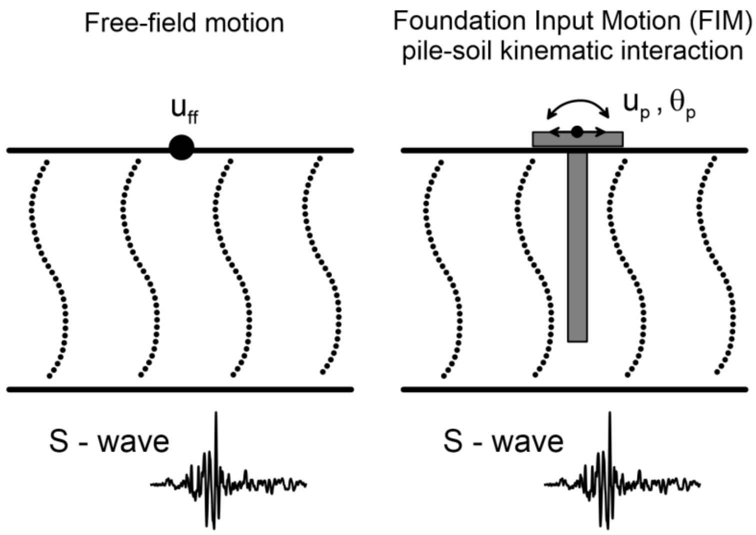

Figure 1: Pile-soil kinematic interaction.

The pile exerts a filtering effect as it can filter out some high frequency components of the free-field motion. The amount of this filtering is influenced by the pile (E_p) and soil (E_s) stiffness and by the pile diameter (d).

To quantify the filtering effect Blaney et al. [1] and later Nikolaou et al. [5] defined the kinematic response coefficients, I_u and I_θ, which represent the ratio of the pile-head horizontal displacement (u_p) (or acceleration, a_p) and rotation (θ_p), respectively, over the free-field displacement (u_{ff}) (or acceleration, a_{ff}) (Eqs. 1 and 2). In Refs. [5, 6] an analytical solution to derive I_u and I_θ for both fixed-head and free-head single piles embedded in a homogeneous soil were developed based on a Winkler model.

$$I_u(\omega) = \frac{a_p(\omega)}{a_{ff}(\omega)} = \frac{u_p(\omega)}{u_{ff}(\omega)} \tag{1}$$

$$I_\theta(\omega) = \frac{\ddot{\theta}_p(\omega)d}{a_{ff}(\omega)} = \frac{\theta_p(\omega)d}{u_{ff}(\omega)} \tag{2}$$

For the special case of an infinitely long fixed-head single pile embedded in a continuously inhomogeneous linear viscoelastic soil with the soil shear stiffness (G_s) varying with depth according to Eq. 3, Di Laora and Rovithis [8] derived, based on FEM results in the frequency domain, the Eq. 4 for I_{uR} (subscript R is used for the case of a fixed-head pile) in which $a_{eff,La}$

is a dimensionless effective frequency (Eq. 5) governing the pile-soil kinematic interaction in the dynamic regime in soil types as in Eq. 3.

$$G_s(z) = G_{sd}\left(a + (1-a)\frac{z}{d}\right)^n \tag{3}$$

In Eq. 3 G_{sd} is the shear modulus at one-diameter depth, a and n control the shape of G_s profile (homogeneous: a = any, n = 0; Gibson: a = 0, n = 1; parabolic: a = 0, n = 0.5).

$$I_{uR} = \frac{1}{1 + 0.02\left(a_{eff,L_a}\right)^3} \tag{4}$$

$$a_{eff,L_a} = \frac{\omega L_a}{V_{s,av}} \tag{5}$$

In Eq. 5, ω is the cyclic frequency of the excitation, L_a is the pile active length and $V_{s,av}$ is an average shear wave velocity within one-half of L_a, respectively (L_a and $V_{s,av}$ can be computed as shown in Refs. [8, 10]).

Nevertheless, as far as the authors know, there are no solutions for I_{uF} and $I_{\theta F}$ (subscript F is used to the case of a free-head pile) for the case of soil conditions other than the homogeneous case. This contribution presents new semi-analytical formulas for I_{uF} and $I_{\theta F}$ in the case of a free-head single pile embedded in a continuously inhomogeneous viscoelastic soil (Eq. 3) and for I_{uR}, I_{uF} and $I_{\theta F}$ in the special case of a single pile (both fixed-head and free-head) embedded in a two-layered viscoelastic soil.

2 PROBLEM STATEMENT

Fig. 2 shows the pile-soil system investigated in the present study. The system consists of a single pile (free and fixed-head) embedded in different soil profiles (homogeneous, Gibson, parabolic and two-layered).

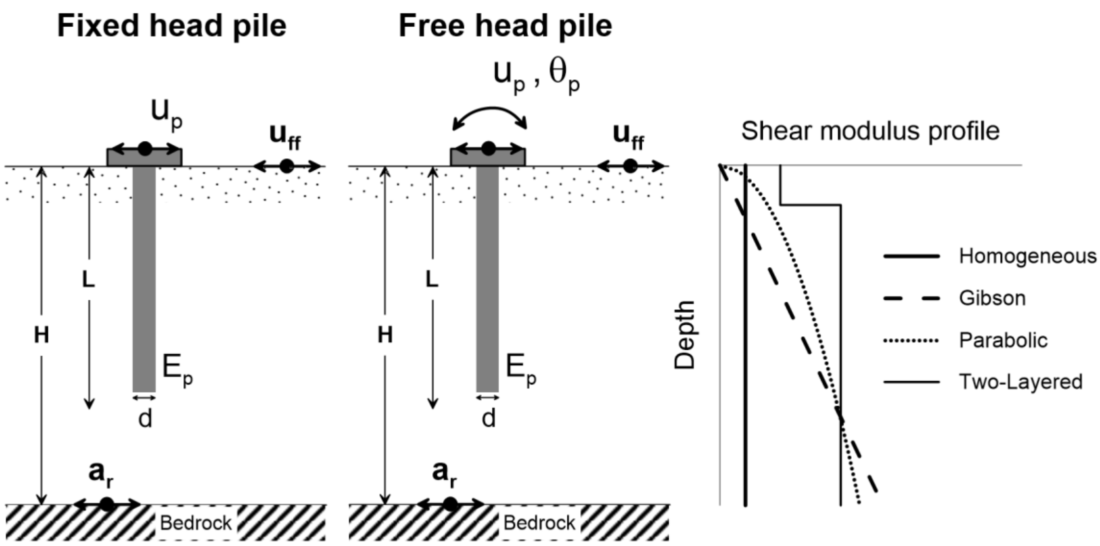

Figure 2: Pile-soil system considered in this study.

The seismic loading (vertically propagating SH waves) is applied at the base of the model. In all the models the following parameters were kept constant: H = 30 m, L = 20 m, E_p = 25

GPa, ν_s (soil Poisson's ratio) = 0.45, γ_s (soil unit weight) = 20 kN/m^3, γ_r (bedrock unit weight) = 23 kN/m^3, γ_p (pile unit weight) = 24 kN/m^3. The remaining properties considered for each of the studied cases are summarized in Tab. 1 (homogenous, Gibson and parabolic profiles) and in Tab. 2 (two-layered soil profiles).

CASE	d [m]	V_{sd} [m/s]	E_p/E_{sd} [-]	a [-]	n [-]	L_a [m]	$V_{s,av}$ [m/s]
1	1	145	205.0	-	0	5.29	145
2	1.5	145	205.0	-	0	7.94	145
3	1	100	431.0	-	0	6.37	100
4	1.5	100	431.0	-	0	9.55	100
5	1	65	1020.2	-	0	7.9	65
6	1.5	65	1020.2	-	0	11.8	65
7	1.5	98	448.8	0.0	1	8.63	83.9
8	1	80	673.5	0.0	1	5.75	67.8
9	1.5	143.9	208.2	0.0	0.5	7.35	135.01
10	1	130	255.0	0.0	0.5	5.13	123.38
11	1.5	71.9	833.8	0.0	0.5	10	72.91
12	1	65	1020.2	0.0	0.5	6.98	66.6

Table 1: Homogeneous, Gibson and parabolic profiles considered in this study. V_{sd} and E_{sd} are the soil shear wave velocity at one-diameter depth and the corresponding Young's modulus.

CASE	V_{s2}/V_{s1} [-]	E_p/E_{s1} [-]	h/h_c [-]
1a, 1b, 1c, 1d	1.5	191.6	0.29, 0.43, 0.57, 0.86
2a, 2b, 2c, 2d, 2e, 2f	2	431.0	0.15, 0.22, 0.29, 0.44, 0.44, 0.66
3a, 3b, 3c, 3d, 3e, 3f, 3g, 3h	3	431.0	0.15, 0.22, 0.29, 0.44, 0.44, 0.66, 1.0, 1.0
4a, 4b, 4c, 4d, 4e, 4f	3.5	431.0	0.15, 0.22, 0.29, 0.44, 0.44, 0.66
5a, 5b, 5c, 5d, 5e	4	191.6	0.29, 0.43, 0.57, 0.75, 0.86
6a, 6b, 6c, 6d, 6e	4	766.3	0.25, 0.38, 0.51, 0.76, 0.76

Table 2: Two-layered soil profiles considered in this study. h/h_c: d = 1.5 m (black); d = 1.0 m (red).

In Tab. 2, V_{s1} and V_{s2} are the shear wave velocities of the upper and lower soil layers ($V_{s2} \geq V_{s1}$), respectively, h is the interface depth (i.e., the thickness of the upper layer 1) and h_c is the critical interface depth (Eq. 6) that have been used in Ref. [9] for developing a simplified formula to compute pile-head kinematic bending in two-layered soils. In Eq. 6 E_{s1} is the Young's modulus of the upper layer. h_c is almost coincident with L_a (computed according to Refs. [8, 10]) in the case of a homogenous profile with $E_s = E_{s1}$. Both h_c and L_a represent the length beyond which pile behaves as infinitely long. Thus, towards the pile a two-layered soil is a soil profile in which the interface is located at a depth h lower than h_c. In fact, if $h \geq h_c$, kinematic interaction factors can be computed with available solutions for the homogeneous soil case.

$$h_c = 1.25d\left(\frac{E_p}{E_{s1}}\right)^{0.25} \tag{6}$$

Pile-soil kinematic interaction analyses have been performed using a BEM (Boundary Element Method) code called KIN SP (Stacul and Squeglia [11]) and a FEM code called

VERSAT-P3D (Wu [12]). These two computer codes perform the analyses in the time domain, thus the input motion applied at the base of the model is in the form of an acceleration time history. The element size along the vertical direction was defined based on the indications provided in Kuhlemeyer and Lysmer [13]. Both radiation and soil damping can be considered. The latter is modelled via a Rayleigh damping formulation.

As described in the introduction, the simplified formula for computing I_{uR} (Eq. 4) was derived based on numerical analyses in the frequency domain under the hypothesis of linear viscoelastic response for the soil material, nevertheless, in Ref. [7] was shown that pile-soil interaction can be reasonably considered insensitive to soil damping, thus even a linear elastic model is sufficient to derive accurate results. Moreover, in Ref. [10], while exploring the FIM under large earthquake-induced non-linear soil response, it was found that the assessment of piles' filtering effect via Eq. 4 is still possible, even if this solution is based on linear elastic or linear viscoelastic soil behavior. Of course, its use in practice is possible if the mobilized soil shear modulus profile (inferred via a pertinent seismic response analysis of the soil profile) can be reasonably fitted (along the pile active length) with Eq. 3.

As pile-soil interaction, and therefore I_{uR}, I_{uF} and $I_{\theta F}$, is not significantly affected by soil damping, the numerical analyses performed in this work have been carried out accounting for radiation damping and considering a very small value for soil damping ratio (Rayleigh type), in the order of 1%. This was done to limit the intrinsic inconvenience related to the frequency dependence of damping via this numerically convenient damping formulation.

3 NUMERICAL RESULTS AND DEVELOPMENT OF NEW SEMI-ANALYTICAL SOLUTIONS

3.1 Validation of the numerical results

The validation of KIN SP and VERSAT-P3D numerical results was carried, as shown in Fig. 3, by comparing the kinematic response factor I_{uR} obtained with these two codes with that inferred via Di Laora and Rovithis [8] solution (Eq. 4) which is based on rigorous analyses in the frequency domain.

Results in Fig. 3 refer to the case of a fixed-head pile embedded in the soil profiles indicated in Tab. 1 (homogeneous and continuously inhomogeneous soils: i.e., Gibson and parabolic). The good agreement between these three different methods allows us to use KIN SP and VERSAT-P3D as numerical tools for developing new semi-analytical formulas.

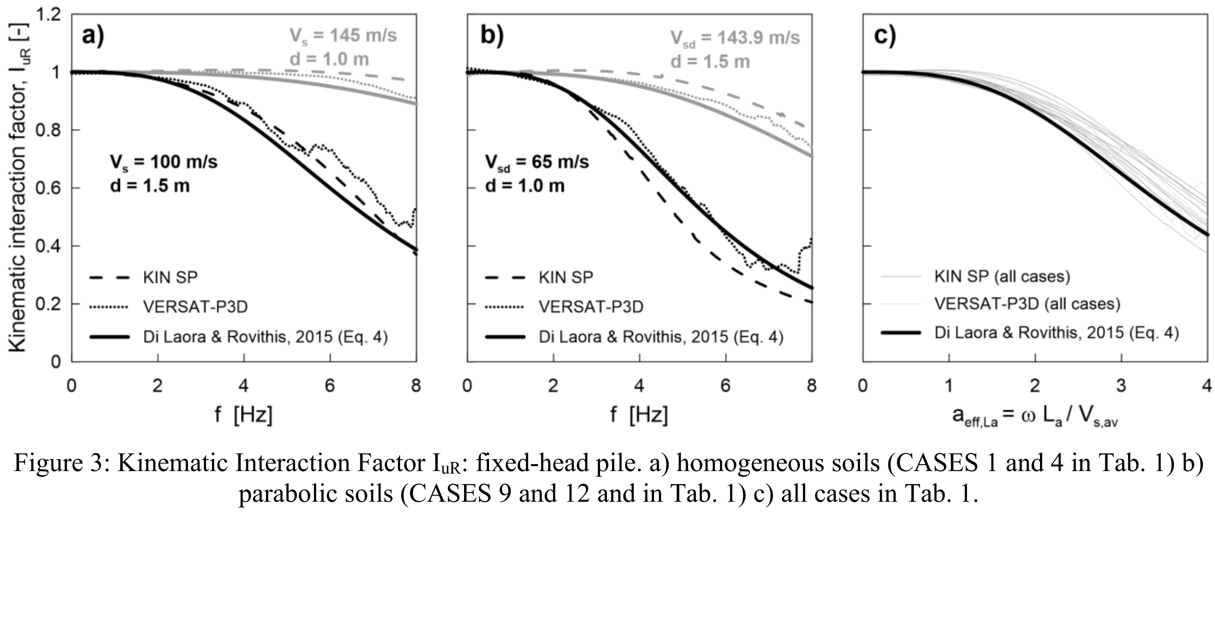

Figure 3: Kinematic Interaction Factor I_{uR}: fixed-head pile. a) homogeneous soils (CASES 1 and 4 in Tab. 1) b) parabolic soils (CASES 9 and 12 and in Tab. 1) c) all cases in Tab. 1.

3.2 Semi-analytical solutions based on numerical results (Inhomogeneous Soils)

As discussed in the introduction, no simple solutions to compute kinematic factors I_{uF} and $I_{\theta F}$ are available for an infinitely long ($L \geq L_a$) free-head single pile embedded in a continuously inhomogeneous soil. KIN SP and VERSAT-P3D have been used to provide new semi-analytical solutions for these kinematic factors. The proposed solutions in Eqs. 7 and 8 are of semi-analytical nature as they are based, from an analytical point of view, on previous Winkler based solutions (as those presented in [5-7]), while coefficients and exponents (i.e., 0.11, 0.20 and 1.65 in Eqs. 7 and 8) have been selected via a best-fit procedure in which KIN SP and VERSAT-P3D numerical results have been used as reference solutions. The analyses were carried out considering the soil profiles in Tab. 1. Figs. 4 and 5 show the good performance of the proposed solutions in providing a reasonable estimate of the kinematic factors I_{uF} and $I_{\theta F}$ in soil profiles in which the shear stiffness varies with depth according to Eq. 3.

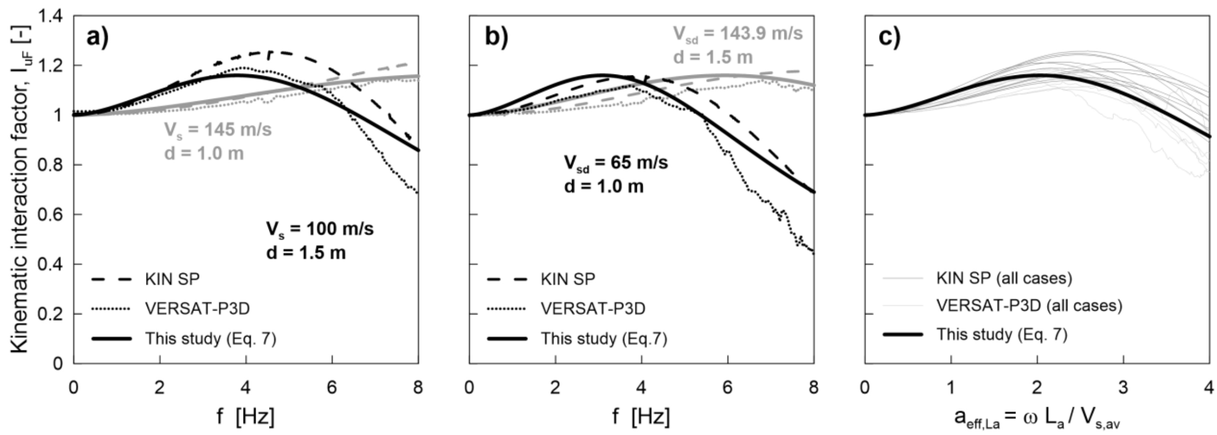

Figure 4: Proposed kinematic interaction factor I_{uF}: free-head pile. a) homogeneous soils (CASES 1 and 4 in Tab. 1) b) parabolic soils (CASES 9 and 12 and in Tab. 1) c) all cases in Tab. 1.

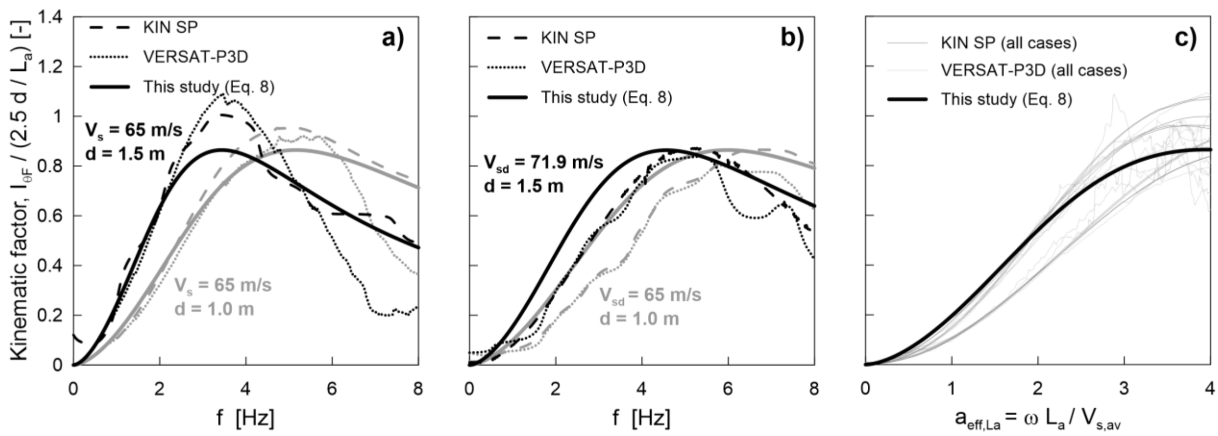

Figure 5: Proposed kinematic interaction factor $I_{\theta F}$: free-head pile. a) homogeneous soils (CASES 5 and 6 in Tab. 1) b) parabolic soils (CASES 11 and 12 and in Tab. 1) c) all cases in Tab. 1.

$$I_{uF} = \frac{1+0.11\left(a_{eff,L_a}\right)^{1.65}}{1+0.02\left(a_{eff,L_a}\right)^{3}} \tag{7}$$

$$I_{\theta F} = \frac{0.20\left(a_{eff,L_a}\right)^{1.65}}{1+0.02\left(a_{eff,L_a}\right)^{3}}\left(\frac{2.5d}{L_a}\right) \tag{8}$$

3.3 Semi-analytical solutions based on numerical results (Two-Layered Soils)

In the case of two-layered soils, there are no solutions for both free and fixed-head piles, thus in the present work new semi-analytical solutions have been developed for the kinematic factors: I_{uR}, I_{uF} and $I_{\theta F}$. The proposed solutions are presented in Eqs. 9, 10 and 11. Numerical analyses were carried out considering the soil profiles in Tab. 2. In Fig. 6 is shown the comparison between the developed semi-analytical solutions and KIN SP and VERSAT-P3D results for two selected soil profiles in Tab. 2.

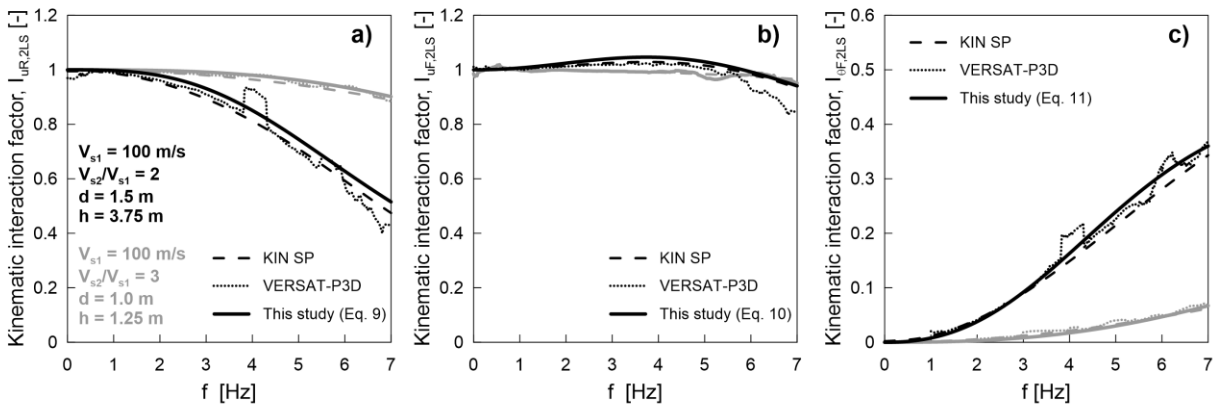

Figure 6: Proposed kinematic interaction factors for two-layered soils: a) I_{uR} b) I_{uF} c) $I_{\theta F}$. Figures refer to CASES 2b and 2d in Tab. 2.

$$I_{uR,2LS} = \frac{1}{1+0.02\left(a_{2LS}\right)^{3}} \tag{9}$$

$$I_{uF,2LS} = \frac{1+A_u\left(a_{2LS}\right)^{B}}{1+0.02\left(a_{2LS}\right)^{3}} \tag{10}$$

$$I_{\theta F,2LS} = \frac{A_\theta\left(a_{2LS}\right)^{B}}{1+0.02\left(a_{2LS}\right)^{3}} \tag{11}$$

Compared to the case of continuously inhomogeneous soil, in the case of two-layered soil the definition of a new dimensionless frequency factor, a_{2LF} (Eq. 12), was necessary, due to the fact that in a two-layered soil, in which the interface is located at a depth lower than the critical one (h_c), both L_a and $V_{s,av}$ cannot be defined.

$$a_{2LS} = \frac{\omega h_c}{V_{s1}}K \tag{12}$$

Coefficients K, A_u, A_θ and exponent B in Eqs. 9-12 were defined following again a best-fit procedure against reference numerical results (Fig. 7). Expressions for computing K, A_u, A_θ and B are presented in Eqs. 13-16. All these parameters were found to be dependent on the dimensionless ratios h/h_c and V_{s2}/V_{s1} which are those ratios governing the kinematic response of a single pile embedded in a two-layered soil as shown in Ref. [9]. It is worth mentioning that Eqs. 13-16 (K, A_u, B and A_θ) are valid and have been derived under the assumption that $V_{s2} \geq V_{s1}$. Moreover, coefficient K can be computed via Eq. 13 only if $0 \leq h/h_c \leq 1$ while A_u, B and A_θ can be computed via Eqs. 14-16 if $[(h/h_c)^{0.10}(V_{s2}/V_{s1})^{0.5}-1] \geq 0$. The latter equations have been tested within $0 \leq [(h/h_c)^{0.10}(V_{s2}/V_{s1})^{0.5}-1] \leq 1$, which includes the most of realistic situations.

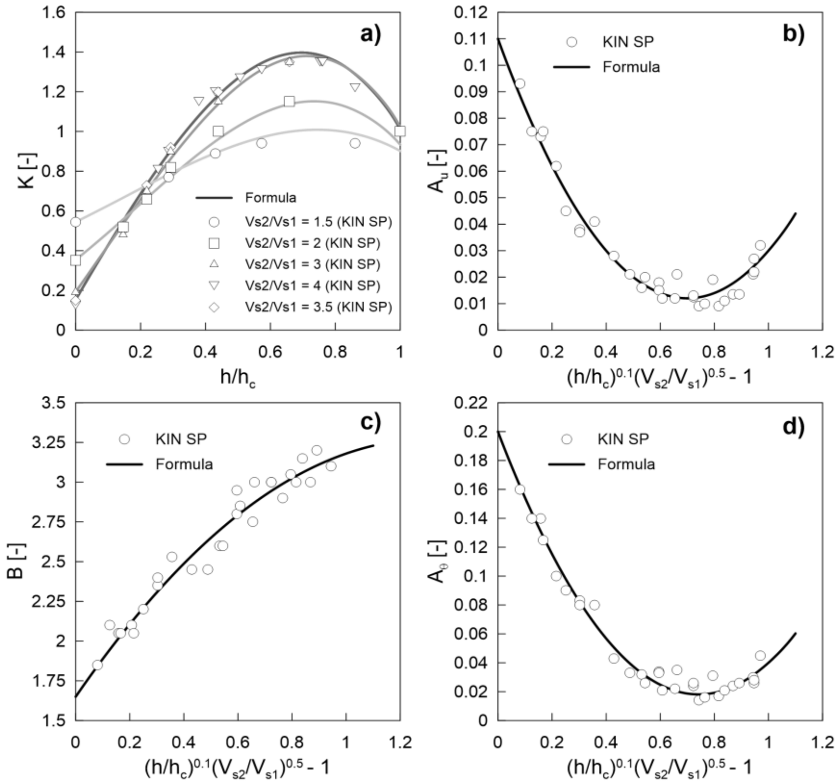

Figure 7: a) Coefficient K (Eq. 13), b) coefficient A_u (Eq. 14), c) exponent B (Eq. 15), d) coefficient A_θ (Eq. 16)

$$K = \left[0.82\left(\frac{V_{s2}}{V_{s1}}\right)^2 - 4.5\left(\frac{V_{s2}}{V_{s1}}\right) + 4.1\right]\left(\frac{h}{h_c}\right)^3 + \left[-0.95\left(\frac{V_{s2}}{V_{s1}}\right)^2 + 4.4\left(\frac{V_{s2}}{V_{s1}}\right) - 4.1\right]\left(\frac{h}{h_c}\right)^2 + \left[\left(\frac{V_{s2}}{V_{s1}}\right) - 0.7\right]\left(\frac{h}{h_c}\right) + \left(\frac{V_{s2}}{V_{s1}}\right)^{-1.5} \tag{13}$$

$$A_u = 0.20\left[\left(\frac{h}{h_c}\right)^{0.1}\left(\frac{V_{s2}}{V_{s1}}\right)^{0.5} - 1\right]^2 - 0.28\left[\left(\frac{h}{h_c}\right)^{0.1}\left(\frac{V_{s2}}{V_{s1}}\right)^{0.5} - 1\right] + 0.11 \tag{14}$$

$$B = -0.94\left[\left(\frac{h}{h_c}\right)^{0.1}\left(\frac{V_{s2}}{V_{s1}}\right)^{0.5} - 1\right]^2 + 2.47\left[\left(\frac{h}{h_c}\right)^{0.1}\left(\frac{V_{s2}}{V_{s1}}\right)^{0.5} - 1\right] + 1.65 \tag{15}$$

$$A_\theta = 0.33\left[\left(\frac{h}{h_c}\right)^{0.1}\left(\frac{V_{s2}}{V_{s1}}\right)^{0.5} - 1\right]^2 - 0.49\left[\left(\frac{h}{h_c}\right)^{0.1}\left(\frac{V_{s2}}{V_{s1}}\right)^{0.5} - 1\right] + 0.20 \tag{16}$$

4 CONCLUSIONS

In this contribution a set of new semi-analytical solutions to compute the transfer functions (i.e., kinematic interaction factors) I_{uR}, I_{uF} and $I_{\theta F}$ for continuously inhomogeneous and two-layered soils have been derived via the numerical results of two computer codes (KIN SP and VERSAT-P3D). These transfer functions, representing the ratio between pile-head and free-field motion as a function of the excitation frequency, are particularly useful for computing the FIM for pile foundations.

Even if the proposed solutions refer to the two extreme cases of free-head and fixed-head single pile, it should be emphasized that these basic formulas can be combined to obtain the transfer functions of a generic pile group (under the assumption that group effects are negligible in the kinematic interaction case) as described in Ref. [14].

The semi-analytical solutions presented herein have been obtained under the hypothesis of linear viscoelastic behavior for the soil material, thus it will be important to check their validity also in the case of highly non-linear soil response. Nevertheless, based on results shown in Ref. [10] it seems that kinematic interaction mechanism is a stiffness-controlled phenomenon and non-linear soil response simply modifies soil deformations at free-field conditions.

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