Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2023) 4030-4043

ECCOMAS Proceedia

COMPDYN 2023
9th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Athens, Greece, 12-14 June 2023

DYNAMIC CHARACTERIZATION OF THE CIRCUS MAXIMUS ARCHEOLOGICAL SITE

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Abstract

Monumental structures and archeological sites are often exposed to intense ambient and anthropic vibrations which might compromise their structural integrity. Monitoring these precious assets allows to assess their overall conditions and highlight possible changes in the behavior of the structure. In this paper, the effects of vibrations on the Circus Maximus archeological area are investigated. A wireless structural health monitoring system was used to record accelerations from July 2022 to January 2023. During this period, several concerts and events were held in the area. The acceleration records were used to perform dynamic characterizations of the site. Different numbers of sensors and configurations were used to assess the influence of spatial resolution. The modal properties were obtained through the frequency domain decomposition technique and compared to those from previous studies. Overall, a reduction of the modal frequencies has been observed which requires further investigations.

Keywords: Monumental Structures, Towers, Anthropic Vibrations, Dynamic Characterization, Operational Modal Analysis.

ISSN:2623-3347 © 2023 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of COMPDYN 2023.
doi: 10.7712/120123.10700.21972

1 INTRODUCTION

Most historical built heritage, such as ancient palaces, monuments, and churches are complex masonry structures whose preservation against degradation and extreme vibration is of primary concern in Europe. In the Italian territories, the vast majority were architected during the Renaissance and extended in different regions, hosting large crowds on occasion. To propose a preservation plan, evaluate the impact of extraordinary excitations, and ensure the safety of them, various Structural Health Monitoring (SHM) strategies are utilized.

Among existing SHM strategies, vibration-based systems are one of the emerging non-invasive technologies to address safety concerns and propose restoration plans for monumental assets by means of system identification schemes. System identification is defined as the evolution of models from experimental studies that can be employed to calibrate numerical simulations, and damage identification algorithms. The usual parameters determined through the experimental studies are the modal properties (i.e., frequency, mode shapes, and damping ratios). They are directly dependent on the inherent characteristics of a structure, such as the stiffness, mass, and material type, which modification results in distinct response of a system. However, the damping ratio is set aside from the damage indicators since its estimation can be largely biased using the conventional techniques [1].

Experimental Modal Analysis (EMA) and Operational Modal Analysis (OMA) are popular tools for structural identification. The former involves recording the input and output, while in the latter, the identification is entirely carried out based on the response. In addition, the former scheme is neither feasible nor practical for built heritage due to the complexity in proper excitation levels and boundary conditions. Thus, the output-only identification methods are the optimal solution in which the infrastructure's service is not halted during the process.

There are several studies on the dynamic identification of historical sites using OMA techniques. In a recent study, the Sant'Agata del Mugello church was densely instrumented to evaluate the spatial resolution impacts on the derived modal properties. The modal anomalies, captured during two experimental campaigns using Enhanced Frequency Domain Decomposition (EFDD) [2], were applied to establish a relation between the dynamic response and the history of restorations. The study disclosed the effect of sensor placement on the extracted modal properties [3].

As a part of the seismic vulnerability assessment of Esfahan Shah mosque, the dynamic identification was performed response using Stochastic Subspace Identification (SSI) [4] and EFDD technique. The modal properties were applied in the calibration of the FE model to perform nonlinear analysis and obtain a relation between the simulated damages and the monument. The study suggested improving the stiffening walls and the two domes connection for better global performance [5].

Saint Torcato church is another historical complex under a hybrid static and dynamic monitoring system. The hybrid SHM monitoring documented the evolution of specific parameters such as cracks width and frequencies before, during, and after the restoration planes. The study took into consideration the impact of environmental agents, namely, temperature variation and humidity, on the specific parameters by long-term monitoring of the asset [6].

As mentioned, in the vibration-based techniques, the damage indicative parameters are mostly natural frequencies and corresponding mode shapes. However, the change in the identified values does not necessarily imply the degradation of physical properties. In fact, the variability of modal properties can be associated with environmental agents [7].

In a study on the Consoli Palace in Gubbio, the role of environmental agents in the accuracy of early damage detection was highlighted. A hybrid SHM system was employed to discard the environmental effects of the damage caused by material degradation and earthquake excitation. The modal properties were identified using FDD, EFDD, and SSI, while the impact of freeze and thaw cycles was quantified. During the long-term monitoring, nonlinear trends for crack widths and frequencies were derived in contrast with previous studies [8].

A similar study was carried out on the renowned masonry monuments, Milan Cathedral, utilizing electrodynamic, temperature and humidity, and tilt measurement sensors. The continuous monitoring was performed using the automated cov-SSI to extract sensitive modal features. The long-term monitoring revealed the negligible effect of relative humidity on the crack width and natural frequencies. In addition, a distinctive correlation was observed between the natural frequencies and temperature fluctuation. The study suggests the use of mode shape geometries and their associated mode complexity as a monitoring parameter due to fewer deviations toward environmental agents [9].

This study aims at the modal identification of the Circo Massimo archeological site in Italy. The historical site is characterized by three sensor placement strategies to record ambient vibration response. Since the major international festivals of Italian capitals are held on this site, the acquired modal properties are compared with their counterparts derived in 2019 to detect anomalies [10]. Additionally, due to the inherent vulnerability of modal parameters against thermal variation, long-term SHM monitoring is carried out to evaluate their impact.

2 METHODOLOGY

In the frequency domain, the general peak picking and Frequency Domain Decomposition (FDD) [11] techniques are widely applied for identification purposes due to high interaction with users. The general peak-picking method assumes the contribution of one mode to the dynamic response at resonance leading to the misidentification of closely spaced modes. On the contrary, the FDD technique has a superior ability to estimate these modes.

In the FDD technique, the first step is the estimation of Power Spectral Density (PSD) from the recorded ambient vibration response. PSD is determined by taking the Fourier transform of correlation between sensor outputs besides assuming the input as a white noise signal. The relation of PSD is summarized below, where the power density function of output is obtained from the input counterpart:

$$G_{yy}(i\omega) = \sum_{k=1}^{n}\sum_{s=1}^{n}\left[\frac{R_k}{(i\omega-\lambda_k)}+\frac{R^*_k}{(i\omega-\lambda^*_k)}\right]C\left[\frac{R_s}{(i\omega-\lambda_s)}+\frac{R^*_s}{(i\omega-\lambda^*_s)}\right] \tag{1}$$

Then, the PSD is decomposed using the Singular Value Decomposition technique as follow:

$$G_{yy}(i\omega_i) = U_i S_i V_i^H \tag{2}$$

The SVD forms two matrices, namely singular values and corresponding singular vectors. The former matrix is diagonal, where the system's natural frequencies are ordered. Furthermore, their associated modes are placed column-wise in the latter matrix. Once the PSD matrix is determined, each peak of the SVD spectrum indicates the relevant frequency of the system [12].

In addition to the FDD method, an automated version is introduced to implement in online SHM monitoring systems. The method is based on the identification of a modal domain at the vicinity of each peak in the SV spectrum by establishing a criterion. The number of modes, domain width, and the criterion to identify them directly affect the specifying physical modes of vibration [13]. The method was developed for the identification of various infrastructures and details can be found on [14], [15], [16].

In the current study, two separate scripts are developed in MATLAB [17] based on the above formulation to perform the analysis in the next sections.

3 CIRCO MASSIMO

The Italian capital is renowned for its ancient elliptical monument, Colosseum, the largest amphitheater ever built. In the southeast of this asset, the Circo Massimo, an ancient chariot-racing stadium, is located where many international gatherings take place. Its wooden perimeters were constructed in the sixth century BC when the "ludi romani" Italian ceremonies were held. Starting from the year of 46 BC, the evolution of masonry elements had initiated, and many retrofits over the years were carried out. Just after the last festival in 549 AD, the site was given up, and numerous natural disasters caused the accumulation of 9-meter debris on the current track. The ancient stadium, the Torre della Moletta, and the circus ruins are depicted in Figure 1.

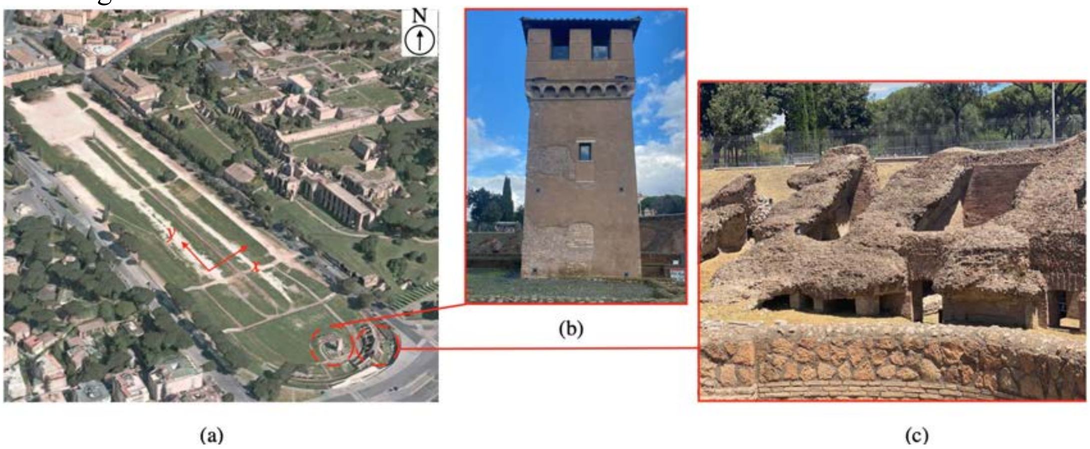

Figure 1: The Circo Massimo. (a) the general view, (b) Torre della Moletta, (c) circus ruins.

As indicated in the Figure 1, the Torre della Moletta is located in the southern part of the circus. The tower can be regarded as an extraordinary case study since the abundant sediment accumulations increase the uncertainty in the soil-structure interaction response. During the last restoration, steel frames connected to the spiral staircase were added to the structure. They are schematized in the following figure:

Figure 2: (a) The steel frame, (b) the spiral staircase.

As mentioned, the structure is exposed to intense ambient vibration due to numerous concert events. Thus, it is necessary to evaluate the impact of such excitation on the health of this archeological site.

4 HEALTH MONITOING SYSTEM

The historical complex is instrumented through the establishment of an extensive monitoring plan. Different locations, such as the circus ruins, the Mithraeum temple, and the Moletta tower, were instrumented, from which a part is on the scope of the current study.

The first sensor layout is labeled as configuration A, carried out on July 9, 2022. The circus ruins and Torre della Moletta are instrumented, as indicated in Figure 3.

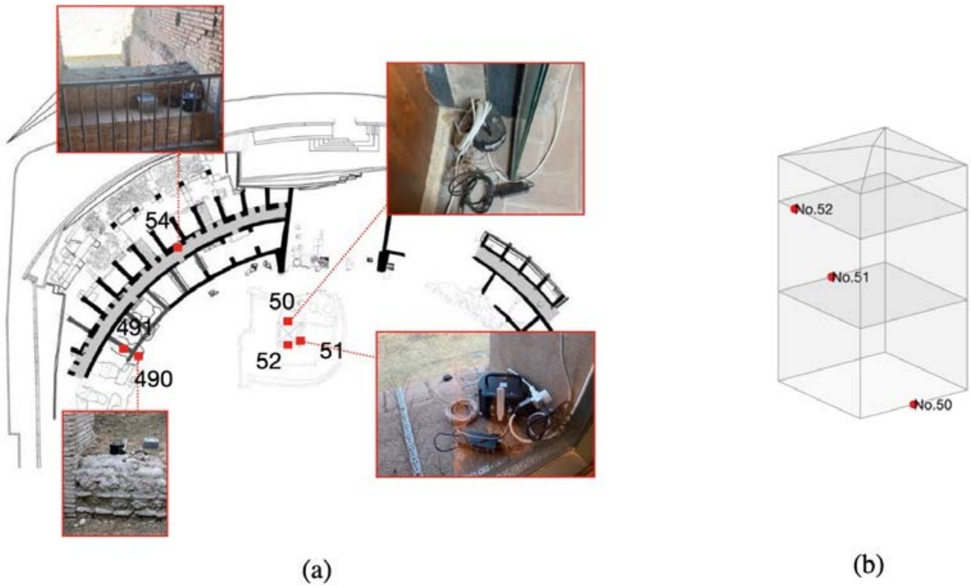

Figure 3: Sensor Configuration A, (a) Circo Massimo, (b) Torre della Moletta.

In sensor configuration B, five sensors are placed exclusively on the tower, and their layout are plotted below:

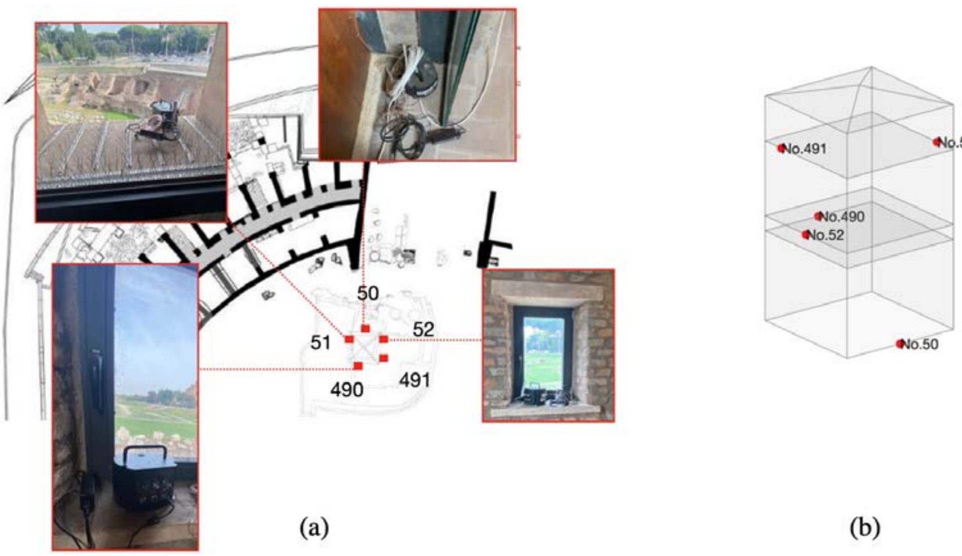

Figure 4: Sensor Configuration B, (a) Circo Massimo, (b) Torre della Moletta.

In the final layout, again, the focus is on the tower by instrumentation on four sensors which are illustrated in Figure 5.

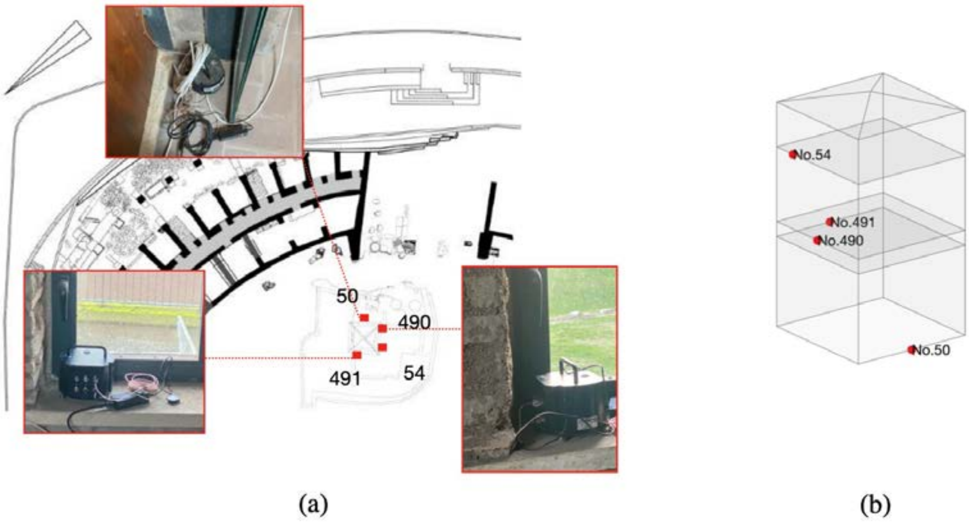

Figure 5: Sensor Configuration C, (a) Circo Massimo, (b) Torre della Moletta.

It is important to mention in section (b) of each figure imaginary rectangular floors are schemed to indicate the sensor's location. However, there is no floor or any kind of diaphragm there.

5 IDENTIFICATION OF MODAL PARAMETERS FROM AMBIENT VIBRATION

In this section, the ambient vibration response for each sensor configuration is set as the input of FDD analysis to extract natural frequencies and their corresponding mode of vibration. In the analysis of sensor configuration A, the data is related to July 9, 2022, from 9:15 to 10:15 UTC. The identified peaks in the SV spectrum using the manual FDD analysis for the Torre della Moletta are plotted in Figure 6.

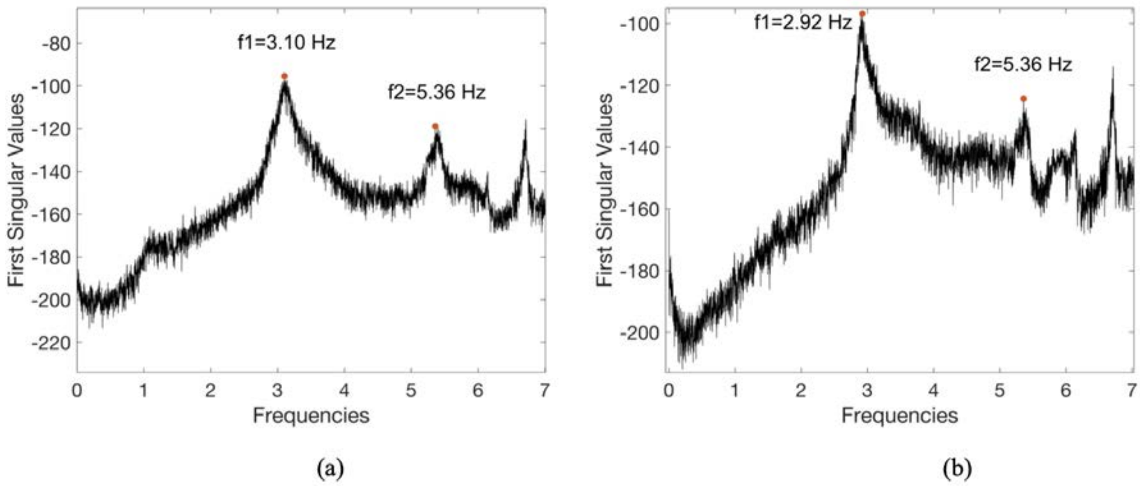

Figure 6: The selected peaks of the SV spectrum for configuration A. (a) x-direction, (b) y-direction.

It is observed that the first mode of vibration in the x and y direction is 3.10 and 2.92 Hz, respectively. Additionally, the same frequency is obtained for the second mode in both directions, indicating a simultaneous mode. The acquired mode shapes are plotted in MATLAB using spline interpolation to smoothen the deformed shape and better depiction in Figure 7.

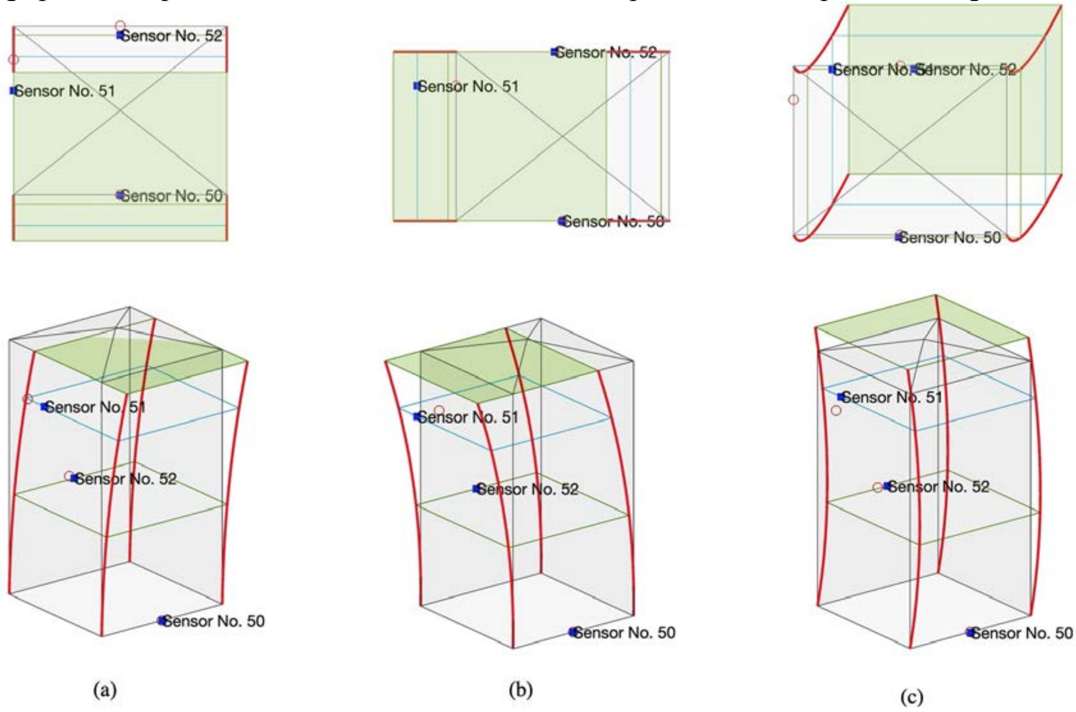

Figure 7: The 3D mode shapes of sensor configuration A. (a) mode in the *x* direction, (b) mode in the *y* direction, (c) mode in both directions.

Following the same scheme, the ambient vibration response of circus ruins related to the interval of 11:00-12:00 are analyzed and the SV spectrum is obtained. The extracted frequencies are compared with the result of similar investigation on 2019 in Table 1.

Table 1: Comparison of extracted frequencies for configuration A.

Torre della Moletta				Circus			
Puzzilli et al. (2019)		Politecnico di Torino 09.07.22		Puzzilli et al. (2019)		Politecnico di Torino 09.07.22	
X-dir	Y-dir	X-dir	Y-dir	X-dir	Y-dir	X-dir	Y-dir
3.25	3.00	3.10	2.92	2.5	2.5	2.22	2.22
5.80	5.80	5.36	5.36	-	-	4.42	4.56

A remarkable reduction in the system's natural frequencies is highlighted by comparing the results of current and former campaigns. This decrease can be induced by damage or interaction with various parameters like temperature variations. Thus, further study is carried out to determine the source of frequency reduction.

To capture the mode shapes of the tower more accurately, configuration B of the sensor's layout was proposed. By performing the manual FDD analysis, the natural frequency of the system is obtained as seen in Figure 8.

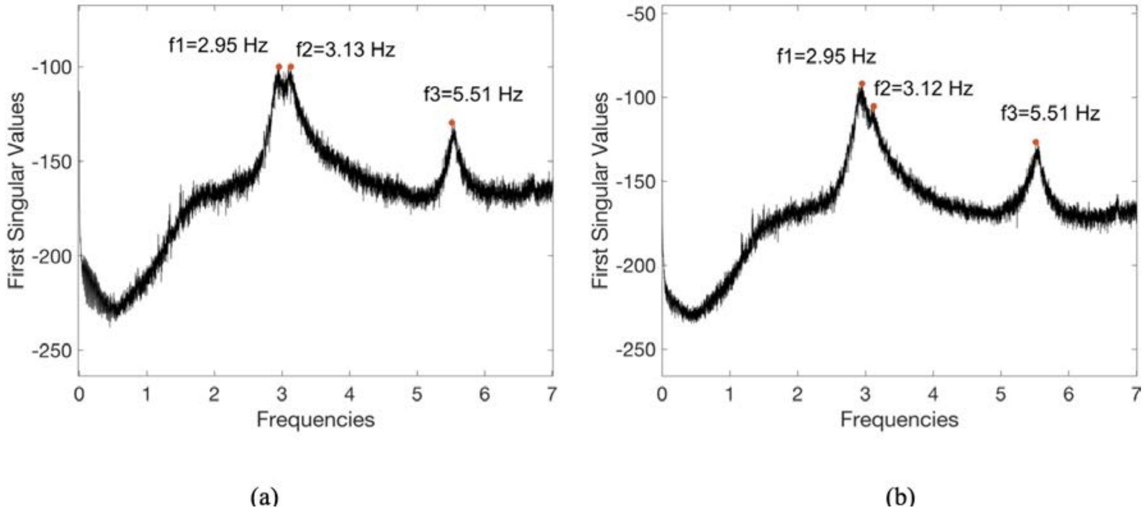

Figure 8: The selected peaks of the SV spectrum for configuration B. (a) x-direction, (b) y-direction.

The new findings are summarized in Table 2.

Table 2: Comparison of extracted frequencies for configuration B.

Puzzilli et al. (2019)		Politecnico di Torino 09.07.22		Politecnico di Torino 21.10.22	
X-dir	Y-dir	X-dir	Y-dir	X-dir	Y-dir
3.25	3.00	3.10	2.92	2.95	2.95
5.80	5.80	5.36	5.36	3.13	3.12
-	-	-	-	5.51	5.51

Analyzing the new data indicates the presence of novel mode shapes in addition to an increase in the natural frequencies concerning July 9. In this part, the role of spatial resolution in the accurate capturing of modes is highlighted. The frequencies in the *x* and *y* directions are equal, implying the presence of simultaneous modes with the probable diagonal movement of the tower.

The 3D mode shapes are plotted in Figure 9.

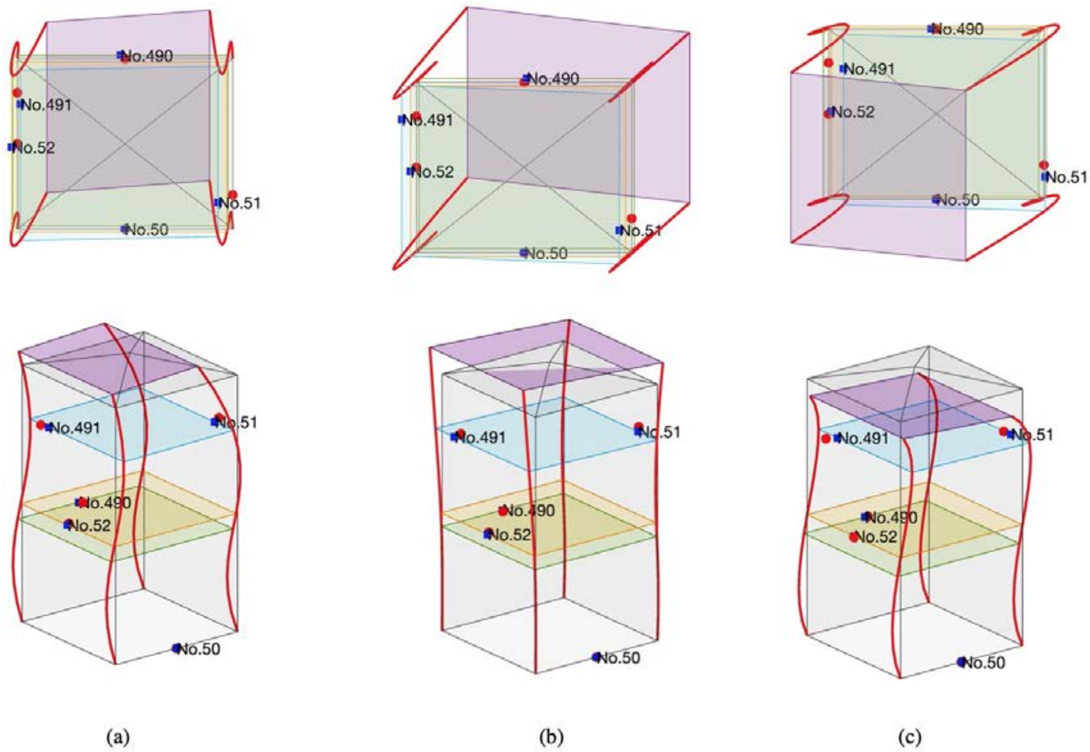

Figure 9: The 3D mode shapes of sensor configuration B. (a) mode 1, (b) mode 2, (c) mode 3.

Figure 9 is significant since the rotation of the tower sides is evident for all acquired modes of vibration on the top level. The twisting phenomenon on the tower can be due to its interaction with the spiral staircase. Thus, it is necessary to perform a further investigation on providing a restoration plan to prevent it.

In the last configuration, the data of December 30 with the similar duration are analyzed. Following the same scheme, the SV plots are not presented for the sake of brevity. The attained modal parameters of configuration C confirm the presence of new mode shapes contradicting the study on July 9. Moreover, there is an increasing trend in the acquired frequencies from July 9 to December 30, raising the hypothesis of environmental impacts on the system.

Table 3: Summary of the extracted frequencies for all configuration.

Puzzilli et al. (2019)		Politecnico di Torino 09.07.22		Politecnico di Torino 21.10.22		Politecnico di Torino 30.11.22	
X-dir	Y-dir	X-dir	Y-dir	X-dir	Y-dir	X-dir	Y-dir
3.25	3.00	3.10	2.92	2.95	2.95	3.01	3.01
5.80	5.80	5.36	5.36	3.13	3.12	3.22	3.22
-	-	-	-	5.51	5.51	5.89	5.89

In the final part of this section, the 3D modes are presented. Since there are not two sensors at the same level, the rotation of the tower is not captured.

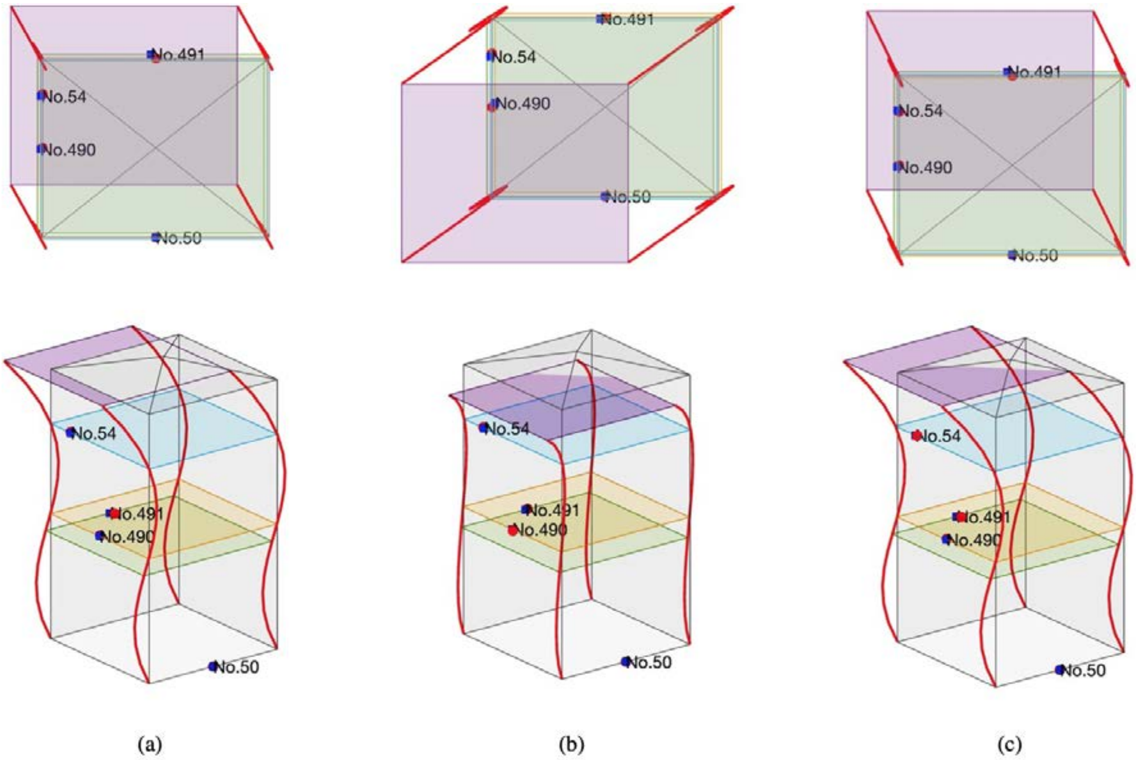

Figure 10: The 3D mode shapes of sensor configuration B. (a) mode 1, (b) mode 2, (c) mode 3.

6 CONTINOUS MONITORING OF THE TOWER

This section aims to assess the effect of temperature variation on the natural frequencies of the tower. For this regard, the AFDD method is employed to analyze the recorded structural response from July 9 to October 20 for sensor configuration A. The analysis for each date is divided into two intervals:

- Daily analysis for 60 minutes acceleration in the interval of 12:30-13:30 UTC;
- Nightly analysis for 60 minutes acceleration in the interval of 2:30-3:30 UTC.

The temperature records pertaining to the CIAMPINO station in the Rome are considered as the thermal variation of the site during the analysis. Before selecting the above intervals, the initial analysis indicated the highest and lowest temperature documented within these intervals.

After extracting the modal properties using the AFDD, the frequencies for daily and nightly intervals are presented in Figure 11. In addition, to prove the efficiency of the modal tracking approach and the fact that all frequencies are related to a certain mode of vibration, the Modal Assurance Criterion (MAC) is implemented. The MAC between the extracted mode shapes is calculated with respect to the one corresponding to July 9 and plotted on the top in Figure 11.

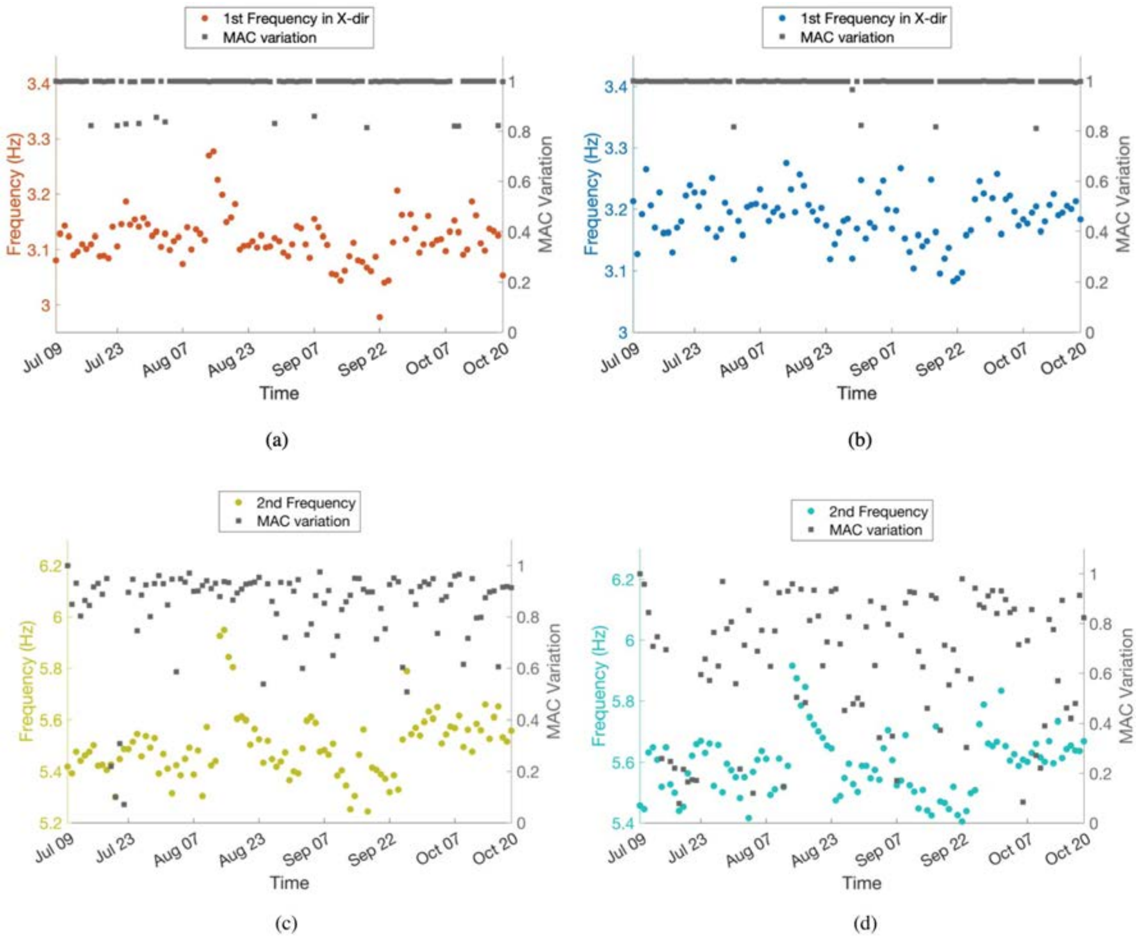

Figure 11: Obtained frequencies, (a) Daily 1st frequency in the *x*-direction, (b) Nightly 1st frequency in the *x*-direction, (c) Daily mutual frequency, (d) Nightly mutual frequency.

In the above figure, the similar observation is made for the first mode in the *y*-direction. According to the figures, it is observed that the acquired data for the second mode during the night has unstable MAC variation, which may originate from the noise made during the time interval. Although it is expected to have some variation of MAC in the higher modes due to environmental effects, it is better to take into consideration some measurements in this regard. As a result, the frequencies with a MAC less than 0.6, indicated in Figure 11 (d), are discarded.

To evaluate the effect of temperature, the determined frequencies for each mode of vibration are plotted against the corresponding temperature in Figure 12. The variation of natural frequencies aligns with the expectation in the masonry structures where the lower the temperature the higher the stiffness. Conducting a statistical analysis, linear regression is performed to correlate the thermal variation with the change of modal frequencies.

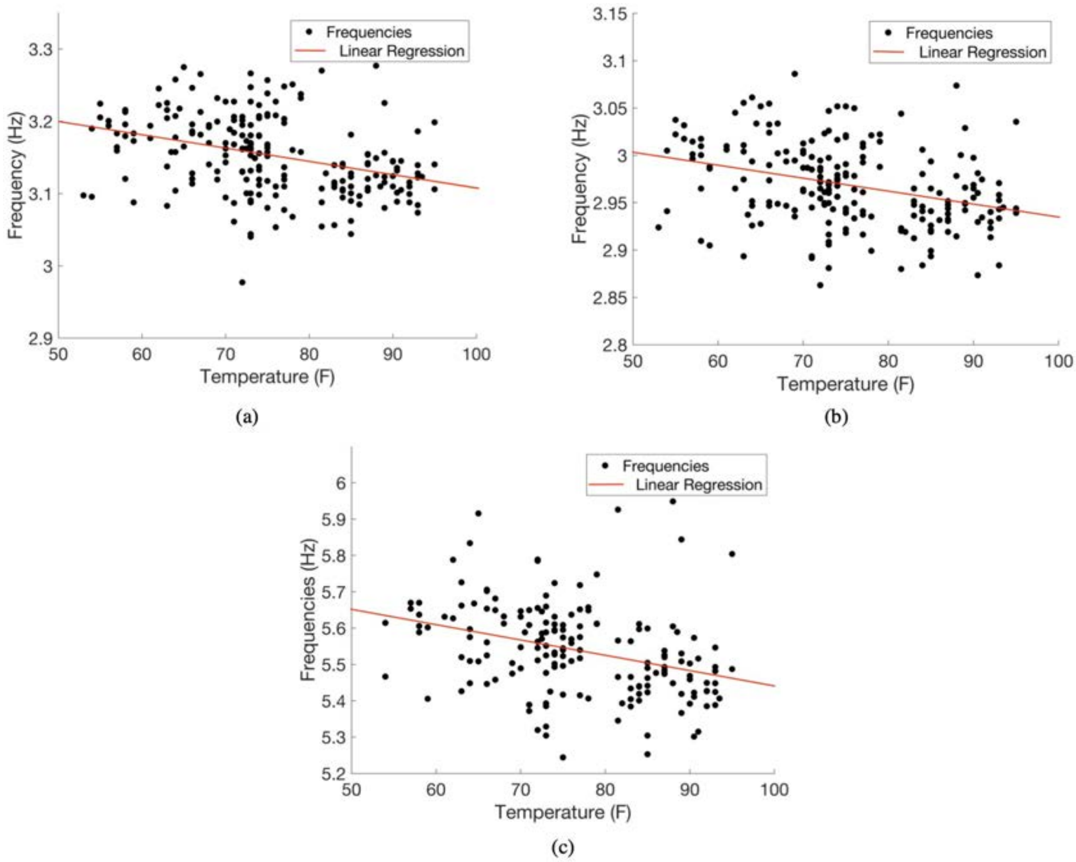

Figure 12: Frequency changes versus temperature variations: (a) Mode 1 in x-dir, (b) Mode 1 in the y-dir, (c) The mutual mode.

The linear regression is performed since a linear trend in the variation of frequencies is observed on the larger scale, which aligns with previous works, as well. The established relations are:

$$F_{1x} = 3.293 - 0.001862T \qquad R^2 = 0.127 \tag{3}$$

$$F_{1y} = 3.071 - 0.001366T \qquad R^2 = 0.108 \tag{4}$$

$$F_{2x} = F_{2y} = 5.863 - 0.004215T \qquad R^2 = 0.116 \tag{5}$$

In the establishment of the frequency-temperature relation, it is evident that the natural frequencies were highly susceptible to change. Besides, identical frequencies with the previous study in 2019 are observed during the continuous monitoring. On September 30, for the interval duration of 23:30-24:00 UTC, the resultant SV spectrum from FDD analysis are derived.

In which, the frequencies are:

Table 4: comparison of frequencies between July 9 and September 30.

Puzzilli et al. (2019)		Politecnico di Torino 09.07.22		Politecnico di Torino 30.09.22	
X-dir	Y-dir	X-dir	Y-dir	X-dir	Y-dir
3.25	3.00	3.10	2.92	3.24	3.01
5.80	5.80	5.36	5.36	5.79	5.79

It can be suggested that the main reason behind the frequency reduction on July 9 was the thermal variation since the 2019 analysis was performed in September too. This suggestion aligns with the continuous monitoring of frequencies.

7 CONCLUSION

An innovative SHM campaign was designed for monitoring Circo Massimo and Torre della Moletta. Three separate sensor configurations have been defined for the dynamic characterization to evaluate the effect of spatial distribution on modal properties. In addition, the thermal influence on modal parameters was assessed through establishing continuous long-term monitoring. The FDD and AFDD were applied to derive natural frequencies and mode shapes. Moreover, the correlation between natural frequencies and temperature variation has been acquired by statistical analysis. The following concluding remarks are drawn:

- The analysis of ambient vibration response on July 9 captured natural frequencies below the previous campaign in 2019. Conventionally, the frequency reduction could be regarded as a damage warning if the proper environmental conditions are available.
- The sensor configurations B and C highlighted the presence of three modes of vibration in each direction with the same natural frequency. This finding underlines the significance of spatial resolution in dynamic identification.
- For sensor layout B, two sensors were placed on the same level to identify torsional modes by plotting 3D mode shapes from FDD analysis. The surprising outcome was the depiction of torsion in all acquired modes. This critical finding could be initiated by interaction with the spiral staircase and needs further elaboration in future works.
- Long-term monitoring indicated the high susceptibility of natural frequencies toward thermal variation. The statistical analysis revealed the decreasing trend of frequencies with the increase in temperature. There is a possibility that the reduction of frequency was related to temperature rather than damage.

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